1. Basis of fitness landscape

Fitness landscape analysis for understanding and designing local search heuristics

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Outline of this part

- **Basis of fitness landscape** :
  - introductory example *(Done)*
  - brief history and background of fitness landscape
  - fundamental definitions

Mono-objective Optimization

- Search space : set of candidate solutions

$$X$$

- Objective fonction : quality criterion (or non-quality)

$$f : X \to \mathbb{R}$$

X discrete : combinatorial optimization
$X \subset \mathbb{R}^n$: numerical optimization

Solve an optimization problem (maximization)

$$X^{\star} = \mathrm{argmax}_X \ f$$

or find an approximation of $X^{\star}$.

Context : black-box optimization

$x \longrightarrow$  $\longrightarrow f(x)$

No information on the objective definition function f

Objective fonction :

- can be irregular, non continuous, non differentiable, etc.
- given by a computation or a simulation

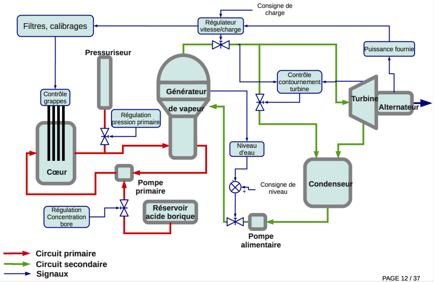
Real-world black-box optimization : first example

PhD of Mathieu Muniglia, Saclay Nuclear Research Centre (CEA), Paris

$x \longrightarrow$

$\longrightarrow f(x)$

$(73, \dots, 8) \longrightarrow$



$\longrightarrow \Delta_z P$

Multi-physic simulator

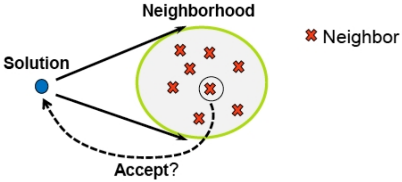
Search algorithms

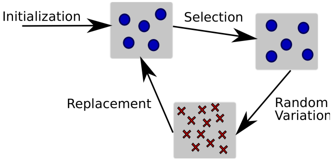
Principle

Enumeration of the search space

- A lot of ways to enumerate the search space
- Using exact method : $A^{\star}$, Branch&Bound, etc.
- Using random sampling : Monte Carlo technics, approx. with guarantees, etc.

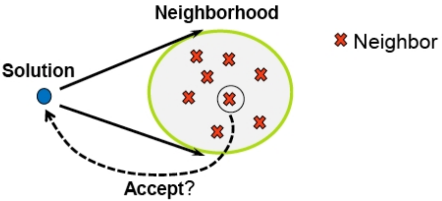
Local search technics :

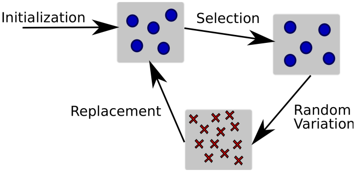




Metaheuristics

Local search methods using neighborhood relation

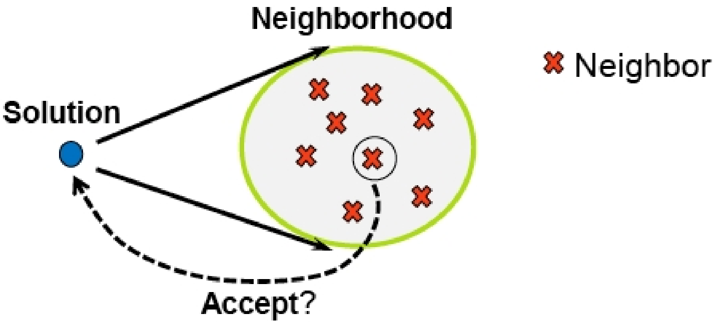




- **Single solution-based** : Hill-climbing technics, Simulated-annealing, tabu search, Iterative Local Search, etc.
- **Population solution-based** : Genetic algorithm, Genetic programming, Ant colony optimization, etc.

Stochatic algorithms with unique solution (Local Search)

- $\mathcal{S}$ set of candidate solutions (search space)
- $f : X \to \mathbb{R}$ objective function
- $\mathcal{N}(x)$ set of neighbor's solutions of x



Main idea behind local search algorithm

Why using a local search strategy based on neighborhood ?

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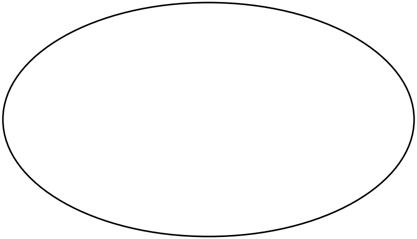

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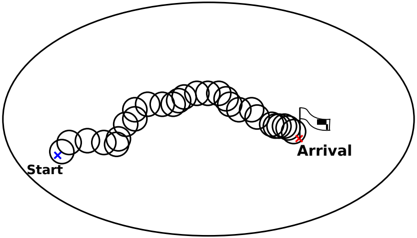

Split the global problem
into a sequence of local problems (smaller)

- Benefit : reduce the complexity

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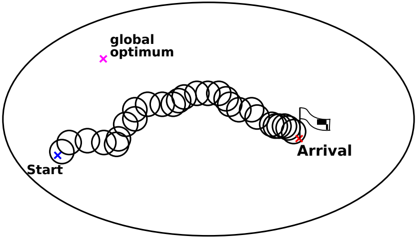


Split the global problem
into a sequence of local problems (smaller)

- Benefit : reduce the complexity
- Risk : do not find optimal solution

Motivations with fitness landscape analysis

To be efficient, the sequence of local optimization problems must be related to the global problem

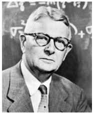
Main motivation : "Why using local search"

- Study the search space from the point of view of local search
  $\Rightarrow$ Fitness Landscape Analysis
- To understand and design effective local search algorithms

"the more we know of the **statistical properties** *of a class of fitness landscapes, the better equipped we will be for the* **design** *of effective search algorithms for such landscapes"*

L. Barnett, U. Sussex, PhD 2003.

Fitness landscape : original plots of S. Wright [Wri32]



S. Wright. "The roles of mutation, inbreeding, crossbreeding, and selection in evolution.", 1932.

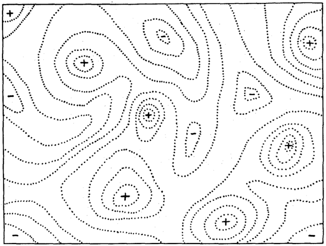


FIGURE 2.—Diagrammatic representation of the field of gene combinations in two dimensions instead of many thousands. Dotted lines represent contours with respect to adaptiveness.

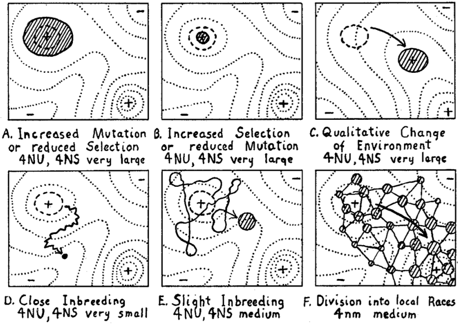
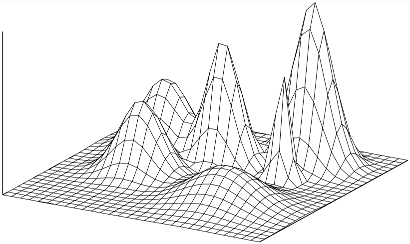


FIGURE 4.—Field of gene combinations occupied by a population within the general field of possible combinations. Type of history under specified conditions indicated by relation to initial field (heavy broken contour) and arrow.

source : Encyclopædia Britannica Online.

Fitness landscapes in (evolutionary) biology



- Metaphorical uphill struggle across a "fitness landscape"
    - mountain **peaks** represent high "fitness" (ability to survive),
    - **valleys** represent low fitness.
- Evolution proceeds :
  population of organisms
  performs an "**adaptive walk**"

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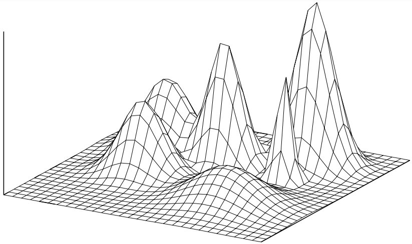

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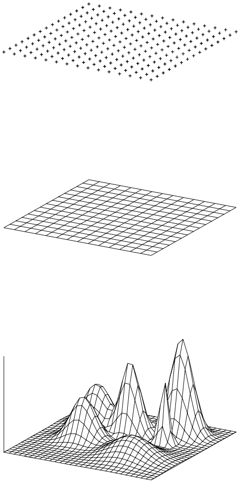
becareful : "2 dimensions instead of many thousands"

Fitness landscapes in biology and others sciences



In biology :

- Model of species evolution

Extended to model dynamical systems :

- statistical physic,
- molecular evolution,
- ecology, etc.

Fitness landscapes in biology

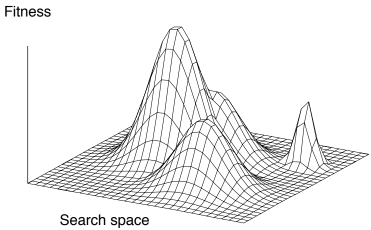
2 sides of Fitness Landscapes

- **Metaphor** : most profound concept in evolutionary dynamics
  - give pictures of evolutionary process
  - be careful of misleading pictures :
    "smooth low-dimensional landscape without noise"
- **Quantitative** concept : predict the evolutionary paths

$$X \longrightarrow X$$

  - Quasispecies equation : mean field analysis
    x_t
  - Stochastic process : Markov chain
    $\Pr(x_{t+1} \mid x_t)$
  - Individual scale : network analysis

Definition of fitness landscape for optimization [Sta02]



Definition

Fitness landscape $(X, \mathcal{N}, f)$:

- **search space** :

$$X$$

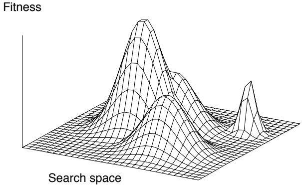
- **neighborhood relation** :

$$\mathcal{N} : X \to 2^X$$

- **objective function** :

$$f : X \to \mathbb{R}$$

What is a neighborhood ?



Neighborhood function :

$$\mathcal{N} : X \to 2^X$$

Set of "**neighbor**" solutions associated to each solution

$$\mathcal{N}(x) = \{y \in X \mid \Pr(y = op(x)) > 0\}$$

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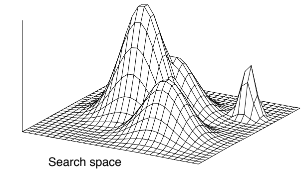

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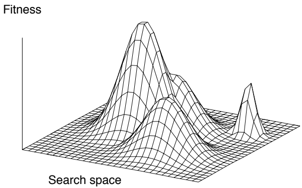

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or

$$\mathcal{N}(x) = \{y \in X \mid \text{distance}(x, y) = 1\}$$

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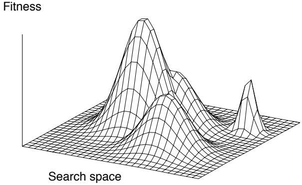


Neighborhood function :

$$\mathcal{N} : X \rightarrow 2^X$$

Set of "**neighbor**" solutions associated to each solution

Important !

Neighborhoood must be based on the operator(s) of the algorithm

Neighborhood $\Leftrightarrow$ Operator

$\mathcal{N}(x) = \{y \in X \mid \Pr(y = op(x)) > 0\}$
or
$\mathcal{N}(x) = \{y \in X \mid \Pr(y = op(x)) > \varepsilon\}$
or
$\mathcal{N}(x) = \{y \in X \mid \text{distance}(x, y) = 1\}$

Typical example : bit strings

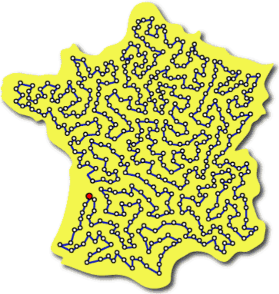
Search space : $X = \{0,1\}^N$

$$\mathcal{N}(x) = \{y \in X \mid d_{Hamming}(x,y) = 1\}$$

Example :
$\mathcal{N}(01101) = \{11101, 00101, 01001, 01111, 01100\}$

Typical example : permutations

Traveling Salesman Problem :
find the shortest tour which cross one time every town



Search space : $X = \{\sigma \mid \sigma \text{ permutations } \}$

$$\mathcal{N}(x) = \{y \in X \mid \Pr(y = op_{2opt}(x)) > 0\}$$

Typical example : triangle program

Triangle.c

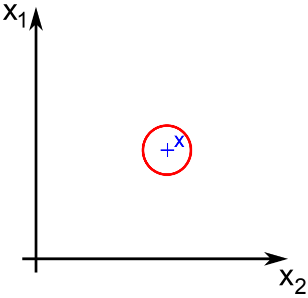
```
int gettri(int side1, int side2, int side3){
  int triang ;
   if( side1 <= 0 || side2 <= 0 || side3 <= 0){
      return 4;
   }
   triang = 0;
   if(side1 == side2){
      triang = triang + 1;
   }
   if(side1 == side3){
      triang = triang + 2;
   }
   if(side2 == side3){
      triang = triang + 3;
   }
   if(triang == 0){
      if(side1 + side2 < side3 || side2 + side3 < side1 || side1 + side3 < side2){
         return 4;
      }
      else {
```

Potential mutation sites (comparisons) in red

William B. Langdon and Mark Harman and Yue Jia, Efficient Multi Objective Higher Order Mutation Testing with Genetic Programming, Journal of Systems and Software, 83 (2010) 2416-2430. [LHJ10]

Not so typical example : continuous optimization

Still an open question...



Search space : $X = [0,1]^d$

$$\mathcal{N}_\alpha(x) = \{y \in X \mid \|y - x\| \leqslant \alpha\} \text{ with } \alpha > 0$$

More than 1 operator...

What can we do with 2 operators (ex : memetic algorithm) ?

$\mathcal{N}_1(x) = \{y \in X \mid y = op_1(x)\}$ $\mathcal{N}_2(x) = \{y \in X \mid y = op_2(x)\}$

More than 1 operator...

What can we do with 2 operators (ex : memetic algorithm) ?

$\mathcal{N}_1(x) = \{y \in X \mid y = op_1(x)\}$ $\quad$ $\mathcal{N}_2(x) = \{y \in X \mid y = op_2(x)\}$

Severals possibilities according to the goal :

- Study 2 landscapes : $(X, \mathcal{N}_1, f)$ and $(X, \mathcal{N}_2, f)$
- Study the landscape of "union" : $(X, \mathcal{N}, f)$

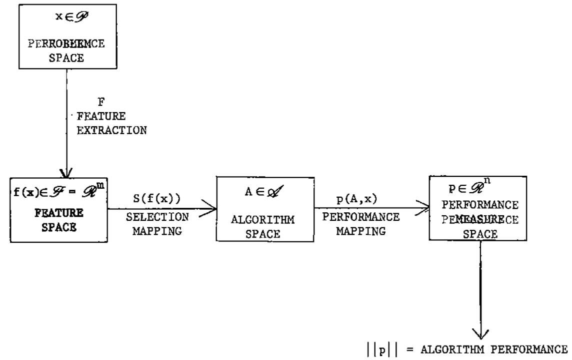
$$\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2 = \{y \in X \mid y = op_1(x) \text{ or } y = op_2(x)\}$$

- Study the landscape of "composition" : $(X, \mathcal{N}, f)$

$$\mathcal{N} = \{y \in X \mid y = op \circ op^{'}(x) \text{ with } op, op^{'} \in \{id, op_1, op_2\}\}$$

Rice framework for algorithm selection

Algorithm selection



Rice, J. R. (1976). The algorithm selection problem. Advances in computers, 15, 65-118. [Ric76]

Position of fitness landscape analysis

Selection of local search algorithm

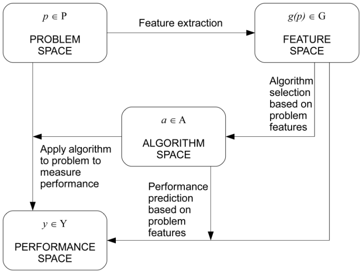


Figure 1.1: A framework for describing the general problems of algorithm selection and performance prediction based on problem features (based Rice's model [132]).

Malan, K. M., Engelbrecht, A. P. (2014). Fitness landscape analysis for metaheuristic performance prediction. In Recent advances in the theory and application of fitness landscapes (pp. 103-132). [ME14]

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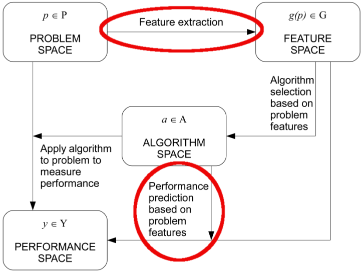

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Fitness landscape analysis : features extraction *vs.* performance

Fitness landscape analysis

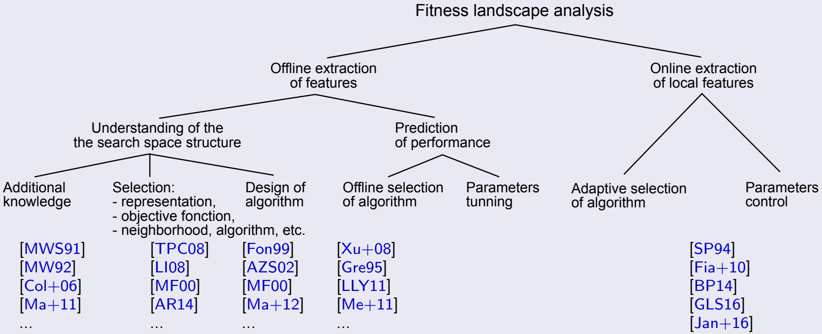
Algebric approach, grey-box :

$$\Delta f = \lambda.(f - \bar{f})$$

Statistical approach, black-box :

Problems $\rightsquigarrow$ Features $\rightsquigarrow$ Algorithm $\rightsquigarrow$ Performances

Goals



Typical use case of fitness landscapes analysis

1. **To compare the difficulty of two search spaces** :
   - One problem, different codings : $(X_1, \mathcal{N}_1, f_1)$ and $(X_2, \mathcal{N}_2, f_2)$ different coding, mutation operator, objective function, etc.

Which one is easier to solve ?

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   - analysis of global geometry of the landscape

   Which algorithm can I use ?

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3. **To tune the parameters** :
   - *off-line* analysis of structure of fitness landscape

     Which is the best mutation operator ? the size of the population ? number of restarts ? etc.

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3. **To tune the parameters** :
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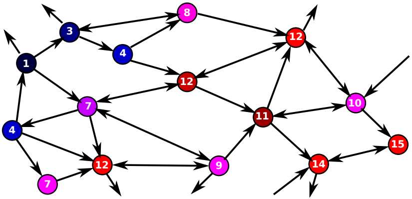
   Which is the best mutation operator ? the size of the population ? number of restarts ? etc.
4. **To control the parameters during the run** :
   - *on-line* analysis of structure of fitness landscape

   Which is the optimal mutation operator according to the estimation of the structure ?

Back to definition

Fitness landscape $(X, \mathcal{N}, f)$ is :

an oriented **graph** $(X, \mathcal{N})$ with valuated nodes given by f.



Remarks :

- Model of the search space
- Non specific to a particular local search

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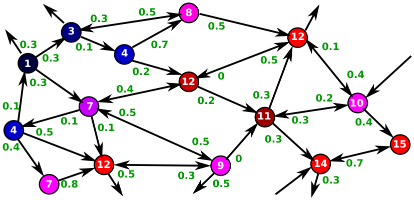


Remarks :

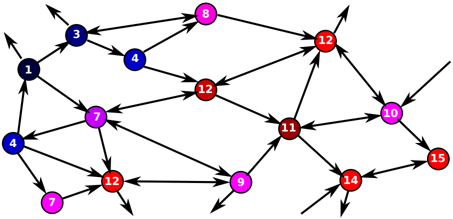
- Model of the search space
- Non specific to a particular local search
- A specific local search puts probability transitions on **edges** according to f and *history* of the search

Fitness landscape and complex systems

Complex system : local *vs.* global properties

- Sample the neighborhood to have information on **local features** of the search space
- From local information : deduce **global feature** such as general shape, difficulty, performance, best algorithm, etc.

$\Rightarrow$ Analysis using complex systems tools



Short summary of this part

Study of the **structure** of the fitness landscape
allows to study the **difficulty**, and
allows to **design** good optimization algorithms

Fitness landscape is a **graph** $(X, \mathcal{N}, f)$:

- nodes are solutions which have a value (fitness),
- edges are defined by the neighborhood relation.

pictured as a real landscape

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Next section : study of the two main geometries

- multimodal and ruggedness
- neutral

References I

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Efficient multi-objective higher order mutation testing with genetic programming.
Journal of systems and Software, 83(12) :2416–2430, 2010.

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