Landscapes and Other Art Forms.

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Blind No More = GRAY BOX Optimization

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With Thanks to: Francisco Chicano, Gabriela Ochoa, Andrew Sutton and Renato Tinós

Know your Landscape! And Go Downhill!

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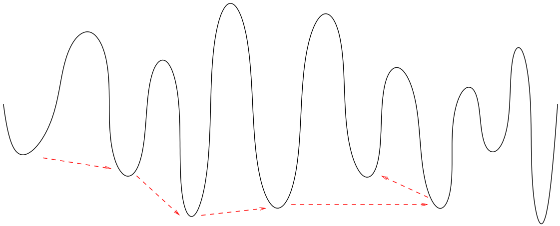
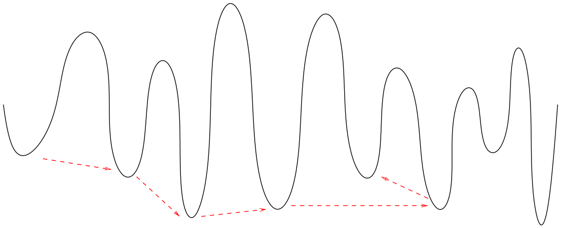
Tunneling Between Local Optima



What if you could ...



“Tunnel” between local optima on a TSP,
or on an NK Landscape or a MAXSAT problem.

Tunneling = jump from local optimum to local optimum

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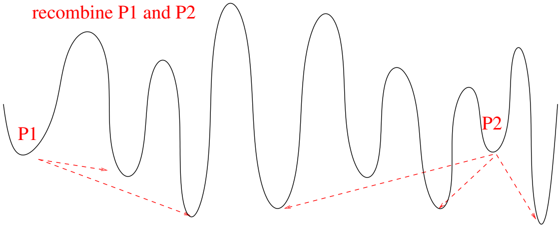


“Tunnel” between local optima on a TSP,
or on an NK Landscape or a MAXSAT problem
AND go the BEST reachable local optima!

Tunneling = jump from local optimum to local optimum

The Partition Crossover Theorem for TSP

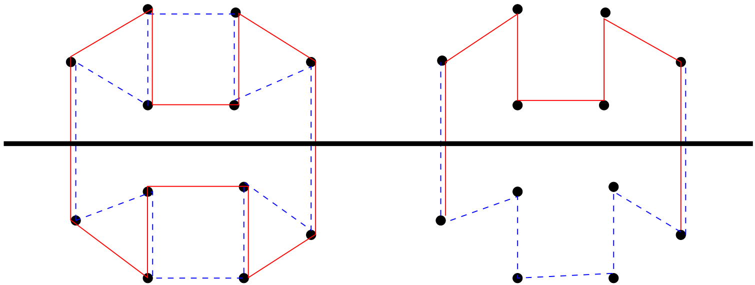
Let G be a graph produced by unioning 2 Hamiltonian Circuits.

Let G' be a reduced graph so that all common subtours are replaced by a single surrogate common edge.

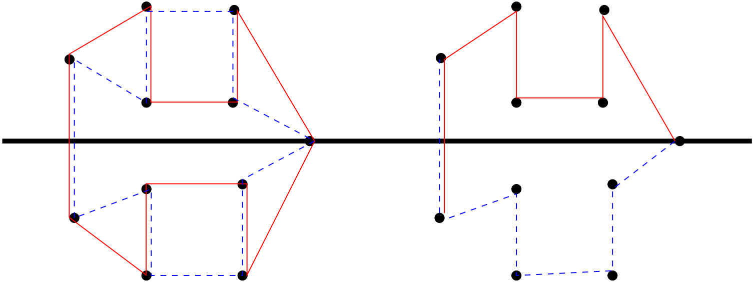
If there is a partition of G' with cost 2, then the 2 Hamiltonian Circuits that make up G can be cut and recombined at this partition to create two new offspring.

The resulting Partition Crossover is Respectful and Transmits alleles.

(Using G' makes the proof easier, but is not necessary.)



As a side effect: f(P1) + f(P2) = f(C1) + f(C2)



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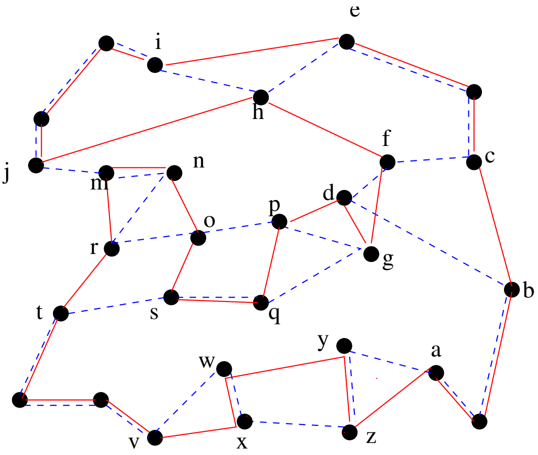
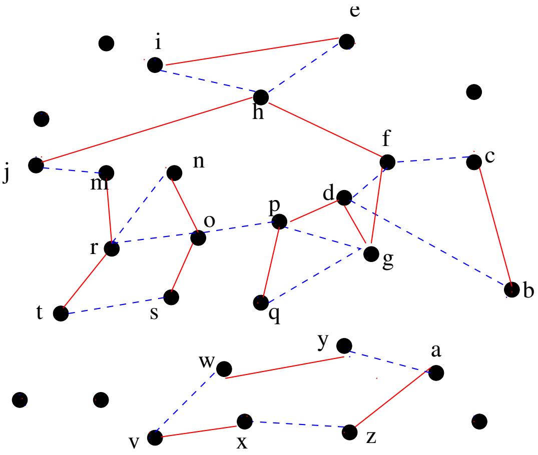
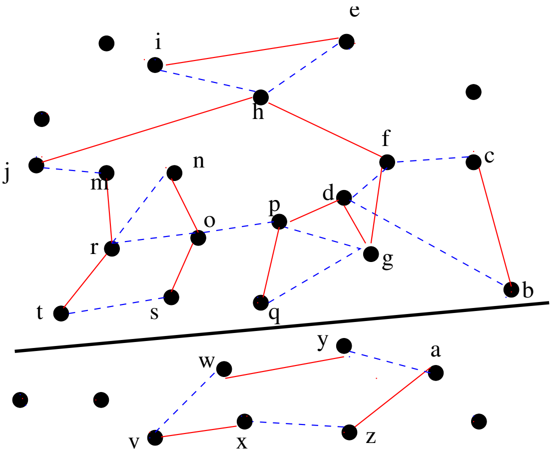
Partition Crossover



Partition Crossover

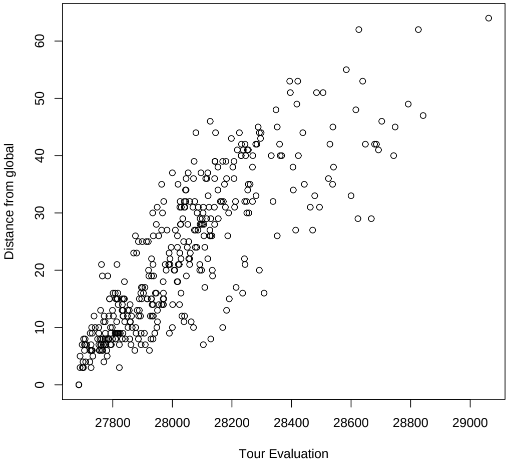


Partition Crossover in O(N) time



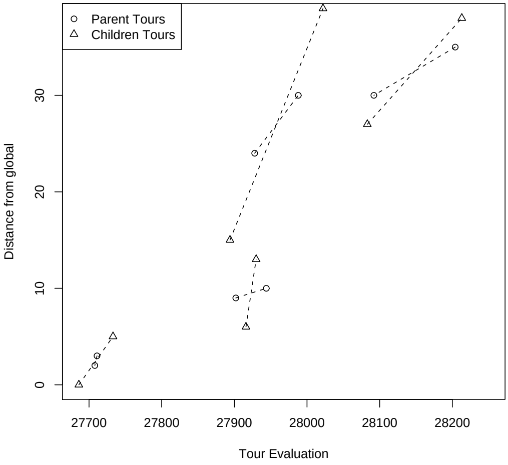
The Big Valley Hypothesis

is sometimes used to explain metaheuristic search

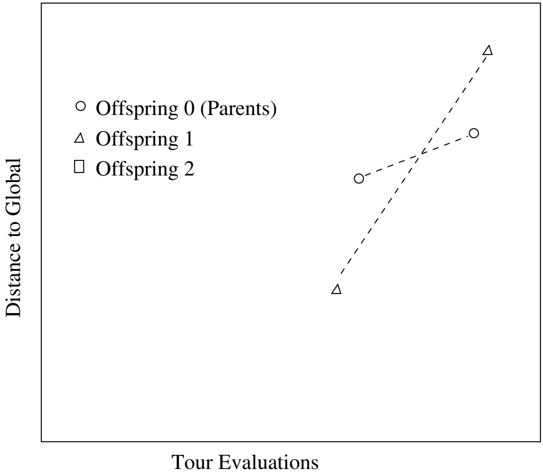


Tunneling Between Local Optima

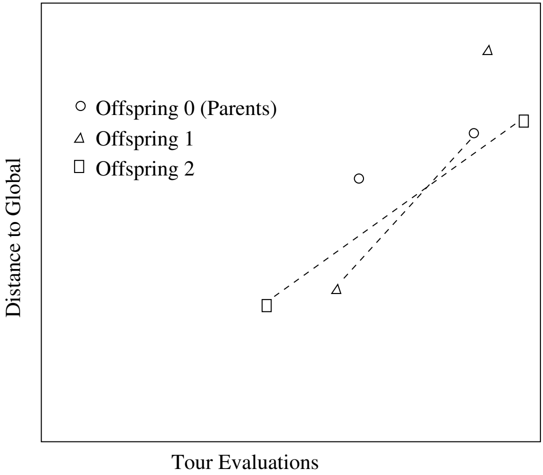
Local Optima are "Linked" by Partition Crossover



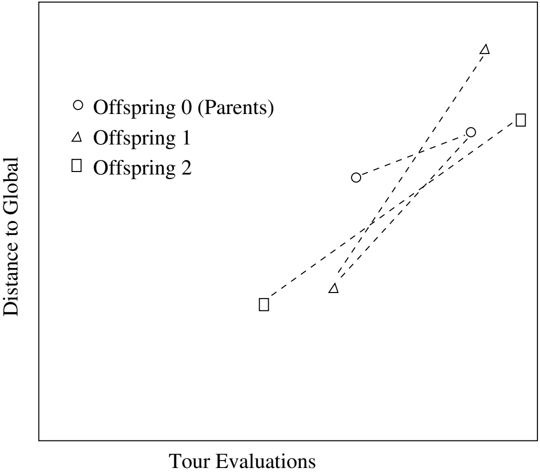
Iterated Partition Crossover



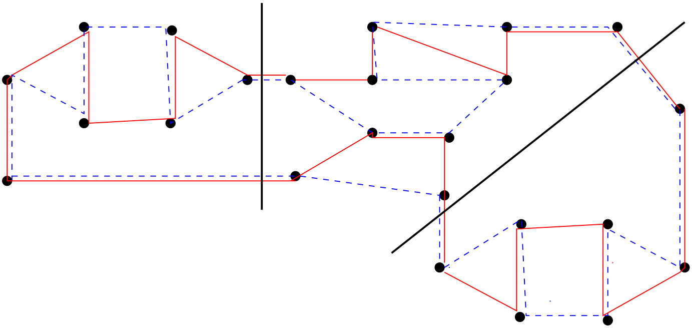
Iterated Partition Crossover



Generalized Partition Crossover



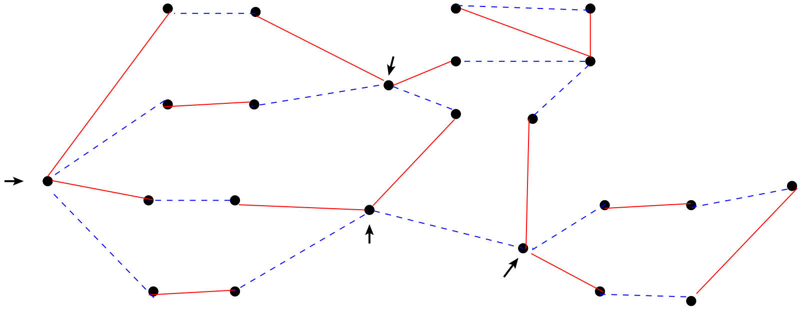
Generalized Partition Crossover



Generalize Partition Crossover is always feasible if the partitions have 2 exits (same color in and out). If a partition has more than 2 exits, the "colors" must match.

This will automatically happen if all of the partitions have cut two.

Generalized Partition Crossover with Splitting



How Many Partitions are Discovered?

Instance	att532	nrw1379	rand1500	u1817
2-opt	3.3 ± 0.2	3.2 ± 0.2	3.7 ± 0.3	5.0 ± 0.3
3-opt	10.5 ± 0.5	11.3 ± 0.5	24.9 ± 0.2	26.2 ± 0.7
LK-search	5.3 ± 0.2	5.2 ± 0.3	10.6 ± 0.3	13.3 ± 0.4

Table: Average number of *partition components* used by GPX in 50 recombinations of random local optima found by 2-opt, 3-opt and LK-search.

With 25 components, 2^{25} represents millions of local optima.

Lin-Kernighan-Helsgaun-LKH

LKH is widely considered the best Local Search algorithm for TSP.

LKH uses deep k-opt moves, clever data structures and a fast implementation.

LKH-2 has found the majority of best known solutions on the TSP benchmarks at the Georgia Tech TSP repository that were not solved by complete solvers: http://www.tsp.gatech.edu/data/index.html.

THE BEST HEURISTIC TSP SOLVERS USE CROSSOVER!

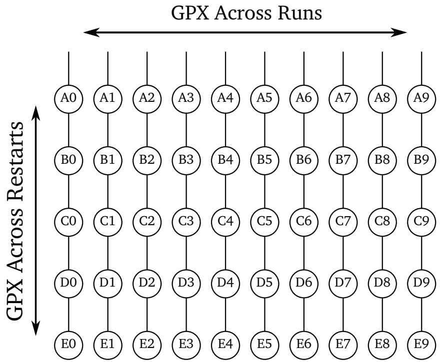
LKH uses "Iterated Partial Transcription"
which is almost the same as GPX but less efficient.

Iterative Partial Transcription and GPX

Instance	C10k.0	C10k.1	C31k.0	C31k.1
LKH-2 no crossover	1.143	1.009	1.489	1.538
LKH-2 w IPT	1.040	0.873	1.280	1.274
LKH-2 w GPX	**1.031**	**0.872**	**1.270**	**1.267**

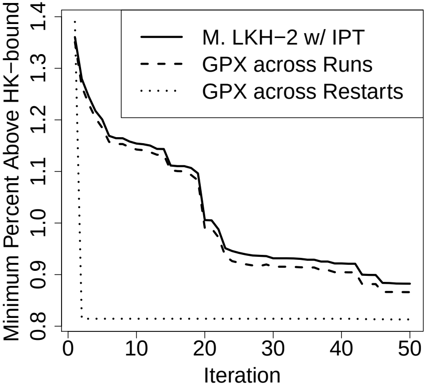
The minimum percentage above the Held-Karp Bound for several clustered instances of the TSP of solutions found by ten random restarts of LKH-2 without crossover, with IPT and with GPX. Best values for each instance are in boldface. Sizes range from 10,000 to 31,000 cities.

GPX Across Runs and Restarts



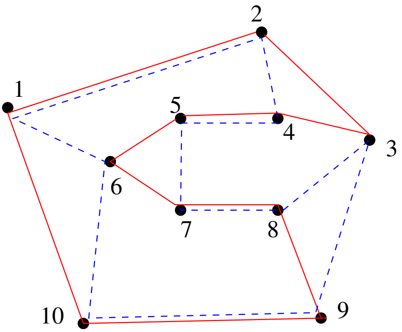
A diagram depicting 10 runs of multi-trial LKH-2 run for 5 iterations per run. The circles represent local optima produced by LKH-2. GPX across runs crosses over solutions with the same letters. GPX across restarts crosses over solutions with the same numbers.

GPX on Clustered Problems

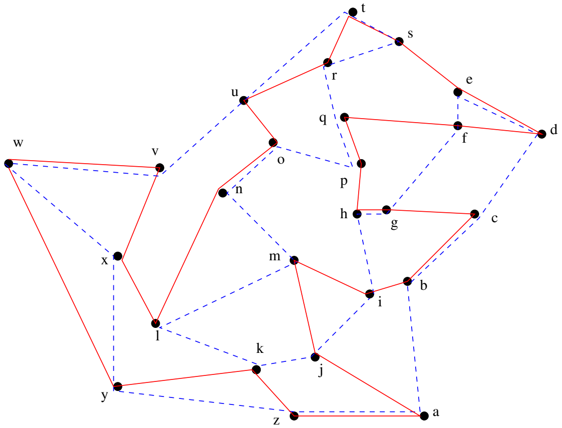


Improvement over time on a 31,000 city Dimacs Clustered Instance.

GPX, Cuts Crossing 4 Edges (IPT fails here)



GPX, Complex Cuts



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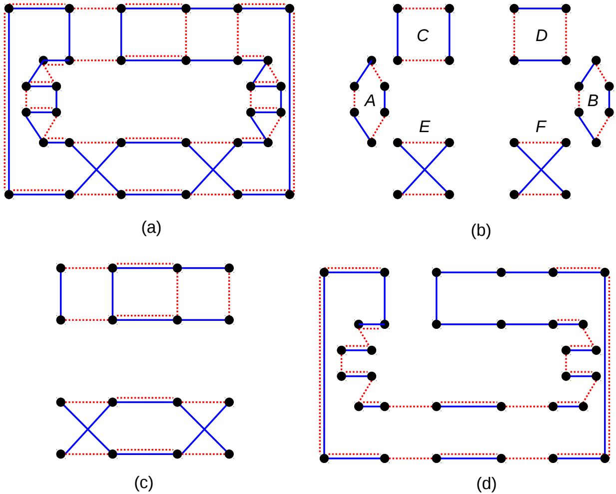

(a) (b)

(c) (d)