Using Squeeziness to test from Finite State Machines

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CREST Information Theory and Software Testing
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- Related to an expression? *easy peasy lemon squeezy.*
What’s the meaning of Squeeziness in Information Theory?

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How bad is FEP?

- FEP can reduce testing effectiveness: we might fail to find a fault despite executing the faulty statement.
- Empirical studies show that many systems suffer from FEP.
What’s this talk about?
The adaption of Squeeziness to a black box scenario.
This talk in a nutshell

Using Squeeziness to test from Finite State Machines
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Using Squeeziness to test from Finite State Machines
Finite State Machines

Graphs with an initial state where transitions are labelled by a pair (input, output).
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FSMs: assumptions

- FSMs are **deterministic**.
- FSMs representing SUTs are **input-enabled**.
FSMs as functions

An FSM $M$ can be seen as a function $f_M : \text{dom}M \rightarrow \text{image}M$ such that for all $\alpha \in \text{dom}M$ (sequence of inputs performed by $M$) $f_M(\alpha) = \beta$ (sequence of outputs observed after applying $\alpha$).

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### FSMs as functions

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**Collisions**

$\alpha_1$ and $\alpha_2$ collide for $M$ if $\alpha_1 \neq \alpha_2$ and $f_M(\alpha_1) = f_M(\alpha_2)$.

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$f_M(i_1) = o_1$

$f_M(i_3) = o_2$

$f_M(i_3i_1) = o_2o_2$

$f_M(i_2i_3) = o_2o_2$
This talk in a nutshell

Using **Squeeziness** to test from Finite State Machines
**Squeeziness as difference of entropies**

**Entropy** of the random variable $\xi_A$

$$\mathcal{H}(\xi_A) = - \sum_{a \in A} \sigma_{\xi_A}(a) \cdot \log_2(\sigma_{\xi_A}(a))$$

If $f : A \rightarrow B$ then **Squeeziness of $f$** is the loss of information after applying $f$ to $A$: $\mathcal{H}(A) - \mathcal{H}(B)$. 
Squeeziness for FSMs

We need to define how to group inputs & outputs. Two alternatives:

- A unique random variable for the whole set of inputs/outputs.
- A random variable for each length of sequences of inputs/outputs.
**Squeeziness for FSMs**

We need to define how to group inputs & outputs. Two alternatives:

- A unique random variable for the whole set of inputs/outputs.
- A random variable for each length of sequences of inputs/outputs.

We choose the second one because it gives an incremental procedure to compute a sequence of *consecutive values* of Squeeziness.
Squeeziness as difference of entropies

Let FSM $M$, $k > 0$ and random variables $\xi_{\text{dom}M,k}$ and $\xi_{\text{image}M,k}$.

$$\text{Sq}_k(M) = \mathcal{H}(\xi_{\text{dom}M,k}) - \mathcal{H}(\xi_{\text{image}M,k})$$
Squeeziness is not monotonic

\[ q_0 \xrightarrow{i_2/o_1} q_2 \xrightarrow{i_1/o_3} q_5 \]
\[ q_0 \xrightarrow{i_1/o_1} q_1 \xrightarrow{i_2/o_2} q_4 \]
\[ q_3 \xrightarrow{i_1/o_1} q_1 \]
\[ q_2 \xrightarrow{i_2/o_4} q_6 \]

Squeeziness for \( k = 1 \) is \( \log_2(2) = 1 \) while for \( k = 2 \) is 0. This is bad because we do not have an obvious stopping rule.
Squeeziness is null for bijective functions

If $f_{M,k}$ is bijective then $Sq_k(M) = 0$. 
Squeeziness is null for bijective functions

If \( f_{M,k} \) is bijective then \( S_q_k(M) = 0 \).

Random variables for outputs are determined

Given FSM \( M, k > 0 \) and \( \xi_{\text{dom} M,k} \), the probability distribution of \( \xi_{\text{image} M,k} \) is completely determined.

\[
\sigma_{\xi_{\text{image} M,k}}(\beta) = \sum_{\alpha \in f_{M}^{-1}(\beta)} \sigma_{\xi_{\text{dom} M,k}}(\alpha)
\]
Maximum entropy principle

Maximum entropy is obtained with a uniform distribution $\xi_{\text{dom}_{M,k}}$.

$$Sq_k(M) = \frac{1}{|\text{dom}_{M,k}|} \cdot \sum_{\beta \in \text{image}_{M,k}} |f_M^{-1}(\beta)| \cdot \log_2(|f_M^{-1}(\beta)|)$$
Maximum entropy principle

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$$\text{Sq}_k(M) = \frac{1}{|\text{dom}_{M,k}|} \cdot \sum_{\beta \in \text{image}_{M,k}} |f_M^{-1}(\beta)| \cdot \log_2(|f_M^{-1}(\beta)|)$$

Maximum loss of information

Probability distribution maximising Squeeziness: uniformly distributed in the bigger inverse image of an element of the outputs $\beta'$ and zero otherwise.

$$\sigma_{\xi_{\text{dom}M,k}}(\alpha) = \begin{cases} \frac{1}{|f_M^{-1}(\beta')|} & \text{if } \alpha \in f_M^{-1}(\beta') \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Sq}_k(M) = \log_2(|f_M^{-1}(\beta')|)$$
Probabilistic Squeeziness

We divide Squeeziness by its maximum value.

$$PSq_k(M) = \frac{\mathcal{H}(\xi_{\text{dom}M,k}) - \mathcal{H}(\xi_{\text{image}M,k})}{\log_2(|f^{-1}_M(\beta')|)}$$
This talk in a nutshell

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Collisions and FEP

Let \( m_i = |f_{M,k}^{-1}(\beta_i)| \) and \( d = \sum_{i=1}^n m_i \). Assuming a uniform distribution, the probability of having a collision is:

\[
P_{\text{Coll}}(M) = \sum_{i=1}^n \frac{m_i \cdot (m_i - 1)}{d \cdot (d - 1)}
\]

Relation between \( P_{\text{Coll}}(M) \) and \( P_{\text{Sq}}(M) \) is not monotonic

There exist \( M_1 \) and \( M_2 \) and \( k > 0 \) such that \( P_{\text{Sq}}(M_1) < P_{\text{Sq}}(M_2) \) but \( P_{\text{Coll}}(M_1) > P_{\text{Coll}}(M_2) \).
Empirical Evaluation via simulations

Simulations to compute $\text{PColl}$, $\text{PSq}$, $\text{Sq}$ assuming uniform distributions over the inputs (methodology similar to [CH12]).

- $d = \text{size of the input space (ranging between } 10^4 \text{ and } 2 \cdot 10^9\text{).}$
- $m = \text{maximum subdomain size (ranging between } 10^2 \text{ and } 10^4\text{).}$
Empirical Evaluation via simulations

Simulations to compute $PC_{oll}$, $PS_{q}$, $S_{q}$ assuming uniform distributions over the inputs (methodology similar to [CH12]).

- $d =$ size of the input space (ranging between $10^4$ and $2 \cdot 10^9$).
- $m =$ maximum subdomain size (ranging between $10^2$ and $10^4$).
- Pearson & Spearman Rank correlation coefficient between $PC_{oll}$ and $PS_{q}$/$S_{q}$. Similar results.

**Strong correlation** between $PC_{oll}$ and $PS_{q}$. Values greater than 0.96 for input sets with $5 \cdot 10^6$ or more elements.
Empirical Evaluation via simulations

Simulations to compute PColl, PSq, Sq assuming uniform distributions over the inputs (methodology similar to [CH12]).

- \( d = \) size of the input space (ranging between \( 10^4 \) and \( 2 \cdot 10^9 \)).
- \( m = \) maximum subdomain size (ranging between \( 10^2 \) and \( 10^4 \)).
- Pearson & Spearman Rank correlation coefficient between PColl and PSq/ Sq. Similar results.
- **Strong correlation** between PColl and PSq. Values greater than 0.96 for input sets with \( 5 \cdot 10^6 \) or more elements.
- **Standard Squeeziness** has a better correlation. Still, PSq can be more useful because it is easier to compare results from different machines and lengths of inputs.
Empirical Evaluation via FSMs: Squeeziness and fault location

- 50 randomly generated FSMs (between 25 and 50 states).
- For each FSM we computed Sq and PSq for all $1 \leq k \leq 25$.
- We generated 100 valid mutants of $M$ presenting FEP.
Empirical Evaluation via FSMs: Squeeziness and fault location

- 50 randomly generated FSMs (between 25 and 50 states).
- For each FSM we computed $S_q$ and $P_Sq$ for all $1 \leq k \leq 25$.
- We generated 100 valid mutants of $M$ presenting FEP.
- **No correlation** between where the fault is produced and the Squeeziness and Probabilistic Squeeziness obtained for the length of the input sequence reaching the mutated transition.
- **Negative result.** We tried something less ambitious.
Empirical Evaluation via FSMs: Squeeziness and probability of FEP

Instead of *predicting* where the fault was, we consider the probability of FEP.

- Same 50 randomly generated FSMs.
- We generated 50 valid mutants of $M$ (with and without FEP).
- We computed the probability of FEP, $Sq$ and $PSq$ for length 25.
- $p(FEP) = \frac{\# \text{ tests reaching wrong state but generating correct output}}{\# \text{ tests reaching wrong state}}$. 
Empirical Evaluation via FSMs: Squeeziness and probability of FEP

Instead of predicting where the fault was, we consider the probability of FEP.

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- We generated 50 valid mutants of $M$ (with and without FEP).
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\[
p(FEP) = \frac{\# \text{ tests reaching wrong state but generating correct output}}{\# \text{ tests reaching wrong state}}.
\]

- High correlations between probability of having FEP with sequences up to 25 and $Sq$ and $PSq$ for $k = 25$. All the values were greater than 0.75 and some close to 1.
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We may consider that $1 - \text{PSq}$ gives the *reliability* of tests: it represents the probability that a correct output indicates that no fault was executed.
Application to testing

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Before running tests, we may compute PSq for different values of $k$. We can choose a value of $k$ such that PSq is low: this makes it less likely to have FEP.
Application to testing

We may consider that $1 - PSq$ gives the *reliability* of tests: it represents the probability that a correct output indicates that no fault was executed. Before running tests, we may compute $PSq$ for different values of $k$. We can choose a value of $k$ such that $PSq$ is low: this makes it less likely to have FEP. Finally, if we have $PSq = 0$ for a certain $k$, we can use this length of tests as a checkpoint (but remember that we do not have monotonicity).
Conclusions and future work

- Squeeziness in a black-box framework.
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- No correlation between Squeeziness for $k$ (length of tests) and faults at length $k - 1$.
- Correlation between Squeeziness and probability of FEP.
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- No correlation between Squeeziness for $k$ (length of tests) and faults at length $k - 1$.
- Correlation between Squeeziness and probability of FEP.
- Future work: Consider observable FSMs and experiments on real FSMs.
THANKS FOR YOUR ATTENTION!!
Questions? Comments?