A Sequence-based Selection Hyper-heuristic Utilising a Hidden Markov Model

Ahmed Kheiri Ed Keedwell

College of Engineering, Mathematics and Physical Sciences

Outline

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Search and Optimisation

Search and optimisation algorithms are concerned with the discovery of the best possible solution in a given time to maximise or minimise an objective (or set of objectives).

Most real-world search and optimisation problems cannot be solved exactly, requiring heuristic approaches such as LSs.

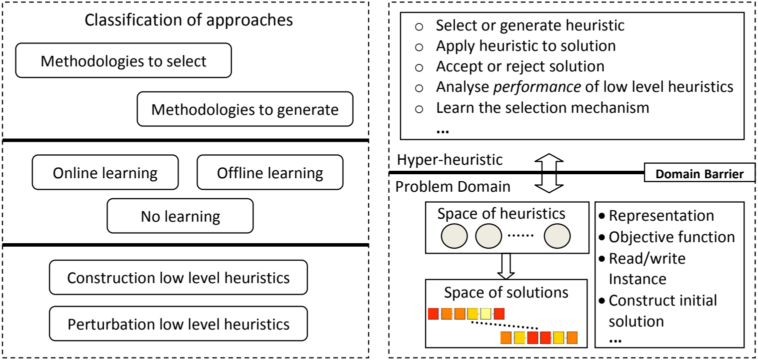
Hyper-heuristics aim to automate the search process.

— Burke et al., (2013) for recent survey

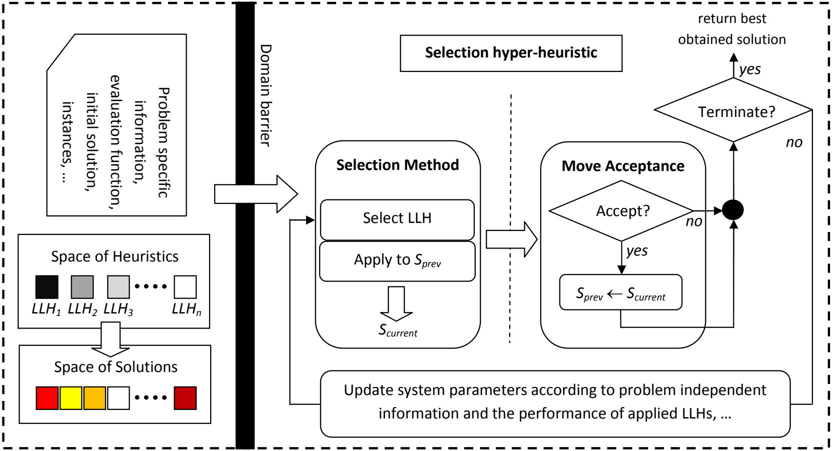
Edmund K. Burke, Michel Gendreau, Matthew Hyde, Graham Kendall, Gabriela Ochoa, Ender Özcan and Rong Qu
Hyper-heuristics: a survey of the state of the art. Journal of the Operational Research Society, 64(12):1695-1724, 2013.

Hyper-heuristic

> "A search method or learning mechanism for *selecting* or *generating* heuristics to solve computational search problems"



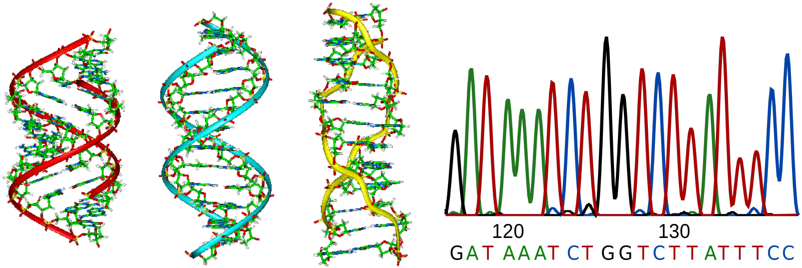
Selection Hyper-heuristic Framework



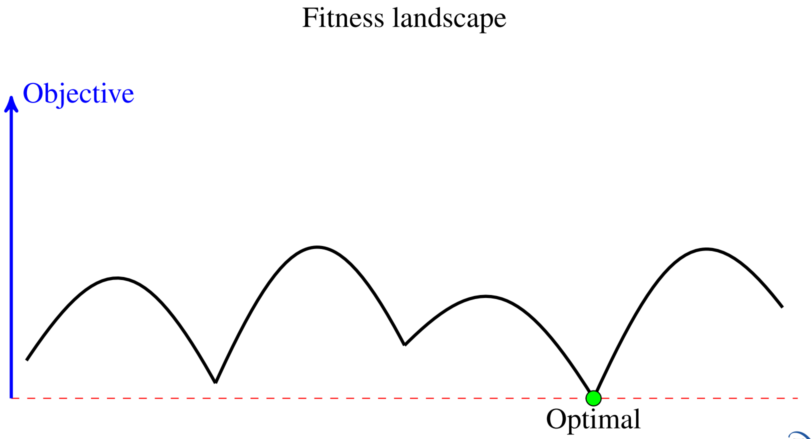
Outline

Sequence-based Selection Hyper-heuristic

Key feature: Sequence-based selection hyper-heuristics aim to analyse the performance of, and construct, sequences of heuristics.

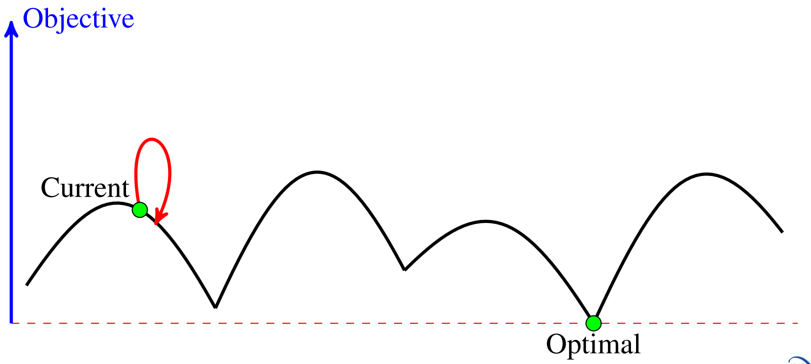


Fitness Landscape and Low Level Heuristics



Fitness Landscape and Low Level Heuristics

Apply a single low level heuristic



Fitness Landscape and Low Level Heuristics

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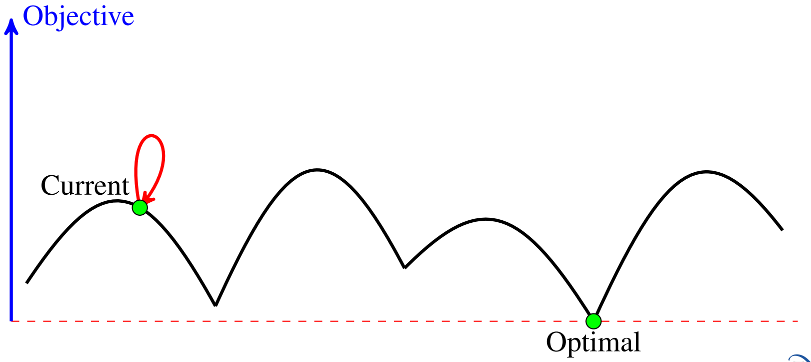
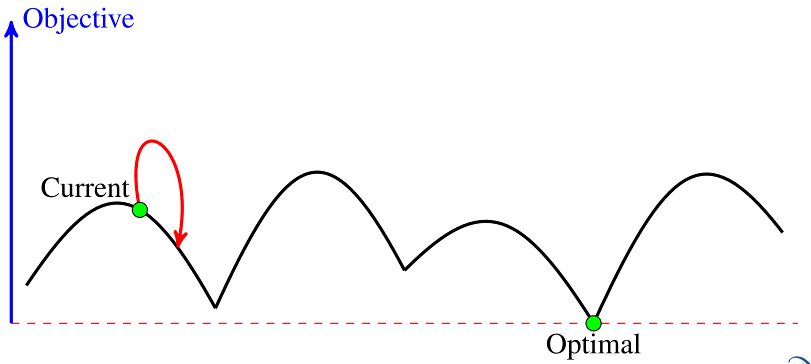


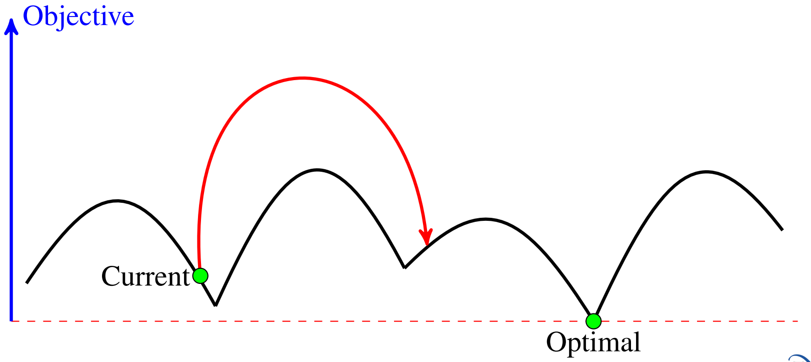
Fitness Landscape and Low Level Heuristics

Apply a sequence of two low level heuristics

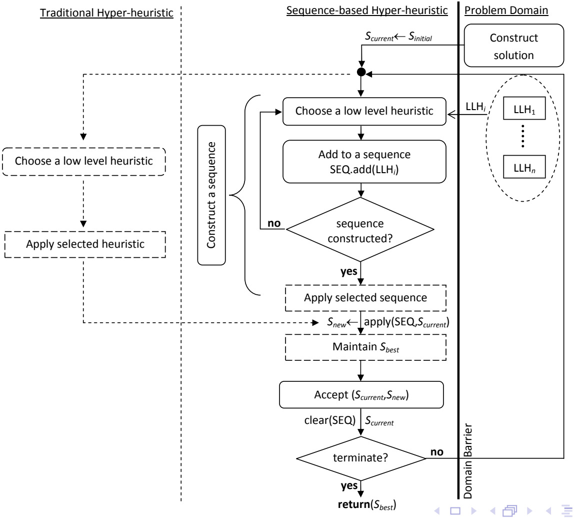


Fitness Landscape and Low Level Heuristics

Apply a sequence of three or more low level heuristics



Sequence-based Selection Hyper-heuristic Framework



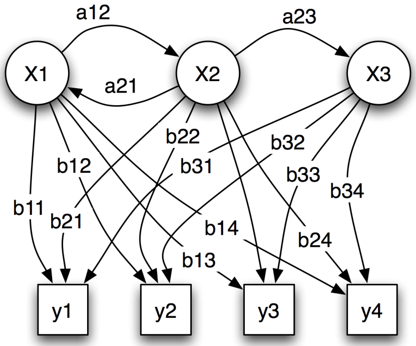
Hidden Markov Model (HMM)

Transition Matrix

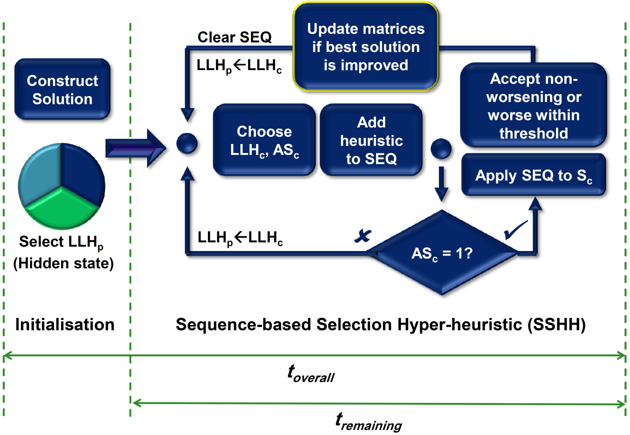
from \ to	X1	X2	X3
X1	a11	a12	a13
X2	a21	a22	a23
X3	a31	a32	a33

Observation Probability Matrix

	y1	y2	y3	y4
X1	b11	b12	b13	b14
X2	b21	b22	b23	b24
X3	b31	b32	b33	b34

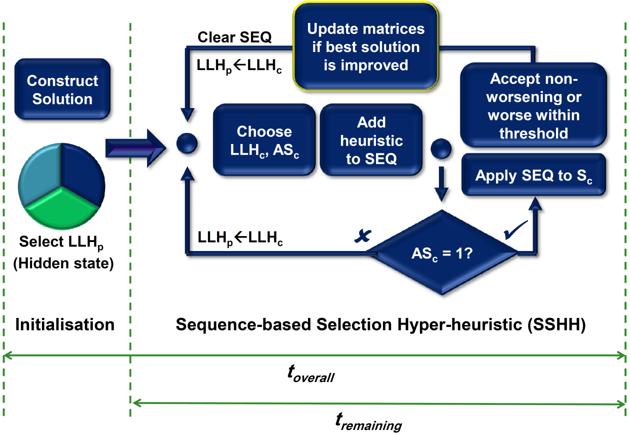


Sequence-based Hyper-heuristic Utilising HMM



AS: Sequence-based Acceptance Strategy (Observation)

Sequence-based Hyper-heuristic Utilising HMM



AS: Sequence-based Acceptance Strategy (Observation)

Sequence-based Hyper-heuristic Utilising HMM

Transition Matrix

from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
LLH2	1/3	1/3	1/3
LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Transition Matrix

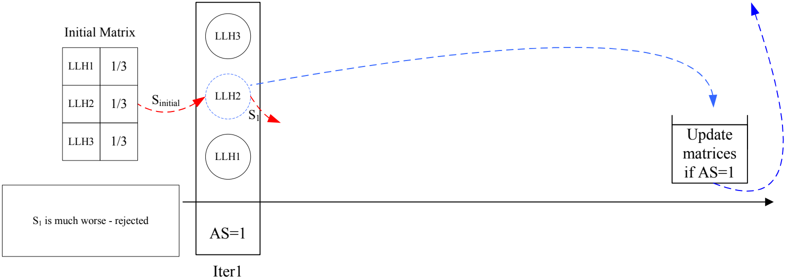
from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
LLH2	1/3	1/3	1/3
LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Initial Matrix

LLH1	1/3
LLH2	1/3
LLH3	1/3



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LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Transition Matrix

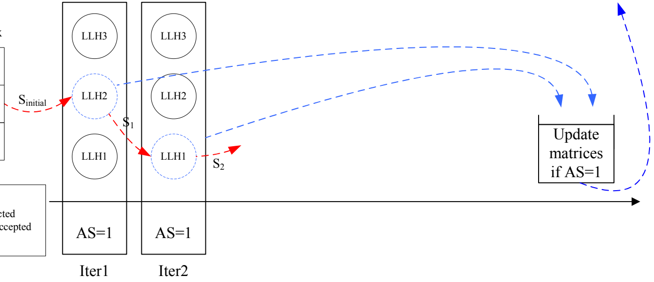
from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
LLH2	2/4	1/4	1/4
LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	2/3	1/3
LLH2	1/2	1/2
LLH3	1/2	1/2

Initial Matrix

LLH1	1/3
LLH2	1/3
LLH3	1/3



S_1 is much worse - rejected
S_2 is better than the best - accepted

Sequence-based Hyper-heuristic Utilising HMM

Transition Matrix

from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
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LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Transition Matrix

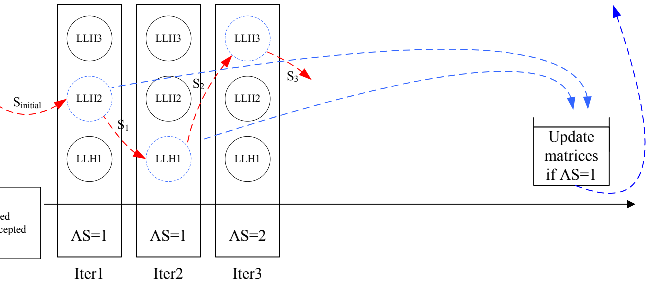
from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
LLH2	2/4	1/4	1/4
LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	2/3	1/3
LLH2	1/2	1/2
LLH3	1/2	1/2

Initial Matrix

LLH1	1/3
LLH2	1/3
LLH3	1/3



S_1 is much worse - rejected
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Sequence-based Hyper-heuristic Utilising HMM

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LLH1	1/3	1/3	1/3
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LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Transition Matrix

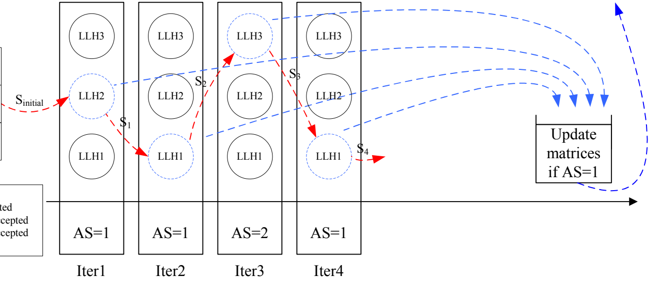
from \ to	LLH1	LLH2	LLH3
LLH1	1/4	1/4	2/4
LLH2	2/4	1/4	1/4
LLH3	2/4	1/4	1/4

AS Matrix

	1	2
LLH1	3/4	1/4
LLH2	1/2	1/2
LLH3	1/3	2/3

Initial Matrix

LLH1	1/3
LLH2	1/3
LLH3	1/3



S_1 is much worse - rejected
S_2 is better than the best - accepted
S_4 is better than the best - accepted

Sequence-based Hyper-heuristic Utilising HMM

Transition Matrix

from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
LLH2	1/3	1/3	1/3
LLH3	1/3	1/3	1/3

AS Matrix

	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Transition Matrix

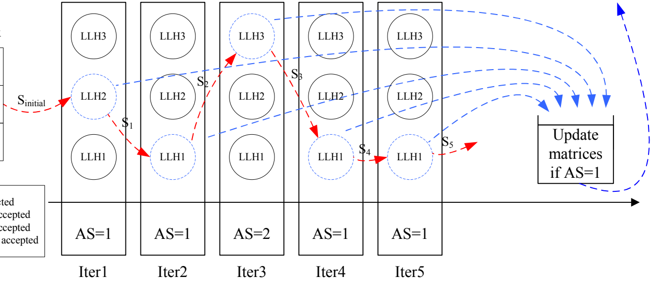
from \ to	LLH1	LLH2	LLH3
LLH1	1/4	1/4	2/4
LLH2	2/4	1/4	1/4
LLH3	2/4	1/4	1/4

AS Matrix

	1	2
LLH1	3/4	1/4
LLH2	1/2	1/2
LLH3	1/3	2/3

Initial Matrix

LLH1	1/3
LLH2	1/3
LLH3	1/3



S_1 is much worse - rejected
S_2 is better than the best - accepted
S_4 is better than the best - accepted
S_5 is not better than the best - accepted

Sequence-based Hyper-heuristic Utilising HMM

The likelihood being in state i at time n:

$$L(i,n) = L(i,n-1) \times a_{llh_{n-1}.llh_n} \times b_{AS}$$

Transition Matrix

from \ to	LLH1	LLH2	LLH3
LLH1	1/3	1/3	1/3
LLH2	1/3	1/3	1/3
LLH3	1/3	1/3	1/3

AS Matrix

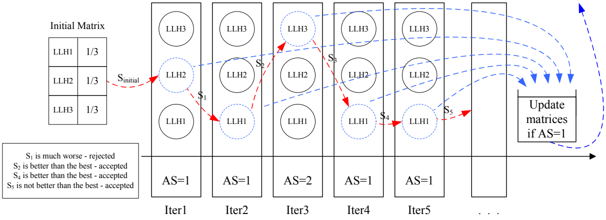
	1	2
LLH1	1/2	1/2
LLH2	1/2	1/2
LLH3	1/2	1/2

Transition Matrix

from \ to	LLH1	LLH2	LLH3
LLH1	1/4	1/4	2/4
LLH2	2/4	1/4	1/4
LLH3	2/4	1/4	1/4

AS Matrix

	1	2
LLH1	3/4	1/4
LLH2	1/2	1/2
LLH3	1/3	2/3



Outline

Hyper-heuristics Flexible Framework - HyFlex

Problem domain	SAT	BP	PS	PFS	TSP	VRP
no. of heuristics	11	8	12	15	13	10
mutational	0-5	0,3,5	11	0-4	0-4	0,1,7
ruin & re-create	6	1,2	5-7	5,6	5	2,3
hill climbing	7,8	4,6	0-4	7-10	6-8	4,8,9
crossover	9,10	7	8-10	11-14	9-12	5,6

Each heuristic is associated with a problem and heuristic dependent parameter. We discretised the choices into 11 different parameters (P).

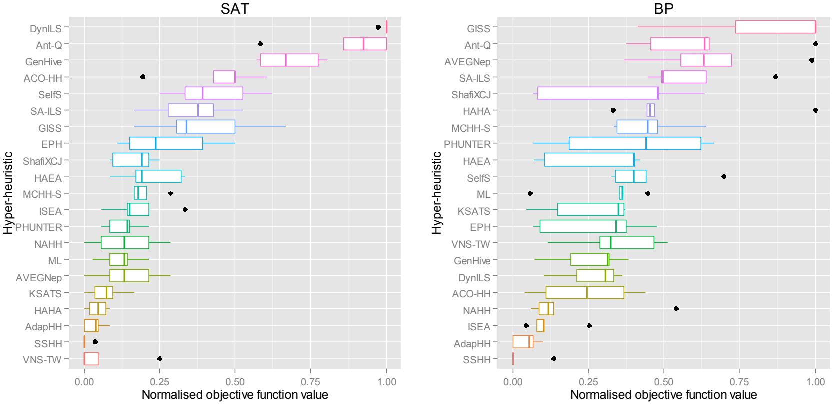
Gabriela Ochoa, Matthew Hyde, Tim Curtois, Jose A. Vazquez-Rodriguez, James Walker, Michel Gendreau, Graham Kendall, Barry McCollum, Andrew J. Parkes, Sanja Petrovic and Edmund K. Burke HyFlex: a benchmark framework for cross-domain heuristic search. In J.-K. Hao and M. Middendorf, editors, Evolutionary Computation in Combinatorial Optimization, volume 7245 of Lecture Notes in Computer Science, pages 136-147. Springer Berlin Heidelberg, 2012.

Hyper-heuristics Flexible Framework - HyFlex

SSHH and Cross-domain Heuristic Search Challenge (CHeSC 2011) competing algorithms (Formula-1 scoring system).

Method	SAT	BP	PS	PFS	TSP	VRP	Overall
SSHH	**39.10**	**45.00**	**22.50**	**3.50**	**41.00**	**14.00**	**165.10**
AdapHH	30.43	38.00	8.00	37.00	34.75	14.00	162.18
VNS-TW	30.93	2.00	35.50	33.50	12.75	5.00	119.68
ML	10.00	8.00	29.50	39.00	10.00	21.00	117.50
PHUNTER	7.00	2.00	11.50	8.00	22.75	32.00	83.25
EPH	0.00	6.00	9.00	21.00	29.75	11.00	76.75
HAHA	26.43	0.00	23.00	3.50	0.00	13.00	65.93
NAHH	10.50	15.00	1.00	22.00	10.00	6.00	64.50
KSATS-HH	20.35	9.00	7.00	0.00	0.00	21.00	57.35
ISEA	3.50	23.00	14.50	3.50	7.00	3.00	54.50
HAEA	0.00	1.00	1.00	8.00	8.00	25.00	43.00
ACO-HH	0.00	16.00	0.00	9.00	7.00	1.00	33.00
GenHive	0.00	10.00	6.50	7.00	2.00	6.00	31.50
SA-ILS	0.25	0.00	16.00	0.00	0.00	4.00	20.25
XCJ	3.50	11.00	0.00	0.00	0.00	5.00	19.50
AVEG-Nep	9.50	0.00	0.00	0.00	0.00	8.00	17.50
DynILS	0.00	9.00	0.00	0.00	8.00	0.00	17.00
GISS	0.25	0.00	8.00	0.00	0.00	6.00	14.25
SelfSearch	0.00	0.00	2.00	0.00	2.00	0.00	4.00
MCHH-S	3.25	0.00	0.00	0.00	0.00	0.00	3.25
Ant-Q	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Hyper-heuristics Flexible Framework - HyFlex (Boxplots)



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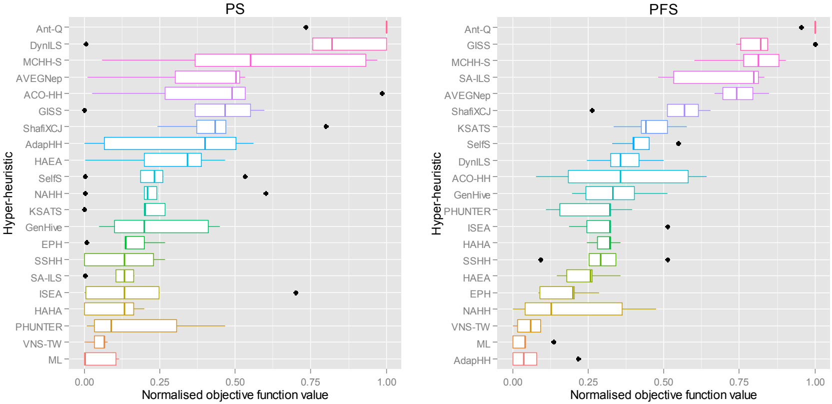

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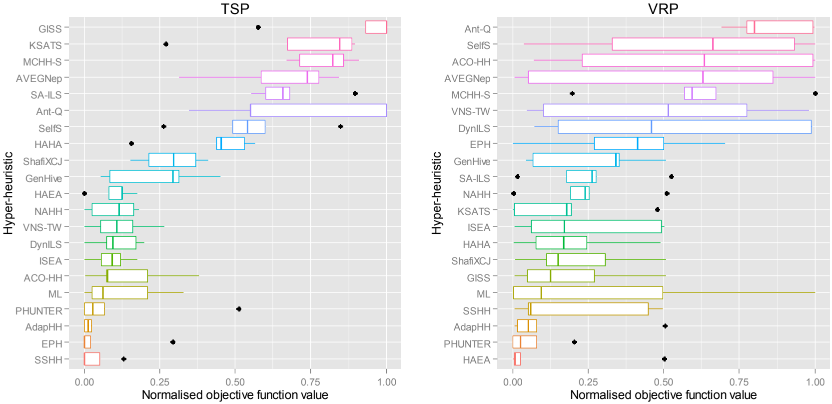
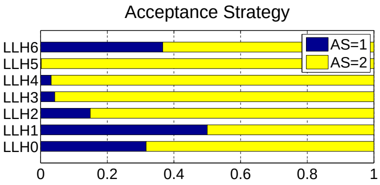
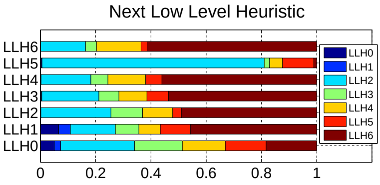
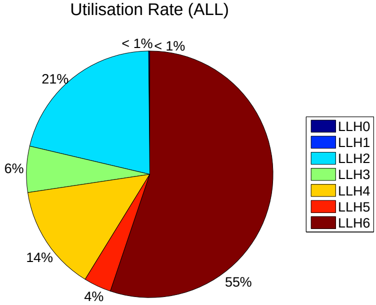
Hyper-heuristics Flexible Framework - HyFlex (Boxplots)

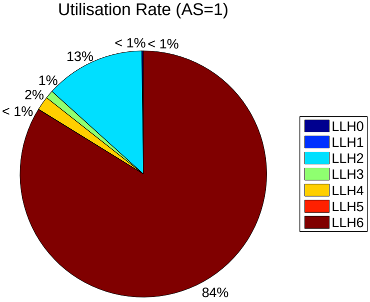


Hyper-heuristics Flexible Framework - HyFlex (BP)

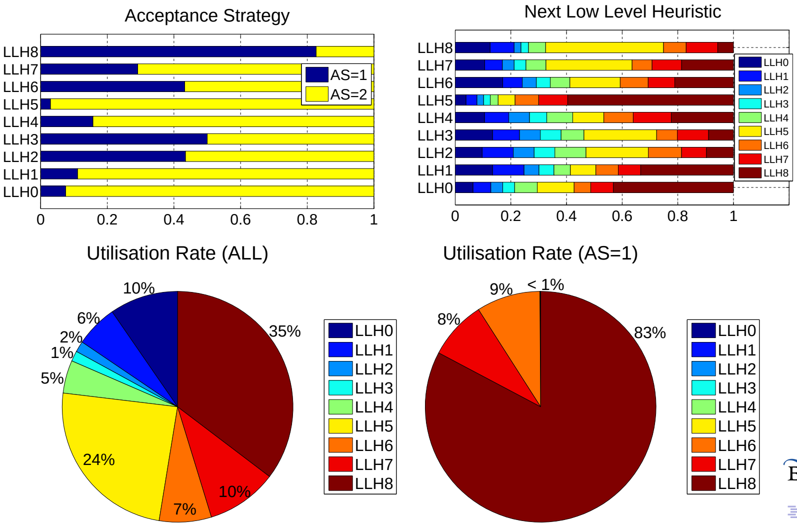








Hyper-heuristics Flexible Framework - HyFlex (TSP)



High School Timetabling Problem

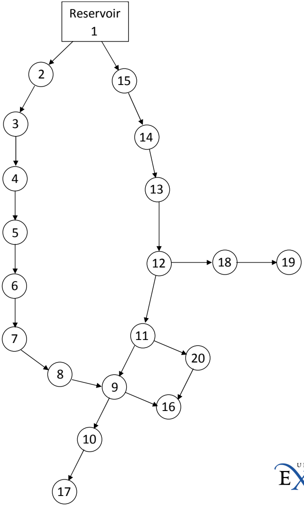
At time of submission:

Instance	SSHH	Best known	Instance	SSHH	Best known
Australia-BGHS98	**0.520**	1.386	Greece-HighSchool1	**0.0**	**0.0**
Australia-SAHS96	**0.2**	0.24	Greece-ThirdHighSchool2010	**0.0**	**0.0**
Australia-TES99	**0.61**	0.125	Greece-WesternUniversity4	**0.4**	**0.4**
Brazil-Instance2	0.10	**0.5**	Italy-Instance4	0.38	**0.34**
Brazil-Instance4	2.117	**0.51**	Kosova-Instance1	**0.3**	**0.3**
Brazil-Instance6	0.101	**0.35**	Netherlands-Kottenpark2003	**0.466**	0.617
Denmark-Falkonergaardens2012	**0.1522**	0.3310	Netherlands-Kottenpark2005	**0.811**	0.1078
Denmark-HasserisGymnasium2012	**12.2628**	12.3124	Netherlands-Kottenpark2009	2.7495	**0.9180**
Denmark-VejenGymnasium2009	**2.2731**	2.4097	England-StPaul	19.1294	**16.2258**
Spain-School	0.517	**0.336**	USA-Westside2009	**0.512**	0.697
Finland-College	0.8	**0.0**	South Africa-Lewitt2009	0.52	**0.0**
Finland-HighSchool	0.7	**0.1**	South Africa-Woodlands2009	9.0	**0.0**
Finland-SecondarySchool	0.89	**0.83**			

Ahmed Kheiri and Ed Keedwell A sequence-based selection hyper-heuristic for operational research problems with a case study in high school timetabling problems. Information Sciences, in preparation.

New York Tunnels Water Distribution Network

Algorithm	Best Solution Cost	Evaluations
SSHH	38.64m	24,996
CGA	38.64m	44,324
SGA	38.64m	54,789
SDE	38.64m	12,855
DDE	38.64m	13,214



Outline

Features of the Algorithm

- The selection method is parameter-free.
- It is capable of discovering sequences of heuristics of any size.
- It discovers automatically when to move from intensification to diversification and vice versa.
- The HMM matrices can be analysed to determine what has been learned about the search space and relationships between low level heuristics and acceptance strategy.
- The sequence-based acceptance strategy allows exploration and exploitation of the heuristic space.

Thank You