Statistical Measurement of Information Leakage

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Introduction

- Information theory for security and anonymity.
- Estimate information leakage from trial runs of a real system.
  - Automatic tool to calculate information leakage.
  - We present an ***if, and only if***, test for ***zero*** leakage.
- Examples: The Mixminion anonymous e-mail systems and e-Passports.

Anonymity Set

- Measure of anonymity is the size of the set of possible IDs
- If you know its one of 10 people set size = 10
- If are 99.9999% sure it’s one person, but it may also be 1 of 99 others then set size = 100 !!

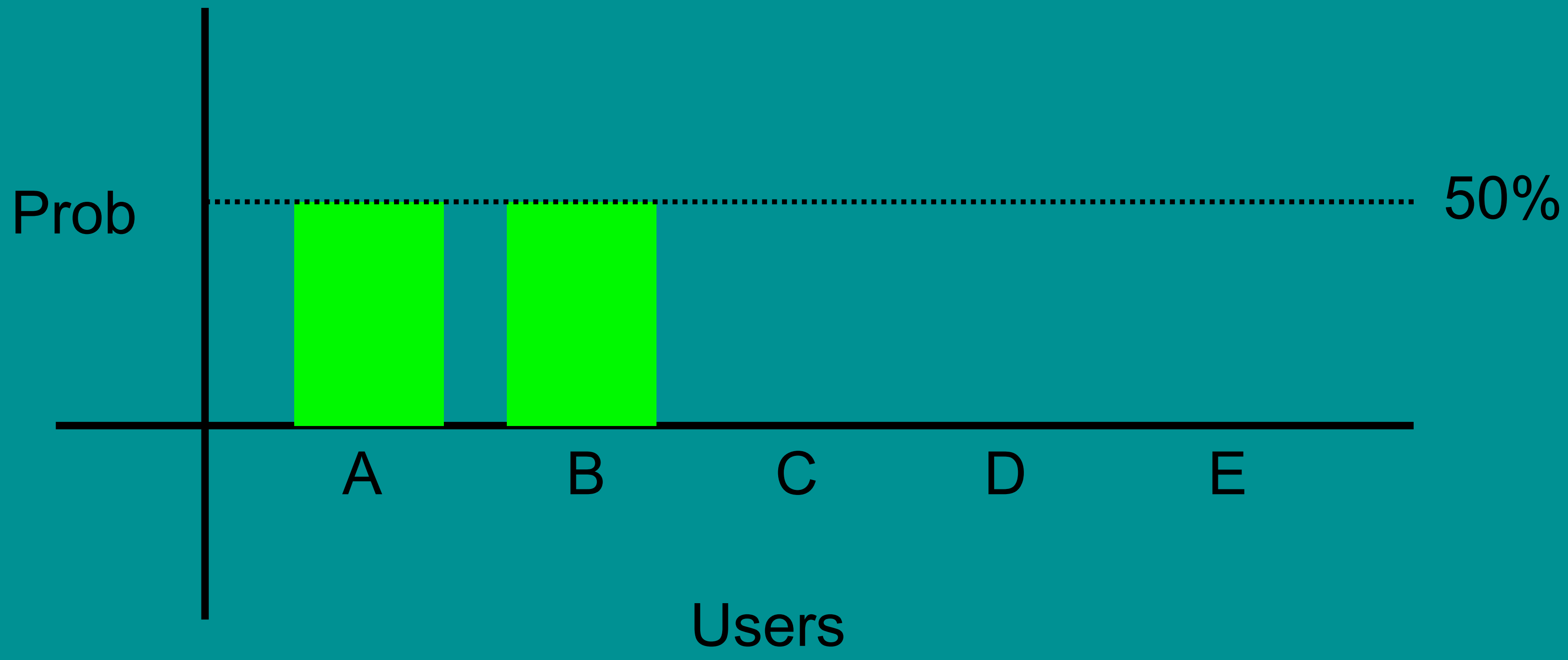
Levels of Anonymity

Reiter and Rubin provide the classification:

- ***Beyond suspicion***: the user appears no more likely to have acted than any other.
- ***Probable innocence***: the user appears no more likely to have acted than to not to have.
- ***Possible innocence***: there is a nontrivial probability that it was not the user.

Probable Innocence

- After a run of the system, who does the observer think did it?



Probable Innocence

- After a run of the system, who does the observer think did it?

Information Theory

$$H(X) = -\sum_{x \text{ in } X} p(x).\log(p(x))$$

Entropy describes the "amount of chaos" or uncertainty in a system

= the number of bits needed to describe the system, on average.

The Horse Race Example

- Race 1: 8 horses, all equally likely to win the race ... very chaotic.

$$H(X) = -(0.125.\log(0.125) +...+)0.125.\log(0.125))$$
$$= \log(0.125) = 3$$

i.e. on average you need three bits of information to send the identify of the winner.

The Horse Race Example

Race 2: 8 horse:

- P(horse1 wins) = 1/2
- P(horse2 wins) = 1/4
- P(horse3 wins) = 1/8
- P(horse4 wins) = 1/16
- P(horse5 wins) = 1/64
- P(horse6 wins) = 1/64
- P(horse7 wins) = 1/64
- P(horse8 wins) = 1/64

The result is much more predicable, much less chaos.

H(X) = 1/2.log(1/2)
+ 1/4.log(1/4) +
....+1/64/.log(1/64)
= 2

i.e. on average we need to send 2 bits

1->0, 2->10, 3->110,
4->1110, 5->111100,
6->111101, 7->111110
8->111111

Information Theory

$$H(X) = -\sum_{x \text{ in } X} p(x).\log(p(x))$$

Entropy describes the “amount of chaos” or uncertainty in a system

= the number of bits needed to describe the system, on average.

A Metric For Anonymity

- The entropy of the set of possible people.
- For anonymity proposed independently in 2002 by Daiz et al. and Danezis et al.
- But what about
  - user actions?
  - attack has some prior knowledge?
  - some users are more likely to be guilty than others?

Conditional Entropy

Conditional Entropy $H(Y|X)$ is the remaining chaos in Y once you know X:

$$H(Y|X) = \sum_x p(x).\ H(Y|X=x)$$
$$= -\sum_{x,y} p(y,x).\log(\ p(y|x)\ )$$

- if you're sending X then $H(Y|X)$ is the average no. of bits needed to send Y as well.
- Proposed for security by McIver and Morgan in 2003

Mutual Information

Mutual Information $I(X;Y)$ is the reduction of uncertainty you get in X if you know Y.

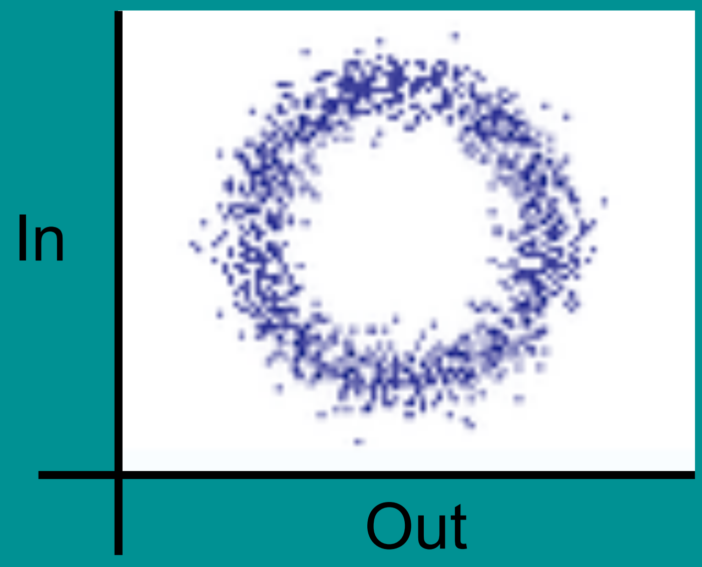
$$I(X;Y) = H(X) - H(X|Y)$$
$$= H(Y) - H(Y|X)$$

If W gives the conditions probabilities of Y given X we also write:

$$I(Q,W) = I(Q;QW)$$
$$= \sum_{x,y} Q(x).W(y|x).\log(W(y|x) / QW(y))$$

Mutual Information vs. Correlation

- Mutual Information measures any link in the data sets.
- Correlations only measures a linear link.



Corr = 0 M.I. ≠0

Conditional Mutual Information

- Mutual Information can be conditional:

$$I(L_1;H \mid L_2) = H(L_1 \mid L_2) - H(L_1 \mid H,L_2)$$

This is the information you learn about H from L1, given you know L2.

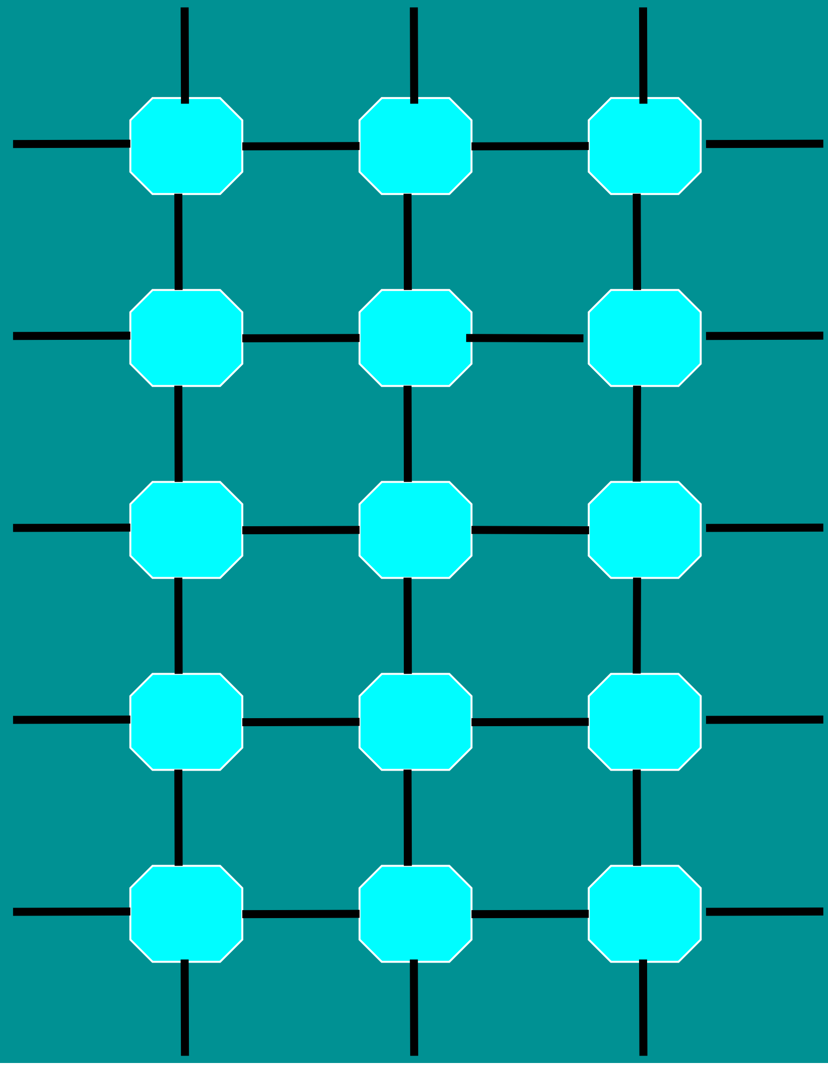
Used by Clark et al. for security.

Assuming a Uniform Distribution Doesn’t Work

Imagine a network of peers.

Each peer randomly picks a nieghbour to act as a proxy for each message.

Communication between peers is undetectable.

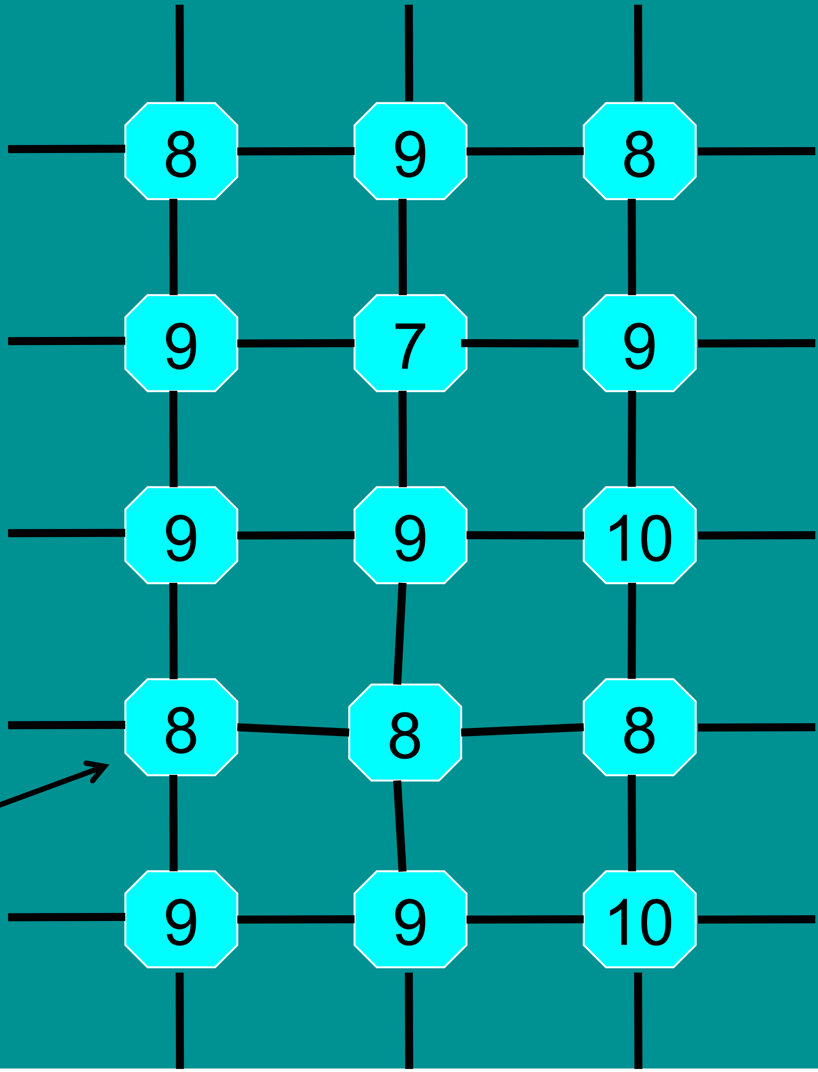


Assuming a Uniform Distribution Doesn't Work

We can observe the messages leaving each peer.

If each peer sends uniformly then we have a 1 in 4 change of guessing the sender

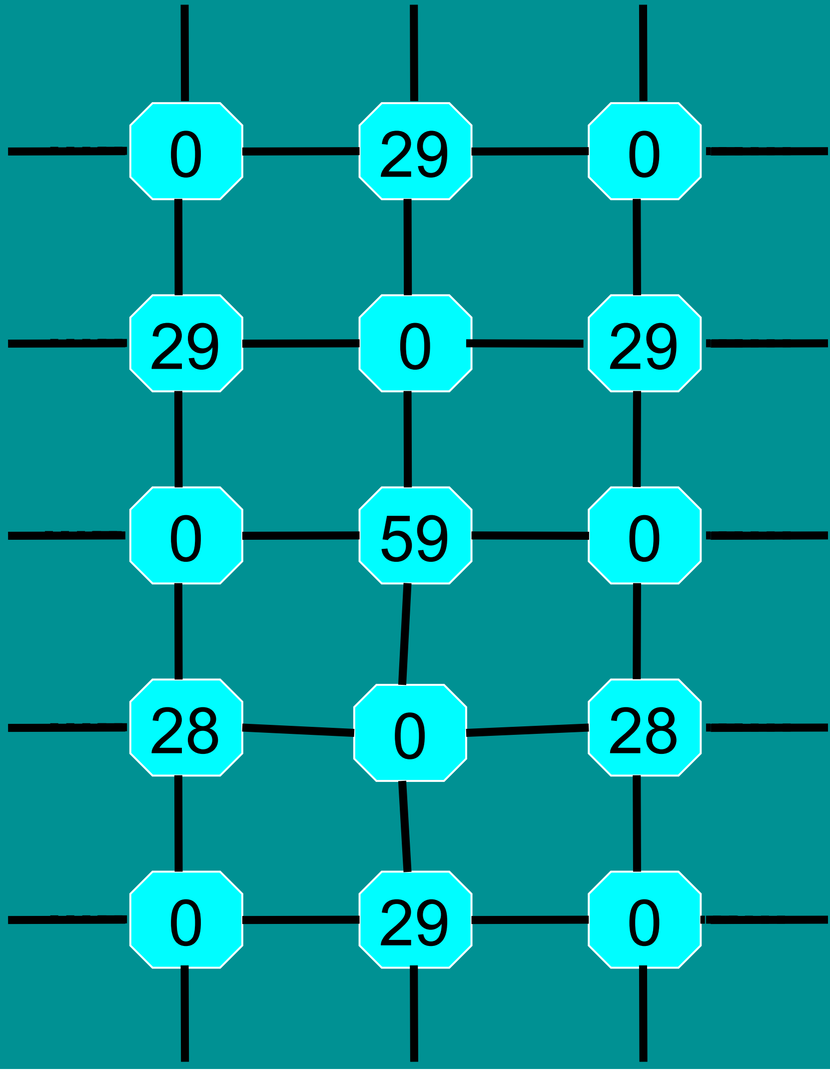
Number of observed messages from node



Assuming a Uniform Distribution Doesn't Work

But if the distribution isn't uniform then a sender has nowhere near this anonymity.

In the worst case the anonymity is zero



Channel Capacity

For a channel:

I(Inputs;Output) = how much the outputs tell you about inputs

The most information it is possible to send over a channel

$$C = \text{Max}_{\text{Inputs}} \; I(\text{Inputs};\text{Outputs})$$

Information Leakage = Capacity (System)

Following Chatzikokolakis et al., Millen, Clark et al. etc.

- Think of the whole system as a channel.
  - secret is the input to the “channel”.
  - observables are the outputs from the “channel”.
- Capacity tells us what an attacker can learn about the users from the observable actions.

Information Theory

Entropy: $H(X) = - \sum_x p(x).\log(p(x))$

the amount of uncertainty in X.

Conditional Entropy: $H(Y|X) = \sum_x p(x).H(Y|X=x)$

the amount of uncertain in Y if you know X

Mutual Information: $I(X;Y) = H(X) - H(X|Y)$

the reduce of uncertainty you get in X if you know Y.

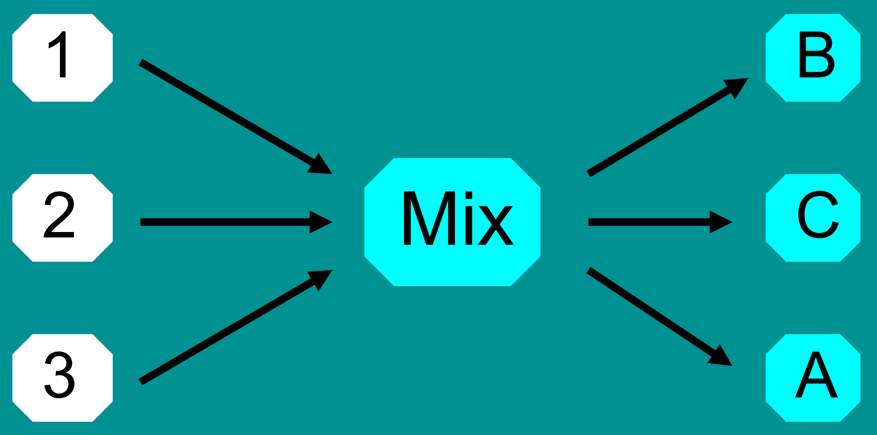
Relative Entropy: $D(p||q) = \sum_x p(x).\log(p(x) / q(x))$

"distance" from one distribution to another.

$$I(p(x),p(y)) = D(p(x,y) \,||\, p(x).p(y))$$

MIXes

- MIXes are proxies that forward messages between them
- The MIX waits until it has received a number of messages, then forwards them in different order.
- E.g. 1 wants to send to A, 2 to B and 3 to C



A Perfect Mix

Message orderings	out A,B,C	out A,C,B	out B,A,C	out B,C,A	out C,A,B	out C,B,A
in 1,2,3	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666
in 1,3,2	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666
in 2,1,3	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666
in 2,3,1	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666
in 3,1,2	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666
in 3,2,1	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666

(a) Probabilites of outputs for each input for a perfect mix node

Information Leakage = Capacity = 0

A Bad Mix

Message orderings	out A,B,C	out A,C,B	out B,A,C	out B,C,A	out C,A,B	out C,B,A
in 1,2,3	0	0.3333	0.3333	0	0	0.3333
in 1,3,2	0.3333	0	0	0.3333	0.3333	0
in 2,1,3	0.3333	0	0	0.3333	0.3333	0
in 2,3,1	0	0.3333	0.3333	0	0	0.3333
in 3,1,2	0	0.3333	0.3333	0	0	0.3333
in 3,2,1	0.3333	0	0	0.3333	0.3333	0

(b) Probabilites of outputs for each input for a flawed mix node

Information Leakage = Capacity = 1 bit

Applying this to Real Systems

- How do we apply information theoretic measures to real systems?
- Leak may be caused by the implementation:
  - Time based attack on RSA (Paul Kocher)
  - Bandwidth attack on Tor (Murdoch & Danezis)
  - CPU Heat attack on Tor Hidden services (Murdoch)

Mixminion: A Type III Anonymous Remailer

Mixminion is the reference implementation of the Type III Anonymous Remailer protocol.

Documentation

The Design Document gives our justifications and security analysis for the Mixminion design:

- PostScript version
- PDF version
- LaTeX source
- BibTeX file
- Roger's design overview slides (PDF)

The Specification aims to give developers enough information to build a compatible version of Mixminion:

- Part 1: Mix Protocol Specification
- Part 2: End-to-end Encoding and Delivery
- Part 3: Mix Directory Specifications
- Addendum: Unresolved specification issues

- Draft nymserver specification (Preliminary version)
- Draft C Client API specification (Preliminary version)

Prob. Observed from Mixminion Node

Message orderings	out A,B,C	out A,C,B	out B,A,C	out B,C,A	out C,A,B	out C,B,A
in 1,2,3	0.0	0.0118	0.0473	0.0118	0.0059	0.9231
in 1,3,2	0.0117	0.0	0.0351	0.0292	0.0	0.924
in 2,1,3	0.005	0.0222	0.0278	0.0444	0.0056	0.8944
in 2,3,1	0.0060	0.012	0.0301	0.0361	0.0060	0.9096
in 3,1,2	0.0067	0.0133	0.04	0.02	0.0067	0.9133
in 3,2,1	0.0061	0.0122	0.0549	0.0244	0.0061	0.8963

Fig. 2. Probabilities of the Message Ordering from Mixminion Experiments

Cover & Thomas: Ways to Finding Capacity

- A “gradient climb” algorithm e.g. Frank-Wolfe.
- Kuhn-Tucker Theorem/Lagrange multipliers.
- The Blahut-Arimoto algorithm

Blahut-Arimoto Algorithm

How do we find the maximising input distribution?

$I(X;Y) = H(X) - H(X|Y)$

$= \sum_{x,y} p(x)\ W(y|x) \log \left(W(y|x)\ /\ \sum_{x'} p(x')W(y|x)\right)$

$= \sum_{x} p(x).D\left(\ W(\_|x)\ \|\ \sum_{x'} p(x')W(\_|x')\ \right)$

$= \sum_{x} p(x).D_x(W\|pW)$

Distribution of y given x

Distribution of y

$\sum_x p(x).D_x(W\|pW) \leq C(W) \leq \mathrm{Max}_x\ D_x(W\|pW)$

Blahut-Arimoto Algorithm.

1) Guess an input distribution $p^0(a)$ e.g., uniform

2) Improve the guess, for all x:

$$p^{n+1}(x) = \frac{\exp(\, D_x(W \,||\, p^n W)\,)}{\sum_{x'} \exp(\, D_{x'}(W \,||\, p^n W)\,)}$$

3) Repeat until $I(p^n,W) - \mathrm{Max}_x\, D_x(W||p^n W) < e$

Can be tweaked for super linear convergence, conditional mutual information etc.

Accelerating Blahut-Arimoto

- Each iteration BA takes a step closer to the maximising distributions.

- Take bigger steps if we are further way :

$$p^{n+1}(x) = \frac{\exp(\, u_n D_x(W \,||\, p^n W)\,)}{\sum_{x'} \exp(\, u_n D_{x'}(W \,||\, p^n W)\,)}$$

$$U_n = D(\, p^n W || p^{n-1} W) \,/\, D(\, p^n || p^{n-1})$$

Method of Analysing Anonymity

- To analyse a system we define the inputs and outputs.
  - Some abstraction might be needed to make the number of observations manageable
- We run tests of the system for each input.
- From these tests we estimate a matrix.
- We estimate capacity, from the matrix.

Terms

Everybody else

W : the matrix of the true system.

$\hat{W}_n$: a matrix estimated from n samples.

Q : the input dist. that maximise M.I.

$\hat{Q}_m(\hat{W}_n)$: the B.A. algorithm applied to W_n.

$C = I(Q,W)$: the true system capacity.

$\hat{C}_{n,m} = I(\hat{Q}_m(\hat{W}_n),\hat{W}_n)$: estimate of capacity ??

Us

Prob. Observed from Mixminion Node

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Fig. 2. Probabilities of the Message Ordering from Mixminion Experiments

Observation from a running Mixminion Node

Message orderings	out A,B,C	out A,C,B	out B,A,C	out B,C,A	out C,A,B	out C,B,A
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Fig. 2. Probabilities of the Message Ordering from Mixminion Experiments

Leakage = 0.0249 bits

Convergence

Theorem: $\hat{C}_{n,m}$ almost surely convergences to C as $n,m \rightarrow \infty$

i.e., for any p_e and error e there exists n‘ & m’ such that for n > n’ and m > m’:

$$p(| C - \hat{C}_{n,m} | > e) < p_e$$

The Distribution of Anonymity

We can get bounds on the error by ask what distribution $\hat{C}_{n,m}$ comes from.

Adapting a statistical method from Rao:

- We find the Taylor expansion of the $\hat{C}_{n,m}$
- We drop the terms smaller than sampleSize^{-2}
- We then calculate the mean and variance.
- We find the distribution using the central limit theory.

Estimated Value

As we can't find the distribution for the maximising distribution we relate our estimate to $I(\hat{Q}_m(\hat{W}_n),W)$

Lemma: The estimate

- is less than or equal to the capacity,
- equals zero if, and only if, the capacity equals zero.

Expectation and Variance

To find a distribution we need to find the expectation:

E(X): the average value

And the variance:

$$\text{Var}(X) = E(\text{mean} - x)^2$$

What We Know

K_{ij} is the number of times the pair (i,j) shows up in our test.

Let the true prob: $p(i,j) = {}^{h}K_{ij}/n$

Then maximum likehood tells us that

- $E(K_{ij} - {}^{h}K_{ij}) = 0$
- $E((K_{ij} - {}^{h}K_{ij})^2) = p(i).\ W(j|i)(1-W(j|i))$
- $E((K_{ij} - {}^{h}K_{ij})^3) = K_{ij}(2W(j|i)^2-3W(j|i)+1)$ ...

Taylor's Theorem

To find the value of a function at x (near a):

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f''(a)(x-a)^3}{3!} + \ldots$$

We take I(X,_) as "f", W_n as "x" and W as "a" to give get an expression for the estimate in terms of the true value.

Taylor Expansion of Entropy

$$I_n(X,Y) = H(X) + H_n(Y) - H_n(X,Y)$$
$$E(I_n(X,Y)) = E(H(X)) + E(H_n(Y)) - E(H_n(X,Y))$$

$$H(X,Y) \quad = - \sum_{x,y} p(x,y)\log(p(x,y))$$
$$H_n(X,Y) = - \sum_{x,y} K_{ij}/n.\log(K_{ij}/n)$$
$$H_n(X,Y) \quad = -\sum_{x,y} {}^hK_{ij}/n - 1/n.\sum_{x,y}(1+ {}^hK_{ij}/n)$$
$$-\sum_{x,y}(K_{ij}-{}^hK_{ij})^2/n.\ {}^hK_{ij})$$
$$+ \sum_{x,y}(K_{ij}+ {}^hK_{ij})^3/6n.{}^hK_{ij}{}^2) +O(n^{-2})$$

$$E(H_n(X,Y)) \ = \ H(X,Y) - I(J-1)/2n +O(n^{-2})$$

For Non-Zero Mutual Information

When the true value is not 0, an estimation of capacity is drawn from a normal distribution with:

$$\text{Mean: } I(\hat{Q}_m(\hat{W}_n), W) + \frac{(I-1)(J-1)}{2n} + O(n^{-2})$$

I = no. of Inputs, J = no. of Outputs

Variance: ...

Variance

$$\frac{1}{n} \cdot \sum_x Q(x).\left(\sum_y W(y|x).\left(\log\left(\frac{Q(x).W(y|x)}{\sum_{x'} Q(x')W(y|x')}\right)\right)^2 - \left(\sum_y W(y|x).\log\left(\frac{Q(x).W(y|x)}{\sum_{x'} Q(x')W(y|x')}\right)\right)^2\right) + O(n^{-2})$$

When I = 0

- The $O(n^{-1})$ term disappears with X and Y are independent.
- In which case we need to find the $O(n^{-2})$ term.
- Following Rao, we observe when I = 0 :

$$\sum_{ij} \left((K_{ij} - E(K_{ij}))^2 / E(K_{ij}) \right) \sim \chi^2$$

and that this approximates mutual information.

Results for I = 0

When the true value is 0, an estimation of capacity (or mutual information) is drawn from the distribution:

$$2n.I \sim \chi^2((\text{noOfInputs}-1)(\text{noOfOutputs}-1))$$

Mean: $(\text{noOfInputs}-1)(\text{noOfOutputs}-1)/2$

Variance: $(\text{noOfInputs}-1)(\text{noOfOutputs}-1)/2n^2$

Upper Bound on the Variance

- In both cases var(C(W)) < I.J / n

- Rule of thumb:
  - If I.J >> n the variance will be low and the results actuate.

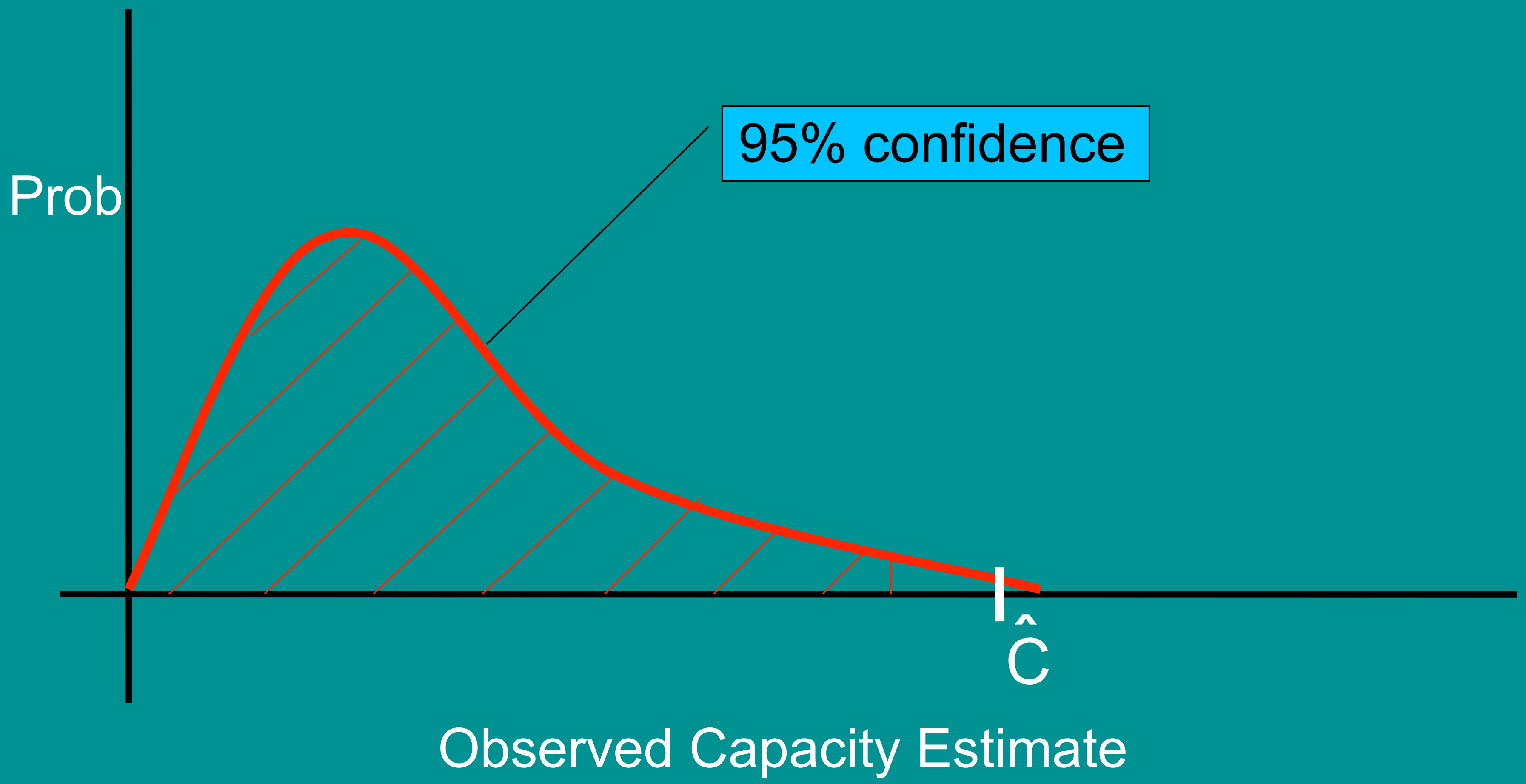
  - If you can get this many samples then statically analysis is useful, otherwise not.

To Analyse a System.

- We define the inputs (I) and outputs (J).
- Run n tests of the system with n >> I.J
- Estimate the matrix and find $\hat{C} = I(\hat{Q}_m(\hat{W}_n),\hat{W}_n)$
- Point Estimate is:

  $$\text{Max}\ (\ 0,\ I(\hat{Q}_m(\hat{W}_n),\hat{W}_n) - (I-1)(J-1)/2n\ )$$

Using the Distributions



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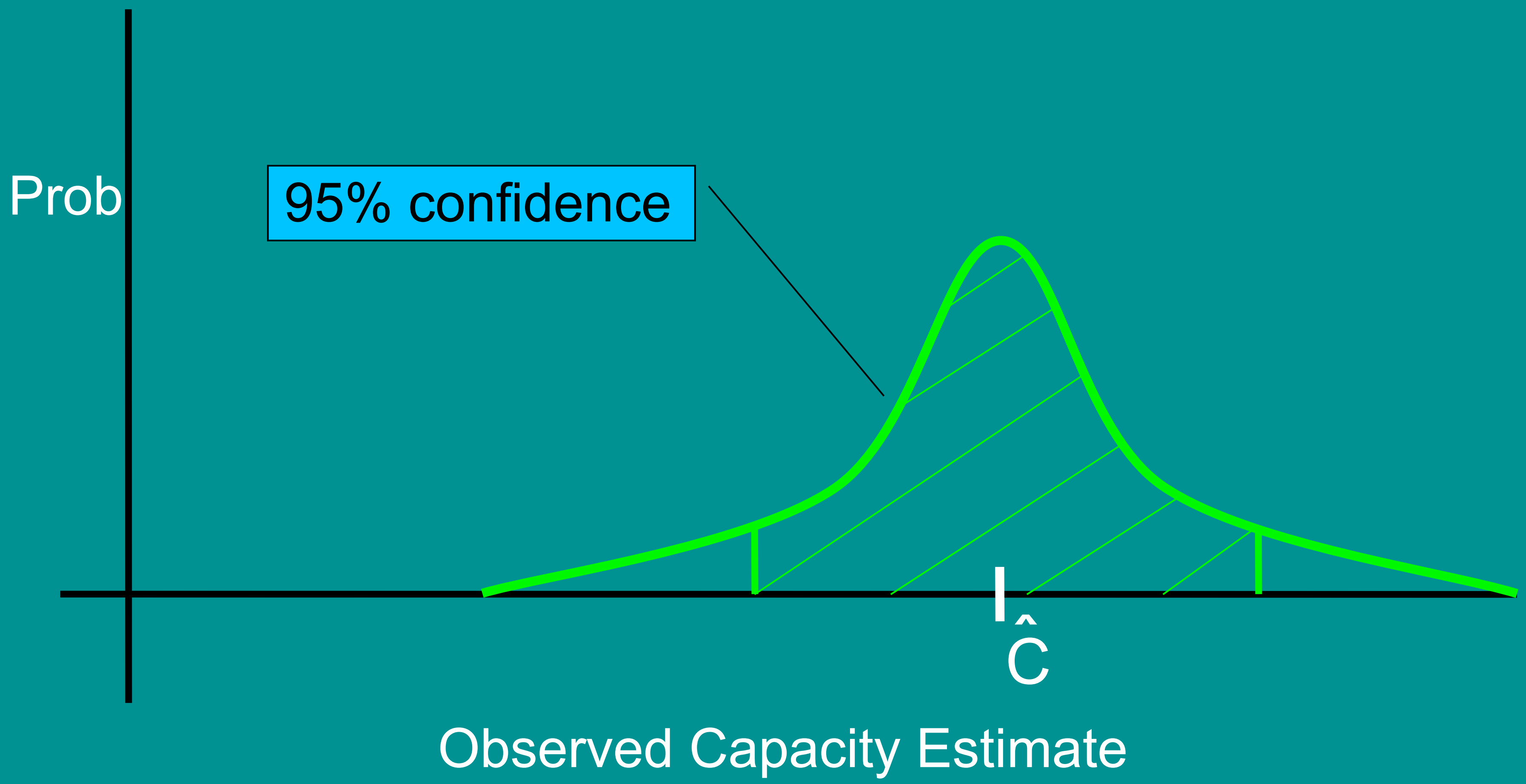

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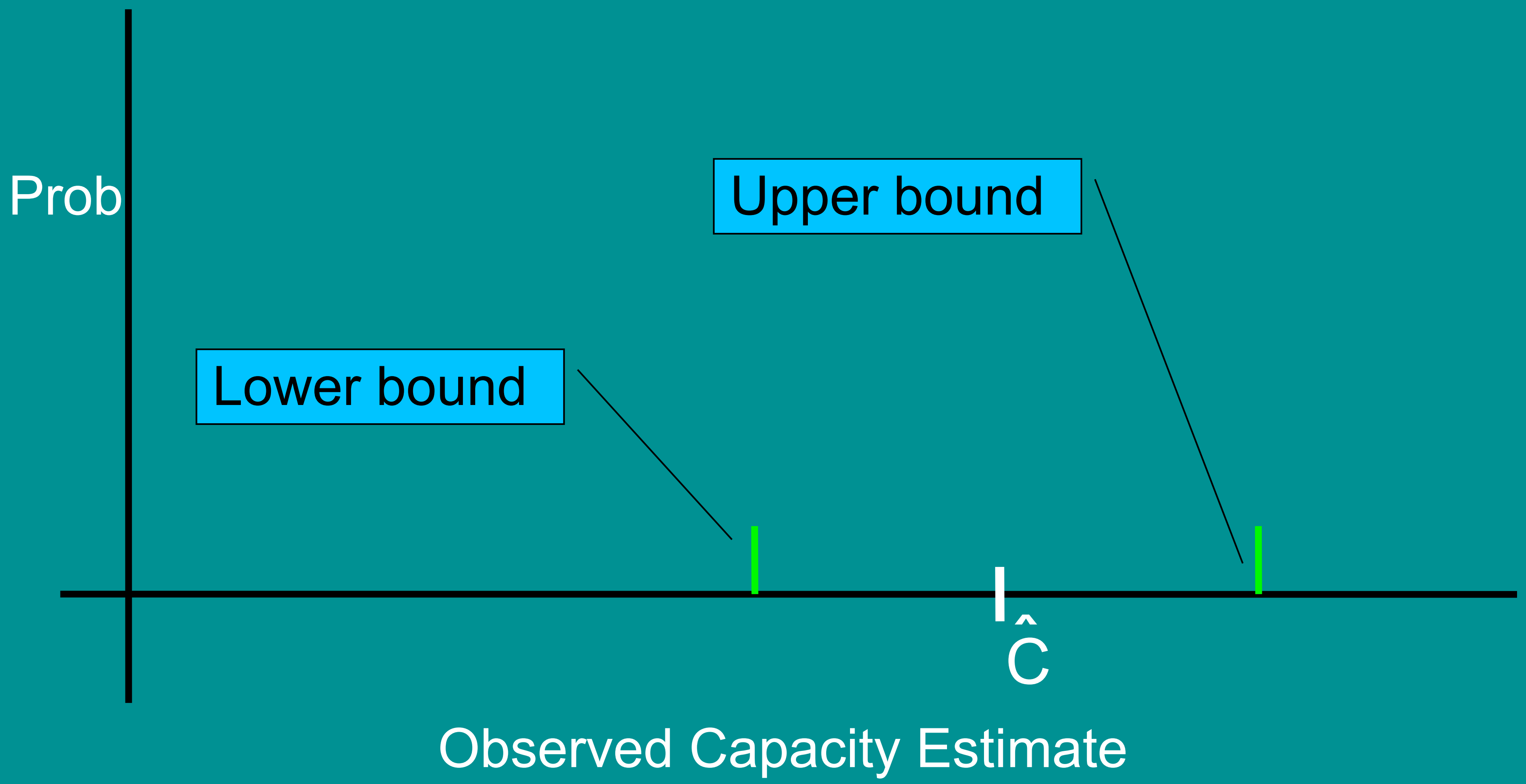

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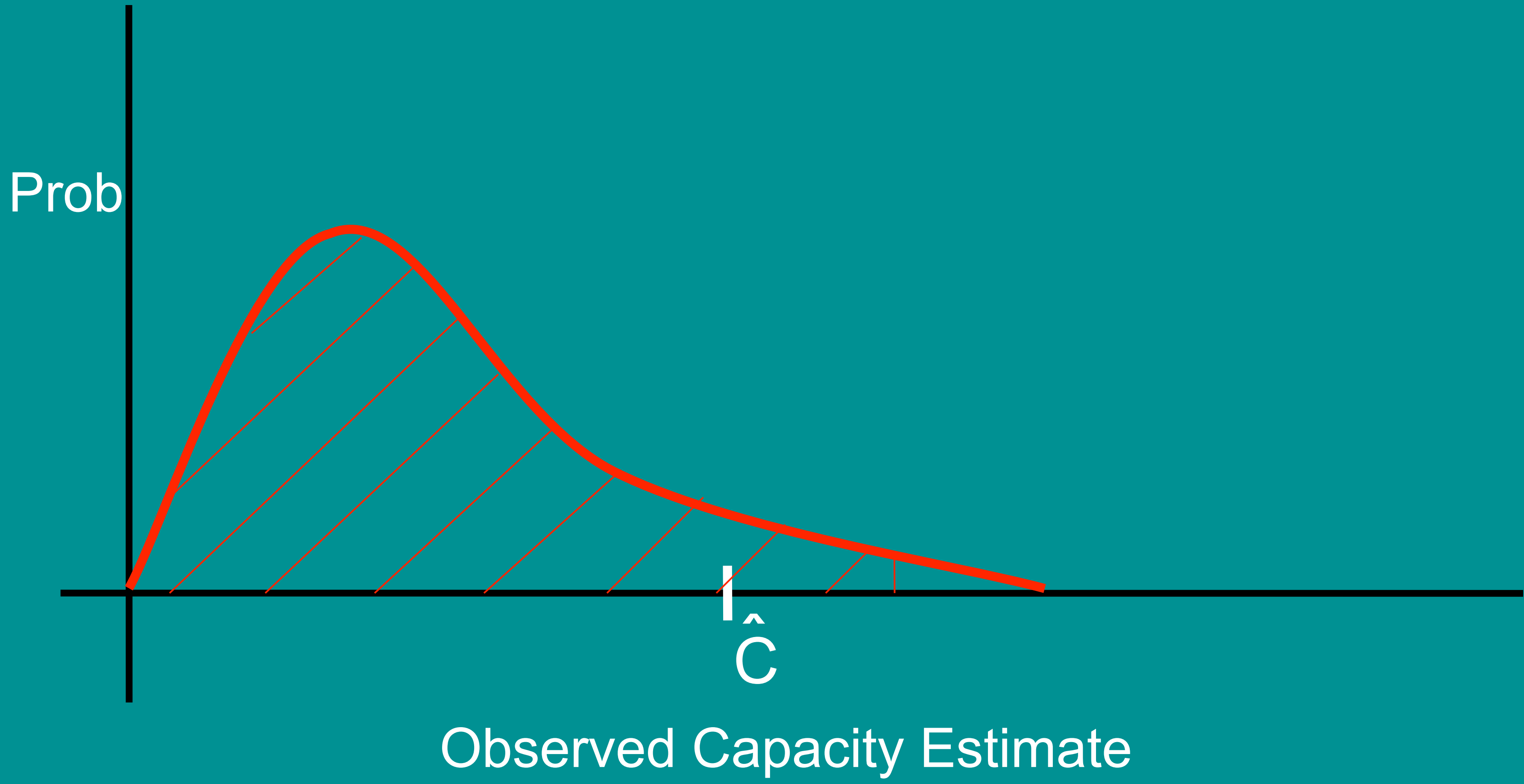

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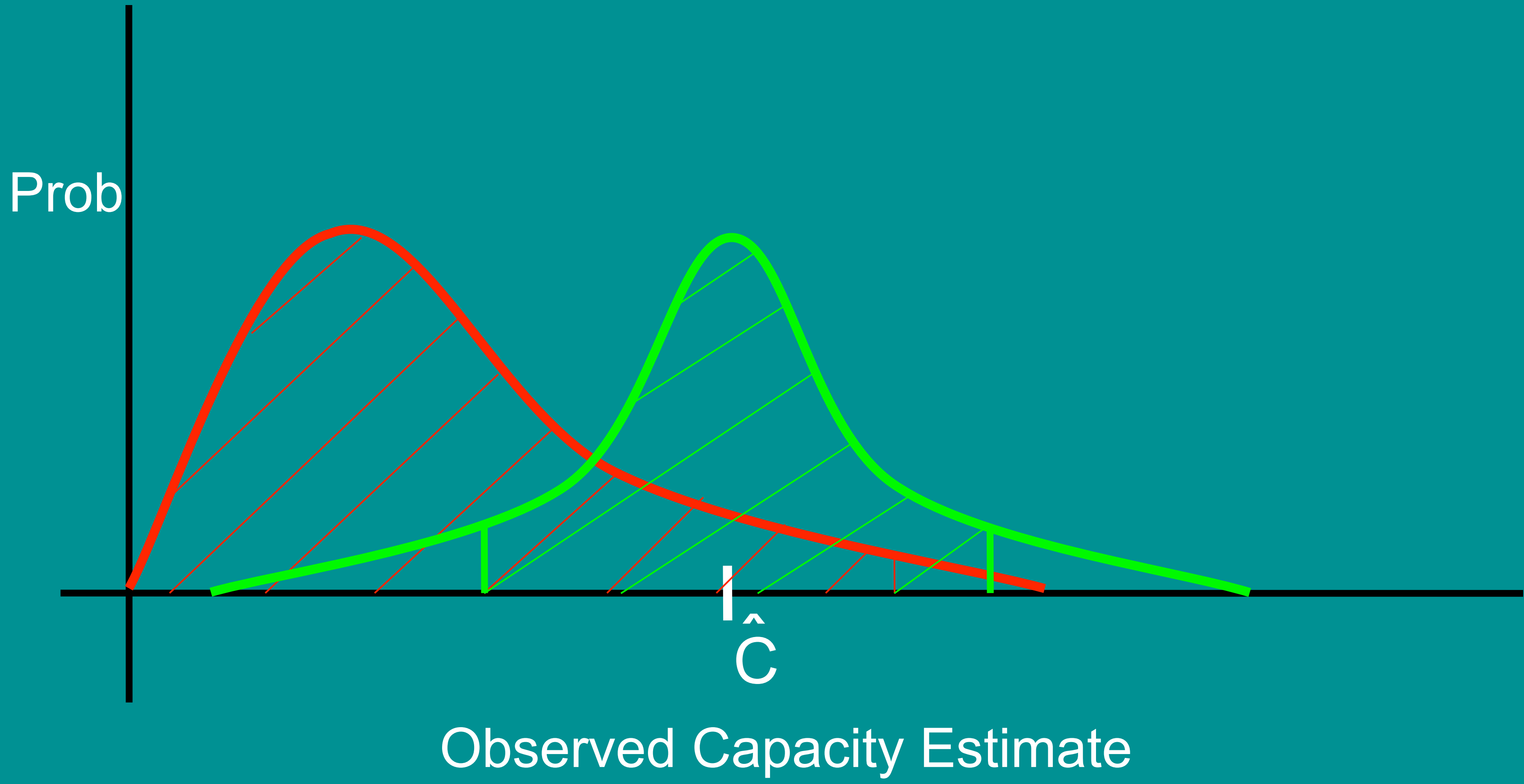

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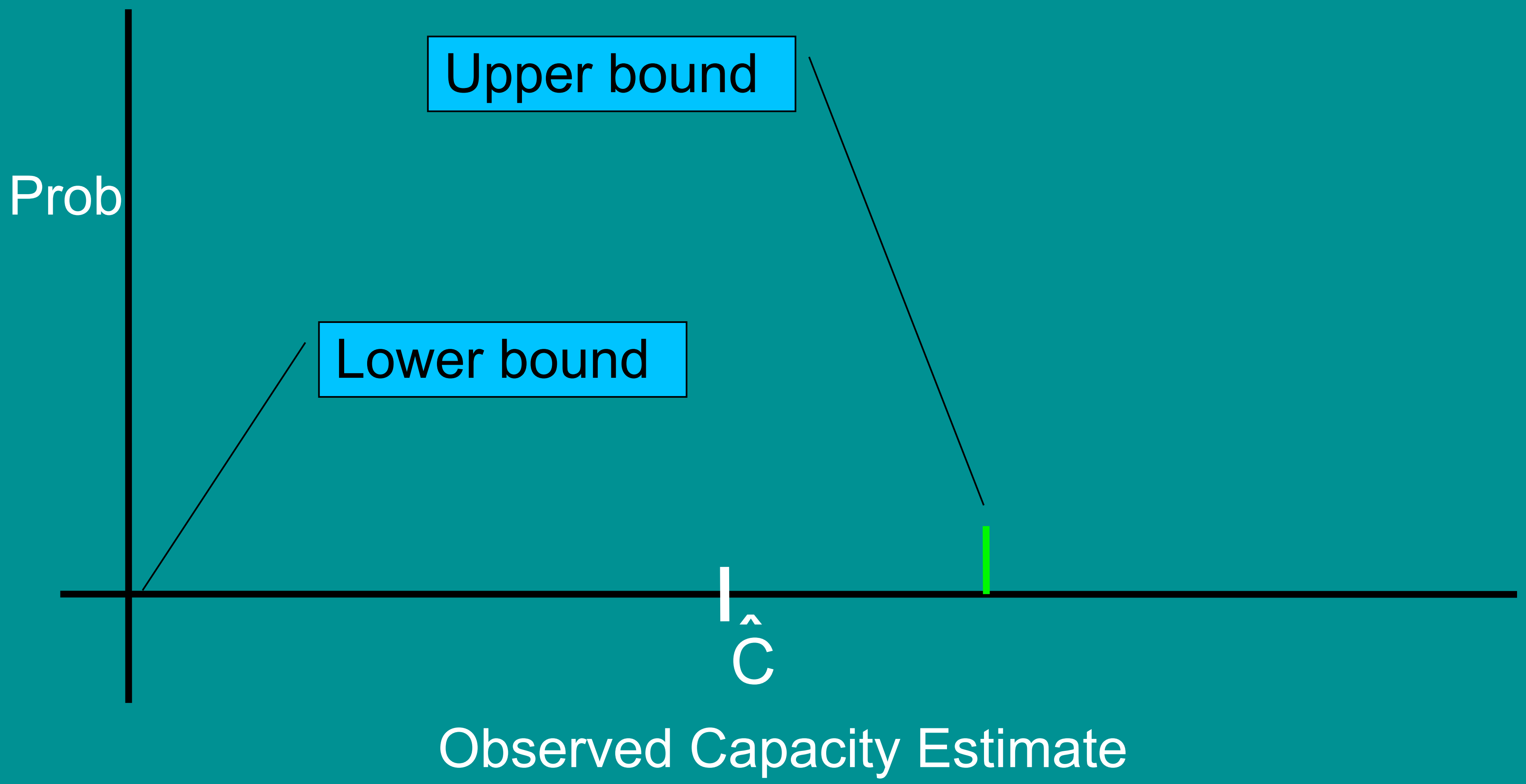

Test for Zero Leakage

- But what if we want to know if the leakage is really zero?

- What distinguishes the zero from the non-zero case is the variance:
  - $O(n^{-1})$ for non zero
  - $O(n^{-2})$ for zero.

- A large enough sample size will always tell these apart, with a given certainty.

Test for Zero Leakage

- Run 40 tests of the system and calculate the observed variance “o” in the tests results.
- Test o against the predicated variance for zero and non-zero observations.
- If it matches the zero predication but not the non-zero we can conclude that there is zero leakage.
- If it only matches the non-zero predication then we can find the confidence interval for the results.
- If it matches both then increase the sample size.

Prob. Observed from Mixminion Node

Message orderings	out A,B,C	out A,C,B	out B,A,C	out B,C,A	out C,A,B	out C,B,A
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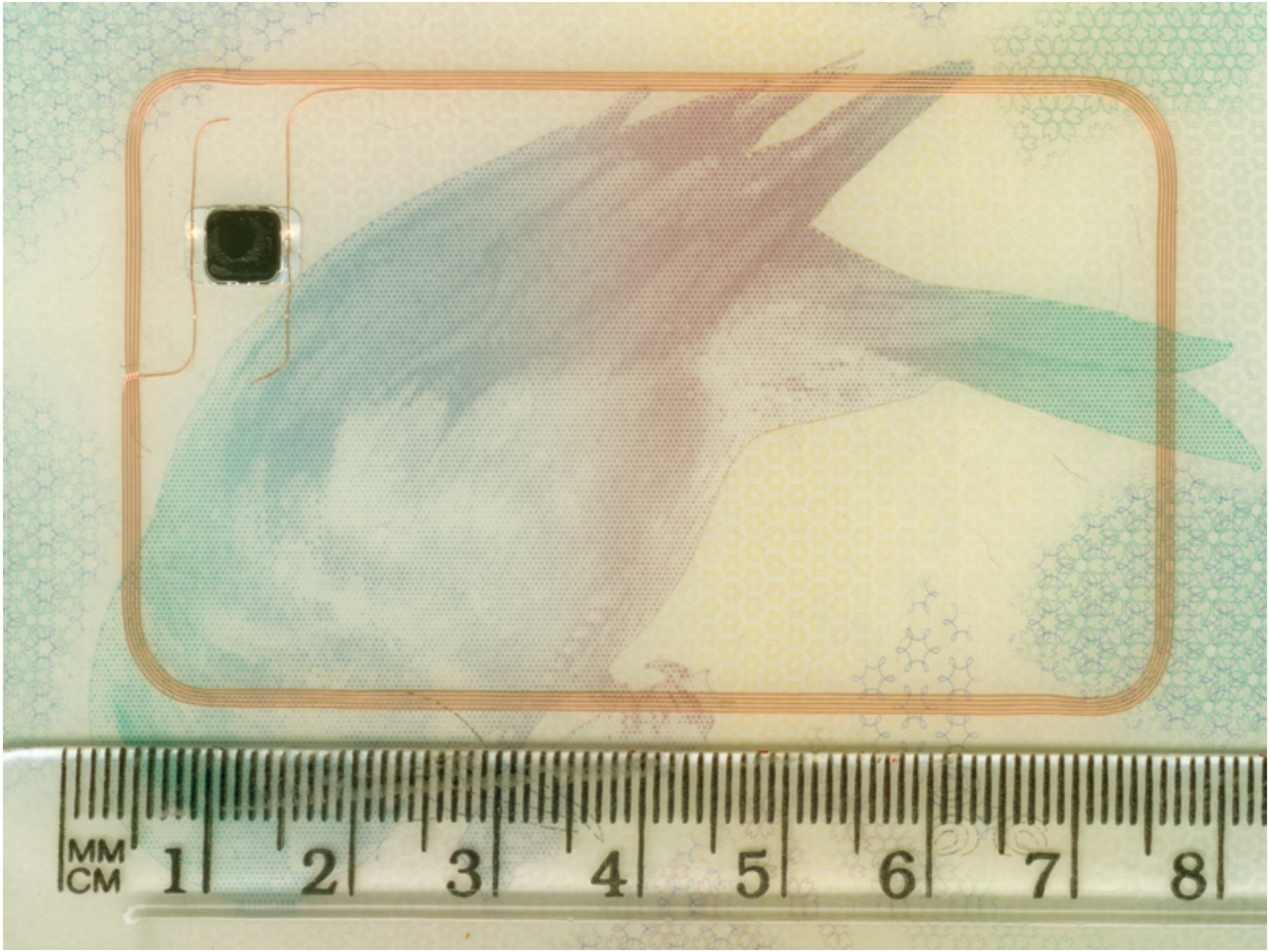
Leakage = 0.0249 bits
Confidence interval for zero leakage = 0, 0.0355

Real Time Test Demo!


MM
CM
1
2
3
4
5
6
7
8

Data on the Passport

- DG1: Machine readable info.
- DG2: Picture
- DG3: Fingerprints (seen on German)
- DG4: Iris Scans (not seen)
- DG7: Signature (not seen)
- DG11+12: Optional (height & home address FR)
- DG14: Extended Access Control Options
- DG15: Active Authentication public key
- DG16: Emergency Contact

Basic Access Control

Reader — Passport

--- GET CHALLENGE →

Pick random N_P

←----- N_P ---------

Pick random $N_{\mathbf{R}},K_R$

--- $\{N_{\mathbf{R}},N_P,K_R\}_{Ke},MAC_{Km}(\{N_{\mathbf{R}},N_P,K_R\}_{Ke})$ →

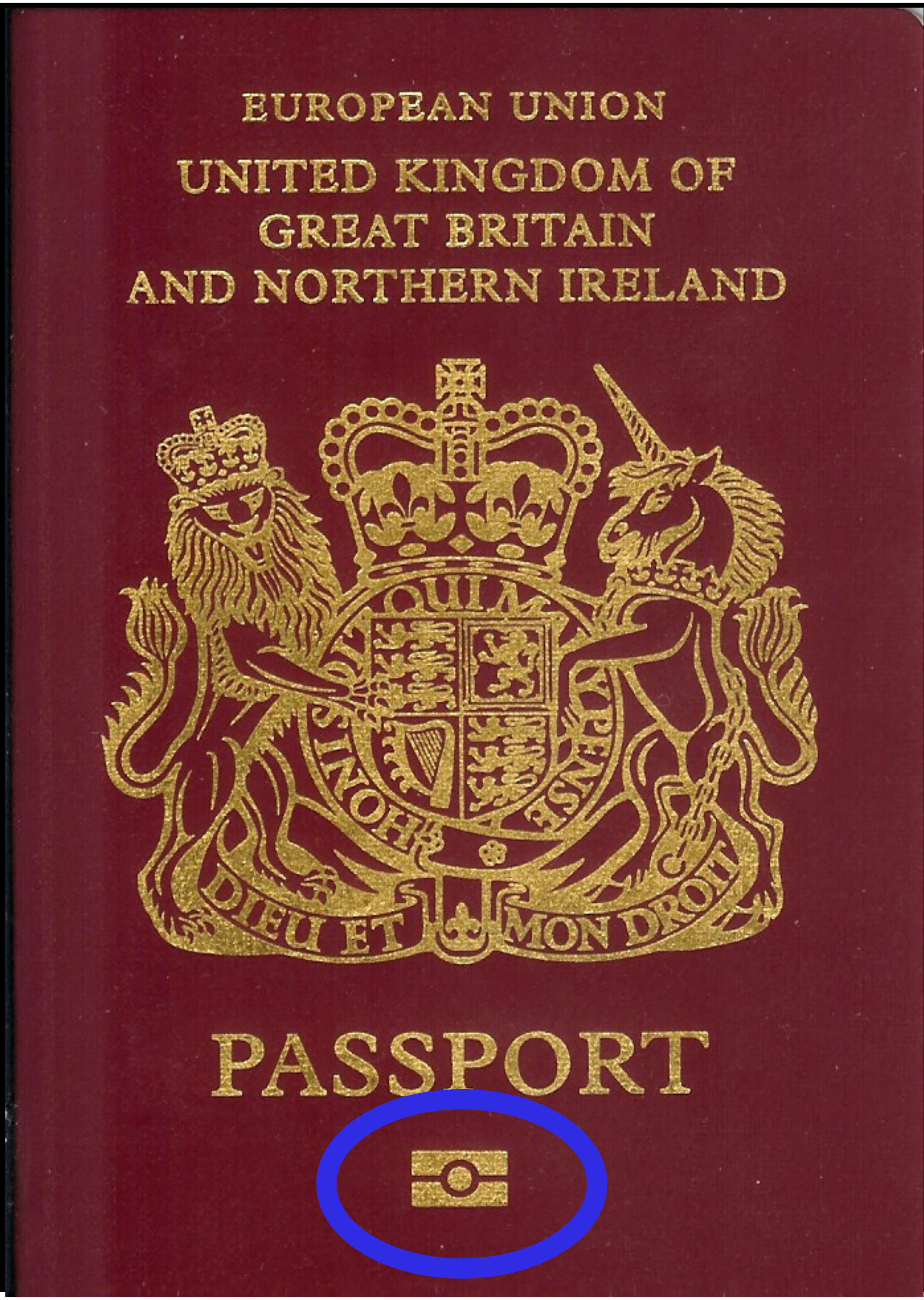
Check MAC, Decrypt,
Check N_P Pick random K_P

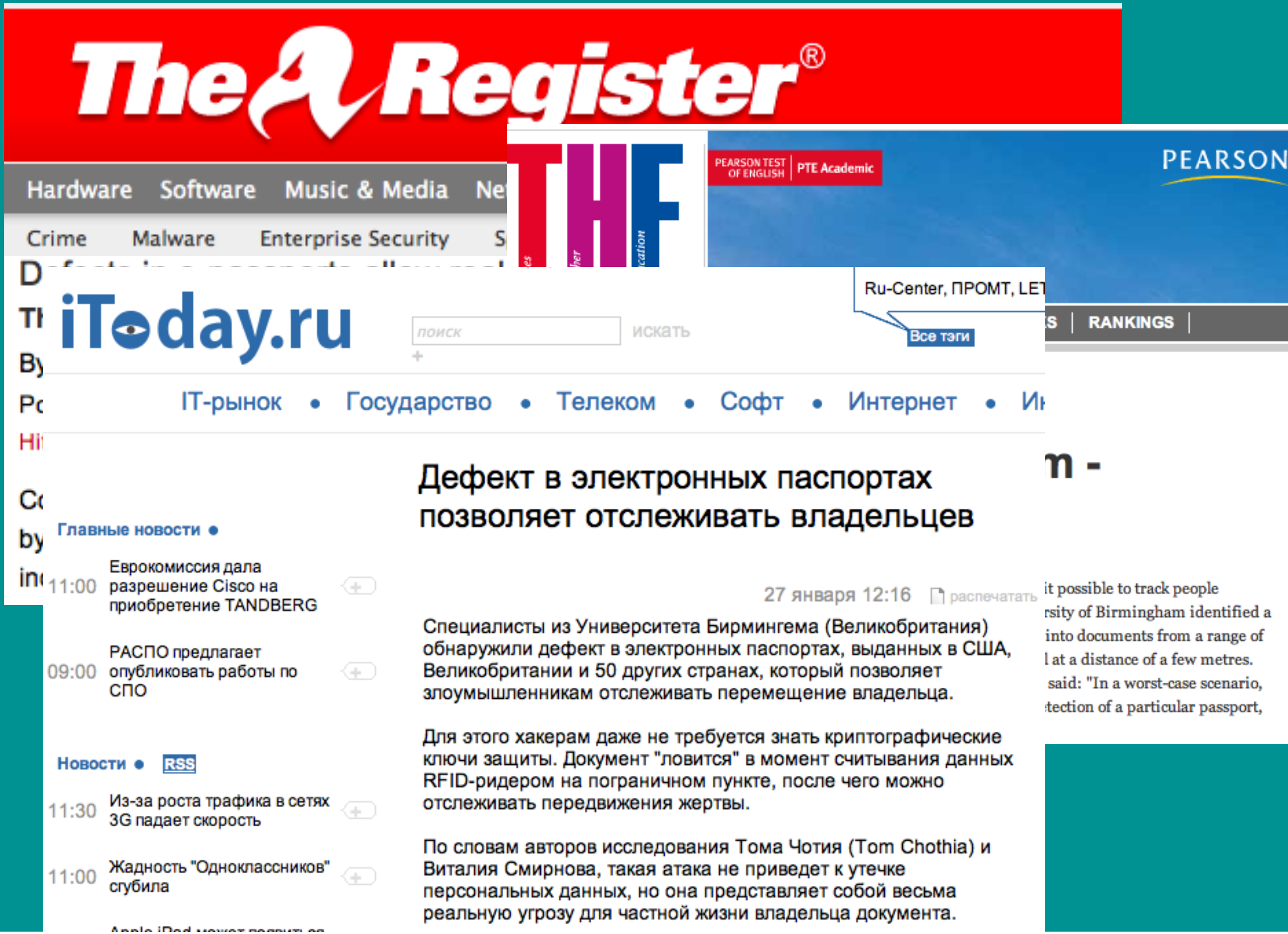
← $\{N_{\mathbf{P}},N_R,K_P\}_{Ke},MAC_{Km}(\{N_{\mathbf{P}},N_R, K_P\}_{Ke})$--

Check MAC, Decrypt,
Check N_R

There is a leak

- Our analysis shows that there is a leak and e-passports can be traced.
- Other methods could have been used to asses the data, but *Information theory* is general enough to be used everywhere.
- Our analysis is based on observations of the real system.


EUROPEAN UNION
UNITED KINGDOM OF
GREAT BRITAIN
AND NORTHERN IRELAND
DIEU ET MON DROIT
HONI SOIT QUI MAL Y PENSE
PASSPORT



Дефект в электронных паспортах позволяет отслеживать владельцев

27 января 12:16 распечатать

Специалисты из Университета Бирмингема (Великобритания) обнаружили дефект в электронных паспортах, выданных в США, Великобритании и 50 других странах, который позволяет злоумышленникам отслеживать перемещение владельца.

Для этого хакерам даже не требуется знать криптографические ключи защиты. Документ "ловится" в момент считывания данных RFID-ридером на пограничном пункте, после чего можно отслеживать передвижения жертвы.

По словам авторов исследования Тома Чотия (Tom Chothia) и Виталия Смирнова, такая атака не приведет к утечке персональных данных, но она представляет собой весьма реальную угрозу для частной жизни владельца документа.

Conclusion

- Information leaks are often due to the implementation.
- We have presented a method to estimate information theoretic measures of information leakage statistically from trail runs of a real system.
- We have used this method to show that Mixminion doesn’t leak data, but e-Passports do.

Further Work

- Proper treatment of continuous data.
- Apply to other forms of information theoretic measurement.
- Better ways to apply this to real systems.

Questions?