4. Fitness landscape analysis for multi-objective optimization problems

Fitness landscape analysis for understanding and designing local search heuristics

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Outline of this part

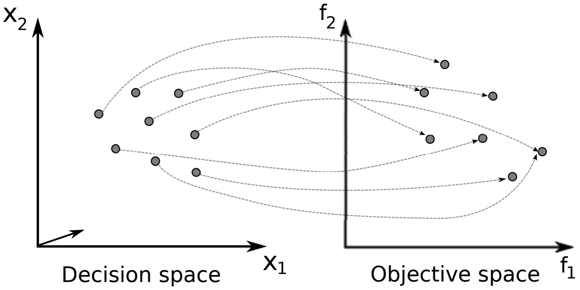
- Basis of fitness landscape *(Done)*
- Geometries of fitness landscapes *(Done)*
- Local optima network : *(Done)*
  - Definition inspired by complex systems science *(Done)*
  - Features of the network, design and performance *(Done)*
  - Performance prediction and portfolio *(Done)*
- **Fitness landscape analysis**
  for multi-objective optimization problems
  - Brief introduction on MO and MO algorithms
  - Fitness landscapes features
  - Performance prediction based on MO features
  - Set-based fitness landscape

Multiobjective optimization

Multiobjective optimization problem

- Search space : *decision space* X
- Objective function : *objective space* $f : X \to \mathbb{R}^m$

component f_i : objective, criterion.

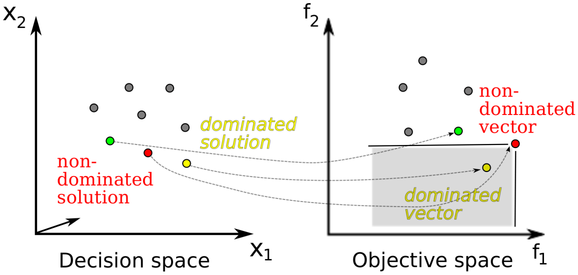


Definition

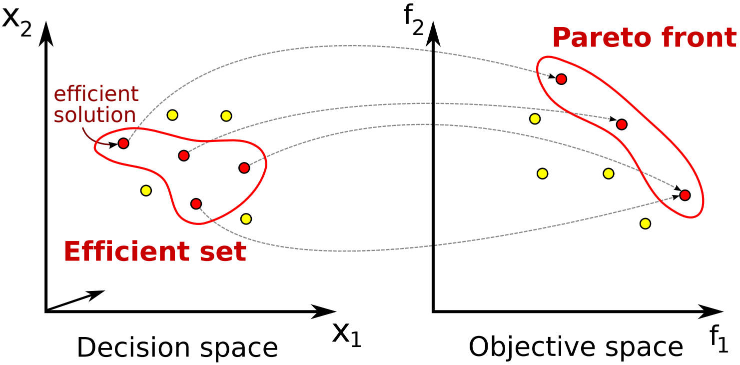
Pareto dominance relation

A solution $x \in X$ dominates a solution $x' \in X$ $(x' \prec x)$ iff

- $\forall i \in \{1, 2, \ldots, M\}$, $f_i(x') \leqslant f_i(x)$
- $\exists j \in \{1, 2, \ldots, M\}$ such that $f_j(x') < f_j(x)$



Pareto set, Pareto front



Goal of multi-objective optimization

Find the whole **Pareto Optimal Set**,

or a good approximation of the Pareto Optimal Set

Why multiobjective optimization ?

Position of multiobjective optimization : Decision making

- No *a priori* on the importance of the different objectives
- *a posteriori* selection by a decision marker :
  Selection of one Pareto optimal solution after a deep study of the possible solutions.

Multi-objective, many-objective optimization

Approximative definition

- Multi-objective : 2, 3 or 4 objectives
- Many-objective : 4, 5 and more objectives

Number of Pareto optimal solutions

Suppose that :

- Probability to improve : p for all objective,
- Objective are independent.

Probability to be non-dominated for m objectives is :

Multi-objective, many-objective optimization

Approximative definition

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- Many-objective : 4, 5 and more objectives

Number of Pareto optimal solutions

Suppose that :

- Probability to improve : p for all objective,
- Objective are independent.

Probability to be non-dominated for m objectives is : $1 - p^m$

Multi-objective, many-objective optimization

Approximative definition

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Number of Pareto optimal solutions

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- Probability to improve : p for all objective,
- Objective are independent.

Probability to be non-dominated for m objectives is : $1 - p^m$

Intuitive goals

Convergence toward the front, and diversity of the solutions.
Many-objective : convergence "easy", diversity "hard"

Note : objective correlation is also important.

Performance assessment

Quality of the approximation of the Pareto front [FKTZ05]

- Goal : indicator function related to the quality of the set.
- No universal indicator

Indicator functions

- Hypervolume indicator
- Epsilon indicator
- Atteinment function :

Performance assessment

Quality of the approximation of the Pareto front [FKTZ05]

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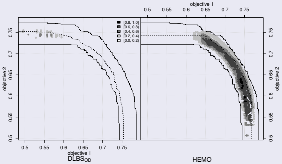
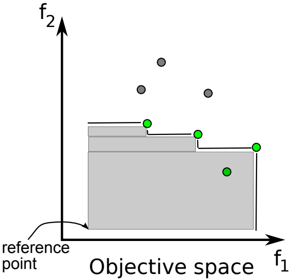
- No universal indicator

Indicator functions

- Hypervolume indicator
- Epsilon indicator
- Atteinment function :
  Probability to reach a point in objective space with algo.



Hypervolume ($\mathtt{I}_H$)



Volume cover
by the set.

Properties

- Compliant with the (weak) Pareto dominance relation
  $\rightarrow A \prec B \Rightarrow \mathtt{I}_H(A) \leqslant \mathtt{I}_H(B)$
- A single parameter :
  the reference point
- Minimal solution-set maximizing $\mathtt{I}_H$
  $\rightarrow$ subset of the Pareto optimal set

$$\arg\max_{\sigma \in \Sigma} \ \mathtt{I}_H(\sigma)$$

Epsilon indicator (I_{ε})

Definition

Smallest coefficient ε to translate the set A to "cover" each point of the set B

$\forall z_b \in B,\ \exists z_a \in A$ such that $z_b \prec (1+\varepsilon)\ .\ z_a$

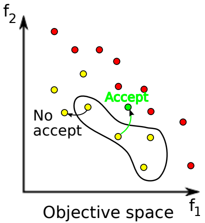
Properties

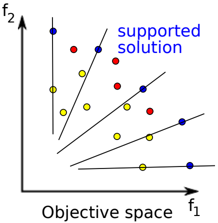
- Compliant with the (weak) Pareto dominance relation (using Pareto front)
  $\rightarrow A \prec B \Rightarrow \mathrm{I}_{\varepsilon}(A) \leqslant \mathrm{I}_{\varepsilon}(B)$
- A parameter : the reference set

Main types of MO local search algorithms

Three main classes :

- Pareto-based approaches : directly or indirectly focus the search on the Pareto dominance relation.
  Pareto Local Search (PLS), Global SEMO, NSGA-II, etc.
- Indicator approaches : Progressively improvement the indicator function : IBEA, SMS-MOEA, etc.
- Scalar approaches : multiple scalarized aggregations of the objective functions : MOEA/D, etc.





Pareto-based approaches

Simple dominance-based EMO with unbounded archive

local search :
Pareto Local Search (PLS)

Pick a random solution $x_0 \in X$
$A \leftarrow \{x_0\}$
repeat
  Select a non-visited $x \in A$
  Create $N(x)$ by flipping each bit of x in turns
  Flag x as visited
  $A \leftarrow$ non-dom. from $A \cup N(x)$
until all-visited $\vee$ maxeval

[Paquete *et al.* 2004][PCS04]

global search :
Global-Simple EMO (G-SEMO)

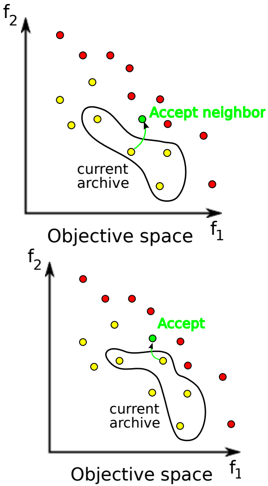
Pick a random solution $x_0 \in X$
$A \leftarrow \{x_0\}$
repeat
  Select $x \in A$ at random
  Create x' by flipping each bit of x with a rate $1/N$

  $A \leftarrow$ non-dom. from $A \cup \{x'\}$
until maxeval

[Laumanns *et al.* 2004][LTZ04]

A Pareto-based approach : Pareto Local Search

- Archive solutions using **Dominance relation**
- Iteratively improve this archive by exploring the neighborhood



SMS-MOEA : $\mathcal{S}$ metric selection- EMOA

[Beume *et al.* 2007][BNE07]

$P \leftarrow$ initialization()
repeat
  $q \leftarrow$ Generate(P)
  $P \leftarrow$ Reduce($P \cup \{q\}$)
until maxeval

Reduction

- Remove the worst solution according to non-dominated sorting, and $\mathcal{S}$ metric

Algorithm 2. Reduce(Q)

1: $\{\mathcal{R}_1, \ldots, \mathcal{R}_v\} \leftarrow$ fast-nondominated-sort(Q) /* all v fronts of Q */
2: $r \leftarrow \text{argmin}_{s \in \mathcal{R}_v}[\Delta_{\mathcal{S}}(s, \mathcal{R}_v)]$ /* $s \in \mathcal{R}_v$ with lowest $\Delta_{\mathcal{S}}(s, \mathcal{R}_v)$ */
3: **return** $(Q \backslash \{r\})$ /* eliminate detected element */

IBEA : Indicator-Based Evolutionary algorithm

[Zitzler *et al.* 2004][ZK04]

$P \leftarrow$ initialization()
repeat
  $P^{'} \leftarrow$ selection(P)
  $Q \leftarrow$ random_variation($P^{'}$)
  Evaluation of Q
  $P \leftarrow$ replacement(P, Q)
until maxeval

Fitness assignment

- Pairwise comparison of solutions in a population w.r.t. indicator I
- Fitness value : "loss in quality" in the population P if x was removed

$$f(x) = \sum_{x^{'} \in P \setminus \{x\}} (-e^{-I(x^{'},x)/\kappa})$$

- Often the ε-indicator is used

Original MOEA/D [ZL07] (minimization)

/ μ sub-problems defined by μ directions */*
$(\lambda^1, \ldots, \lambda^\mu) \leftarrow$ initialization_direction()
Initialize $\forall i = 1..\mu$ $B(i)$ the neighboring sub-problems of sub-problem i
/ one solution for each sub-problem */*
$(x^1, \ldots, x^\mu) \leftarrow$ initialization_solution()
repeat
  for $i = 1..\mu$ **do**
    Select x and x' randomly in $\{x_j \ : \ j \in B(i)\}$
    $y \leftarrow$ mutation_crossover(x, x')
    for $j \in B(i)$ **do**
      if $g(y|\lambda_j, z_j^\star) < g(x_j|\lambda_j, z_j^\star)$ **then**
        $x_j \leftarrow y$
      end if
    end for
  end for
until max_eval

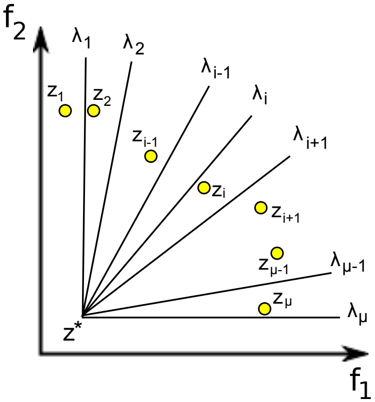
$B(i)$ is the set of the T closest neighboring sub-problems of sub-problem i
$g(\ |\lambda_i, z_i^\star)$: scalar function of sub-pb. i with λ_i direction, and $z_i^\star$ reference point

MOEA/D steady-state variant

Another MOEA/D (minimization)

/* μ *sub-problems defined by* μ *directions* */
$(\lambda^1, \ldots, \lambda^\mu) \leftarrow$ initialization_direction()
Initialize $\forall i = 1..\mu$ $B(i)$ the neighboring sub-problems of sub-problem i
/* *one solution for each sub-problem* */
$(x^1, \ldots, x^\mu) \leftarrow$ initialization_solution()
repeat
  Select i at random $\in 1..\mu$
  Select x randomly in $\{x_j \ : \ j \in B(i)\}$
  $y \leftarrow$ mutation_crossover(x_i, x)
  for n_r sub-problems $\in B(i)$ **do**
    if $g(y|\lambda_j, z_j^\star) < g(x_j|\lambda_j, z_j^\star)$ **then**
      $x_j \leftarrow y$
    end if
  end for
until max_eval

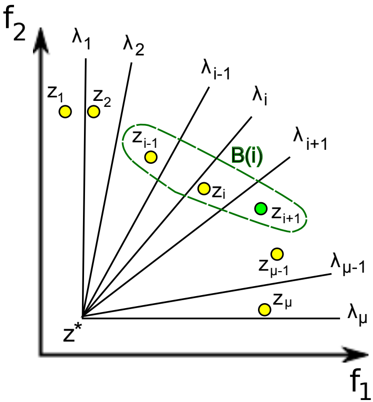
Representation of steady-state MOEA/D



Population at iteration t

- Minimization problem
- One solution x_i for each sub pb. i
- Representation of solutions in objective space : $z_i = g(x_i|\lambda_i, z_i^\star)$
- Same reference point for all sub-pb. $z^\star = z_1^\star = \ldots = z_\mu^\star$
- Scalar function g :
  Weighted Tchebycheff
- Neighborhood size $\sharp B(i) = T = 3$

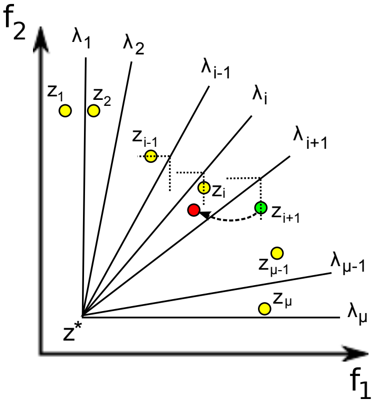
Representation of steady-state MOEA/D



From the neigh. $B(i)$ of sub-pb. i, x_{i+1} is selected

- Minimization problem
- One solution x_i for each sub pb. i
- Representation of solutions in objective space : $z_i = g(x_i | \lambda_i, z_i^\star)$
- Same reference point for all sub-pb. $z^\star = z_1^\star = \ldots = z_\mu^\star$
- Scalar function g :
  Weighted Tchebycheff
- Neighborhood size $\sharp B(i) = T = 3$

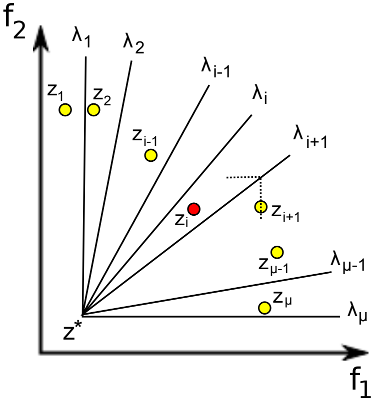
Representation of steady-state MOEA/D



The mutated solution y is created

- Minimization problem
- One solution x_i for each sub pb. i
- Representation of solutions in objective space : $z_i = g(x_i|\lambda_i, z_i^\star)$
- Same reference point for all sub-pb. $z^\star = z_1^\star = \ldots = z_\mu^\star$
- Scalar function g :
  Weighted Tchebycheff
- Neighborhood size $\sharp B(i) = T = 3$

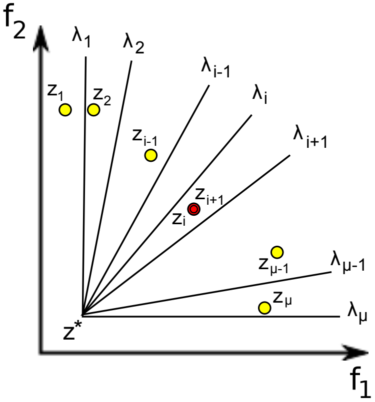
Representation of steady-state MOEA/D



According to scalar fonction,
y is worst than x_{i-1},
y is better than x_i and replaces it.

- Minimization problem
- One solution x_i for each sub pb. i
- Representation of solutions in objective space : $z_i = g(x_i | \lambda_i, z_i^\star)$
- Same reference point for all sub-pb. $z^\star = z_1^\star = \ldots = z_\mu^\star$
- Scalar function g :
  Weighted Tchebycheff
- Neighborhood size $\sharp B(i) = T = 3$

Representation of steady-state MOEA/D



According to scalar fonction,
y is also better than x_{i+1}
and replaces it for the next iteration.

- Minimization problem
- One solution x_i for each sub pb. i
- Representation of solutions in objective space : $z_i = g(x_i | \lambda_i, z_i^\star)$
- Same reference point for all sub-pb. $z^\star = z_1^\star = \ldots = z_\mu^\star$
- Scalar function g :
  Weighted Tchebycheff
- Neighborhood size $\sharp B(i) = T = 3$

Join work with

Arnaud Liefooghe, Fabio Daolio, Hernan Aguirre, Kiyoshi Tanaka, Clarisse Daehnens, Laetitia Jourdan, Manuel López-Ibáñez, Mathieu Basseur, Adrien Goëffon

Slides are shared with those co-authors, special thanks to Arnaud and Fabio

Fitness landscape for Multi-Objective optimization

Context

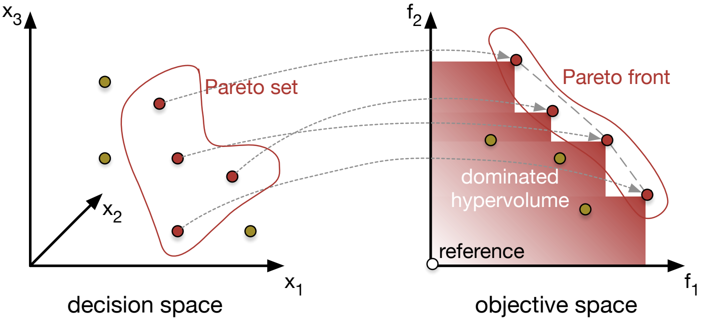
- MO problems are hard :
  underlying single-objective functions, join functions, etc.
- Learning the problem structure :
  to understand and improve algorithms
- Questions :
  - What are the relevant problem features ?
  - Replace one-dimensional by d-dimensional objective function ?

Problem features from the Pareto set

$\#po$ proportion of solutions that belong to the Pareto set

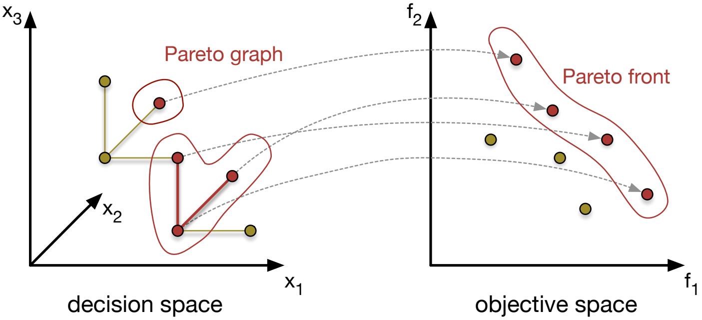
$\#supp$ proportion of supported solutions in the Pareto set

hv hypervolume-value of the Pareto front $\left(z^{\text{ref}} = (0, \ldots, 0)\right)$



Problem features from the Pareto set topology

dist_avg average distance between solutions from the Pareto set
dist_max maximum distance between solutions from the Pareto set
#cc relative number of connected components in the Pareto set
#lcc proportional size of the largest connected component



Problem features related to multi-modality and ruggedness

> Features sampled along adaptive walks :

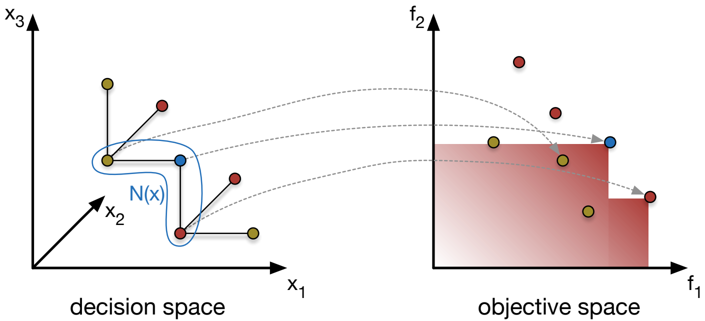
#lnd_avg_aws proportion of locally non-dominated solutions

ladapt length of a Pareto-based adaptive walk

> Features sampled along random walks :

hv_r1_rws autocorrelation coeff. of solution hypervolume

nhv_r1_rws autocorrelation coeff. of neighborhood hypervolume



Summary of problem features

	BENCHMARK **parameters**	
n	number of (binary) variables	
k_n	proportional number of variable interactions (epistatic links) : k/n	
m	number of objectives	
ρ	correlation between the objective values	
	FEATURES FROM **enumeration**	
#po	proportion of Pareto optimal (PO) solutions	*knowles2003*
#supp	proportion of supported solutions in the Pareto set	*knowles2003*
hv	hypervolume-value of the (exact) Pareto front	*aguirre2007*
#plo	proportion of Pareto local optimal (PLO) solutions	*paquete2007*
#slo_avg	average proportion of local optimal solutions per objective	
podist_avg	average Hamming distance between Pareto optimal solutions	*liefooghe2013*
podist_max	maximal Hamming distance between Pareto optimal solutions (diameter of the Pareto set)	*knowles2003*
po_ent	entropy of binary variables from Pareto optimal solutions	*knowles2003*
fdc	fitness-distance correlation in the Pareto set (Hamming dist. in solution space *vs.* Manhattan dist. in objective space)	*knowles2003*
#cc	proportion of connected components in the Pareto graph	*paquete2009*
#sing	proportion of isolated Pareto optimal solutions (singletons) in the Pareto graph	*paquete2009*
#lcc	proportional size of the largest connected component in the Pareto graph	*verel2011a*
lcc_dist	average Hamming distance between solutions from the largest connected component	
lcc_hv	proportion of hypervolume covered by the largest connected component	
#fronts	proportion of non-dominated fronts	*aguirre2007*
front_ent	entropy of the non-dominated front's size distribution	

Summary of problem features (cont'd)

	FEATURES FROM RANDOM WALK **sampling**	
hv_avg_rws	average (single) solution's hypervolume-value	
hv_r1_rws	first autocorrelation coefficient of (single) solution's hypervolume-values	*liefooghe2013*
hvd_avg_rws	average (single) solution's hypervolume difference-value	
hvd_r1_rws	first autocorrelation coefficient of (single) solution's hypervolume difference-values	*liefooghe2013*
nhv_avg_rws	average neighborhood's hypervolume-value	
nhv_r1_rws	first autocorrelation coefficient of neighborhood's hypervolume-value	
#lnd_avg_rws	average proportion of locally non-dominated solutions in the neighborhood	
#lnd_r1_rws	first autocorrelation coefficient of the proportion of locally non-dominated solutions in the neighborhood	
#lsupp_avg_rws	average proportion of supported locally non-dominated solutions in the neighborhood	
#lsupp_r1_rws	first autocorrelation coefficient of the proportion of supported locally non-dominated solutions in the neighborhood	
#inf_avg_rws	average proportion of neighbors dominated by the current solution	
#inf_r1_rws	first autocorrelation coefficient of the proportion of neighbors dominated by the current solution	
#sup_avg_rws	average proportion of neighbors dominating the current solution	
#sup_r1_rws	first autocorrelation coefficient of the proportion of neighbors dominating the current solution	
#inc_avg_rws	average proportion of neighbors incomparable to the current solution	
#inc_r1_rws	first autocorrelation coefficient of the proportion of neighbors incomparable to the current solution	
f_cor_rws	estimated correlation between the objective values	
	FEATURES FROM ADAPTIVE WALK **sampling**	
hv_avg_aws	average (single) solution's hypervolume-value	
hvd_avg_aws	average (single) solution's hypervolume difference-value	
nhv_avg_aws	average neighborhood's hypervolume-value	
#lnd_avg_aws	average proportion of locally non-dominated solutions in the neighborhood	
#lsupp_avg_aws	average proportion of supported locally non-dominated solutions in the neighborhood	
#inf_avg_aws	average proportion of neighbors dominated by the current solution	
#sup_avg_aws	average proportion of neighbors dominating the current solution	
#inc_avg_aws	average proportion of neighbors incomparable to the current solution	
length_aws	average length of Pareto-based adaptive walks	*verel2011a*

NK-landscapes [Kauffman 1993] [Kau93]

general-purpose family of multi-modal pseudo-boolean optimization functions
superposition of n Walsh functions of order k+1

$$\begin{array}{lll} \max & f(x) = \frac{1}{n}\sum_{j=1}^{n} c_j(x_j, x_{j_1}, \dots, x_{j_k}) & \\ \text{s.t.} & x_j \in \{0,1\} & , \quad j \in \{1,\dots,n\} \end{array}$$

> problem size n (decision space dimension)

> problem non-linearity $k <$ n
(multi-modality, epistatic interactions)

$x_1x_2x_4$	c_1
000	0.9
001	0.6
010	0.1
011	0.2
...	...

$x_1x_2x_3$	c_2
000	0.4
001	0.8
010	0.3
011	0.2
...	...

$x_2x_3x_4$	c_3
000	0.2
...	...
101	0.9
110	0.1
111	0.5

$x_1x_2x_4$	c_4
000	0.1
001	0.2
010	0.8
011	0.0
...	...

MNK-landscapes [Aguirre and Tanaka 2007][AT07]

$$\begin{array}{lll} \max & f_i(x) = \frac{1}{n}\sum_{j=1}^{n} c_j^i(x_j, x_{j_1}, \ldots, x_{j_k}) & , \quad i \in \{1, \ldots, m\} \\ \text{s.t.} & x_j \in \{0,1\} & , \quad j \in \{1, \ldots, n\} \end{array}$$

- m **independent** objective functions
- different variable interactions for each objective function

$x_1x_2x_4$	c_1^1
000	0.9
001	0.6
010	0.1
011	0.2
...	...

$x_1x_2x_3$	c_2^1
000	0.4
001	0.8
010	0.3
011	0.2
...	...

$x_2x_3x_4$	c_3^1
000	0.2
...	...
101	0.9
110	0.1
111	0.5

$x_1x_2x_4$	c_4^1
000	0.1
001	0.2
010	0.8
011	0.0
...	...

$x_1x_2x_4$	c_1^2
000	0.1
001	0.2
010	0.7
011	0.3
...	...

$x_1x_2x_3$	c_2^2
000	0.2
001	0.6
010	0.4
011	0.2
...	...

$x_2x_3x_4$	c_3^2
000	0.6
...	...
101	0.1
110	0.3
111	0.9

$x_1x_2x_4$	c_4^2
000	0.9
001	0.2
010	0.6
011	0.1
...	...

ρMNK-landscapes [Verel et al. 2011] [VLJD11]

$$\begin{array}{lll} \max & f_i(x) = \frac{1}{n}\sum_{j=1}^{n} c_j^i(x_j, x_{j_1}, \ldots, x_{j_k}) & , \quad i \in \{1, \ldots, m\} \\ \text{s.t.} & x_j \in \{0, 1\} & , \quad j \in \{1, \ldots, n\} \end{array}$$

- c_j^i's follow a **multivariate uniform law** of correlation ρ
- same variable interactions for each objective function

$x_1x_2x_4$	c_1^1
000	0.9
001	0.6
010	0.1
011	0.2
...	...

$x_1x_2x_3$	c_2^1
000	0.4
001	*0.8*
010	0.3
011	0.2
...	...

$x_2x_3x_4$	c_3^1
000	0.2
...	...
101	0.9
110	0.1
111	0.5

$x_1x_2x_4$	c_4^1
000	0.1
001	0.2
010	0.8
011	0.0
...	...

$x_1x_2x_4$	c_1^2
000	0.8
001	0.7
010	0.2
011	0.3
...	...

$x_1x_2x_3$	c_2^2
000	0.6
001	*0.7*
010	0.1
011	0.2
...	...

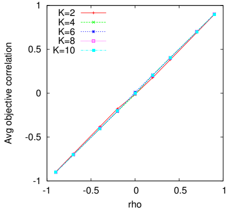
$x_2x_3x_4$	c_3^2
000	0.5
...	...
101	0.6
110	0.2
111	0.3

$x_1x_2x_4$	c_4^2
000	0.4
001	0.3
010	0.7
011	0.1
...	...

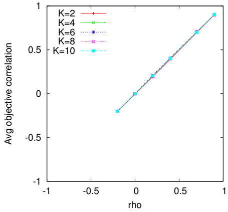
ρMNK-landscapes

$$
\begin{array}{lll}
\max & f_i(x) = \frac{1}{n} \sum_{j=1}^{n} c_j^i(x_j, x_{j_1}, \ldots, x_{j_k}) & , \quad i \in \{1, \ldots, m\} \\
\text{s.t.} & x_j \in \{0, 1\} & , \quad j \in \{1, \ldots, n\}
\end{array}
$$

m = 2



m = 5

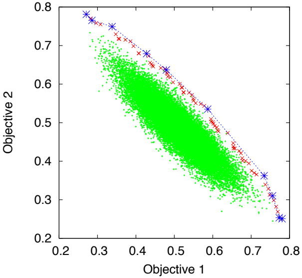


Benchmark parameters :

- problem size n (decision space dimension)
- problem non-linearity $k <$ n (multi-modality, epistatic interactions)
- number of objective functions m (objective space dimension)
- objective correlation $\rho > -\frac{1}{m-1}$

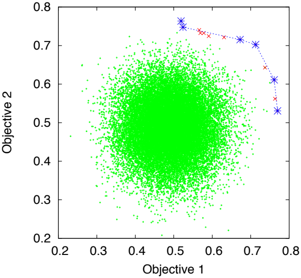
`http://mocobench.sf.net`

Some intuitions on objective correlation ρ



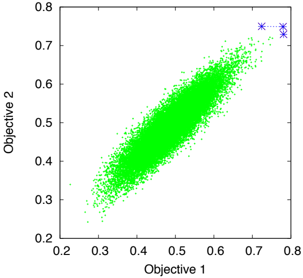
conflicting objectives

$\rho = -0.9$



independent objectives

$\rho = 0.0$



correlated objectives

$\rho = 0.9$

$\boxed{m = 2 \qquad n = 18 \qquad k = 4}$

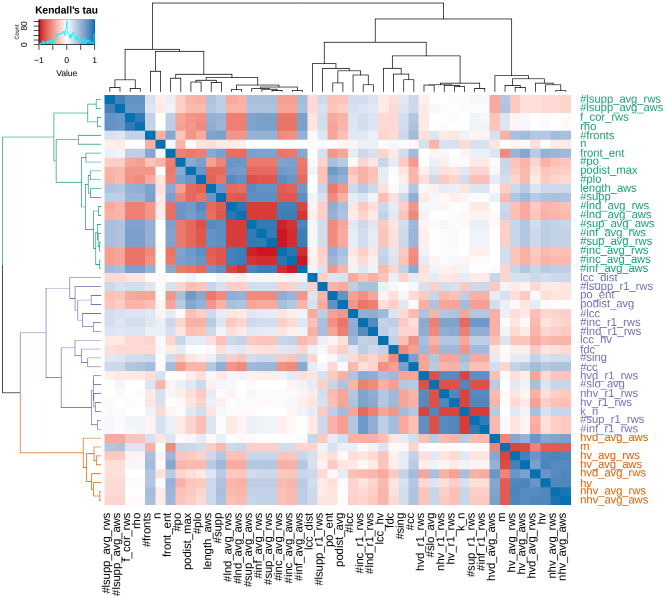
Experimental setup for enumerable instances

Small-size ρ*MNK-landscapes*, factorial design, 30 instances/unit

- problem size $n \in \{10, 11, 12, 13, 14, 15, 16\}$
- problem non-linearity $k \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- number of objectives $m \in \{2, 3, 4, 5\}$
- obj. corr. $\rho \in \{-0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1\}$, $\rho > \frac{-1}{m-1}$

60 480 problem instances overall

Pairwise feature association (enumerable instances)



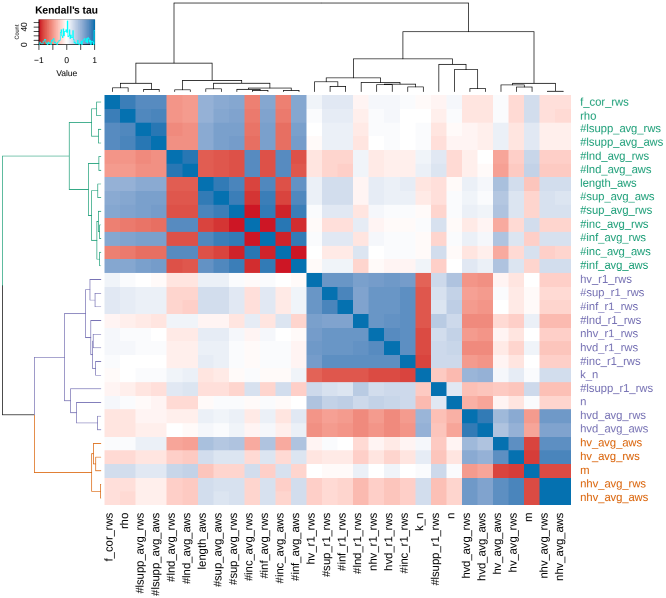
Experimental setup for large-size instances

Large-size ρ*MNK-landscapes*, constrained random LHS DOE

- problem size $n \in \{64, 256\}$
- problem non-linearity $k \in \{0, 8\}$
- number of objectives $m \in \{2, 5\}$
- objective correlation $\rho \in [-1, 1]$, $\rho > \frac{-1}{m-1}$

1 000 problem instances overall

Pairwise feature association (large instances)



Experimental setup for large-size instances

Large-size *ρMNK-landscapes*, constrained random LHS DOE

- problem size $n \in \{64, 256\}$
- problem non-linearity $k \in \{0, 8\}$
- number of objectives $m \in \{2, 5\}$
- objective correlation $\rho \in [-1, 1]$, $\rho > \frac{-1}{m-1}$

1 000 problem instances overall

GSEMO and IPLS *algorithms*

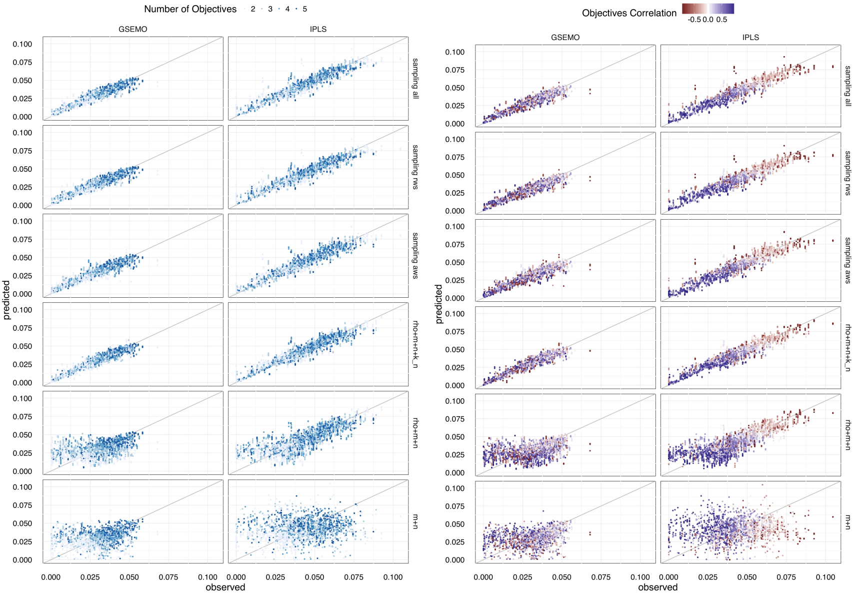
- 30 independent runs per instance
- Fixed budget of 100 000 evaluation calls
- epsilon approximation ratio to best-found non-dominated set

Prediction accuracy (large instances)

cross validation with repeated subsampling, 50 iterations, 90/10 split

feature set	MAE	MSE	R^2	rank
		GSEMO		
all	**0.003049**	**0.000017**	**0.891227**	1
sampling all	0.003152	**0.000018**	**0.883909**	1.3
sampling rws	0.003220	0.000019	0.878212	2
sampling aws	0.003525	0.000023	0.854199	3
ρ+m+n+k/n	**0.003084**	**0.000017**	**0.892947**	1
ρ+m+n	0.009062	0.000148	0.065258	4
m+n	0.010813	0.000206	-0.303336	5
		IPLS		
all	**0.004290**	**0.000034**	**0.886568**	1
sampling all	**0.004359**	**0.000035**	**0.883323**	1
sampling rws	**0.004449**	0.000036	**0.879936**	1.3
sampling aws	0.004663	0.000039	0.871011	2
ρ+m+n+k/n	**0.004353**	**0.000033**	**0.889872**	1
ρ+m+n	0.008415	0.000119	0.600965	3
m+n	0.016959	0.000472	-0.568495	4

Predicted vs observed values (out-of-folds)



Portfolio accuracy

cross validation with repeated subsampling, 50 iterations, 90/10 split

Portfolio : { GSEMO, IPLS }

feature set	error rate	rank
all	**0.0128**	1
sampling all	**0.0138**	1
sampling rws	**0.0150**	1
sampling aws	**0.0144**	1
ρ+m+n+k/n	**0.0134**	1
ρ+m+n	0.0824	2
m+n	0.1328	3
const=GSEMO	0.0880	
const=IPLS	0.7250	

Practice : Bi-objective squares problem

Squares Problem (SP)

Find the position of 5 squares
in order to maximize inside squares the number of brown points
and the number of blue points

Fitness function

$f(x) =$ (number of brown points, number of blue points)

Fitness landscape analysis

Execute line by line the `main` function of the code `ex05.R` :
random walk and Pareto adaptive walk

Set-based multiobjective fitness landscapes [VLD11]

The key idea

EMO algorithms are local search algorithms performing on sets

Set-based multiobjective fitness landscape

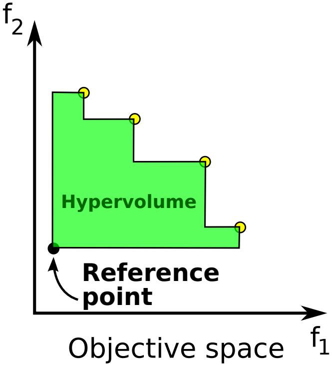
$$(\Sigma, \mathtt{N}, \mathtt{I})$$

- **Set-domain search space** (Σ)
  $\Sigma \subset 2^X$ is a set of feasible solution-sets
  (where X is the set of feasible solutions)
- **Set-domain neighborhood relation** $(\mathtt{N})$
  $\mathtt{N} : \Sigma \to 2^\Sigma$ is a neighborhood relation between solution-sets
- **Set-domain fitness function** $(\mathtt{I})$
  $\mathtt{I} : \Sigma \to \mathbb{R}$ is a unary quality indicator,
  i.e. a fitness function measuring the quality of solution-sets

Set-domain fitness function

Quality indicator

$\mathrm{I}(\sigma) \in \mathbb{R}$: quality of a set of solutions σ



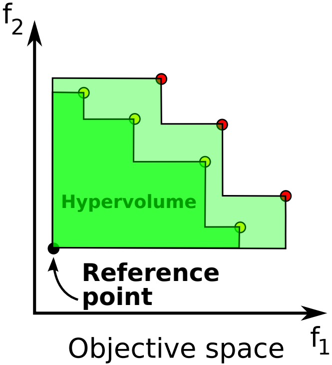
Hypervolume (I_H)

- Compliant with the (weak) Pareto dominance relation :
  $$\sigma \prec \sigma' \Rightarrow \mathrm{I}_H(\sigma) \leqslant \mathrm{I}_H(\sigma')$$
- A single parameter :
  the reference point
- Minimal set maximizing I_H :
  subset of the Pareto optimal set

Set-domain fitness function

Quality indicator

$\mathrm{I}(\sigma) \in \mathbb{R}$: quality of a set of solutions σ



Hypervolume (I_H)

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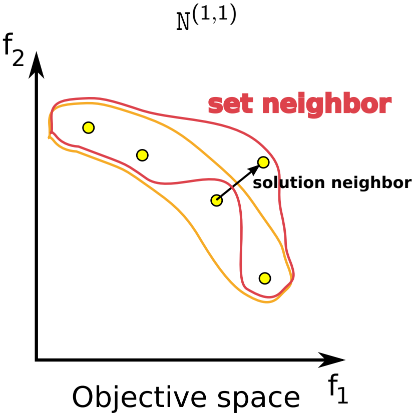
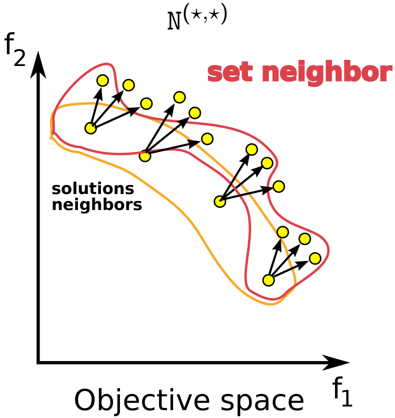
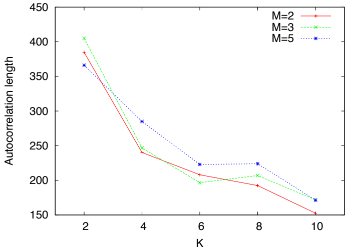
- A single parameter :
  the reference point
- Minimal set maximizing I_H :
  subset of the Pareto optimal set

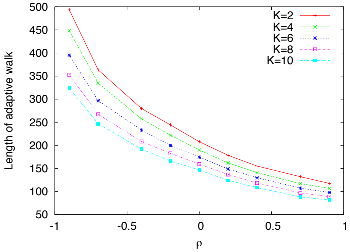
Set-domain neighborhood relations [BGLV13]





Ruggedness and multimodality





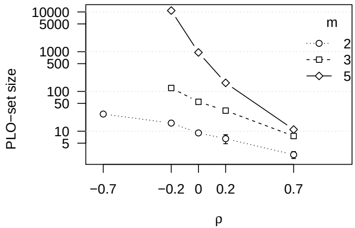
Autocorrelation length (τ)

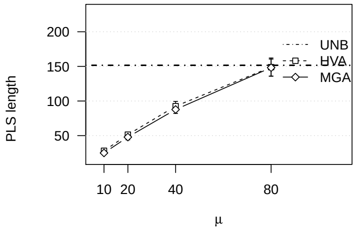
- non-linearity degree K
  $\rightarrow$ τ decreases wrt. K
- number of objectives M
  $\rightarrow$ no influence
- objective correlation ρ
  $\rightarrow$ no influence

Adaptive walk length ($\mathcal{L}$)

- non-linearity degree K
  $\rightarrow$ $\mathcal{L}$ decreases wrt. K
- number of objectives M
  $\rightarrow$ no clear trend
- objective correlation ρ
  $\rightarrow$ $\mathcal{L}$ decreases wrt. ρ ! !

Pareto Local Optima set *vs.* archive size





Pareto set-size

for unbounded solution-set (archive)

- objective correlation ρ
  $\rightarrow$ decreases wrt. ρ
- number of objectives M
  $\rightarrow$ increases wrt. M

Pareto Local search Length

for bounded solution-set (archive)

- Increases with archive size

Fitness landscapes analysis for MO optimization

Summary

- A number of features are shown to be related to problem difficulty
- Better knowledge on problem difficulty for MO optimization
- Performance can be predicted using such relevant features (for some algo/problems)
- And now,
  - toward a better design ?
  - toward a algorithm portfolio ?
  - toward more theoretical works ?
  - design low complexity features ?...

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