Why Types Matter

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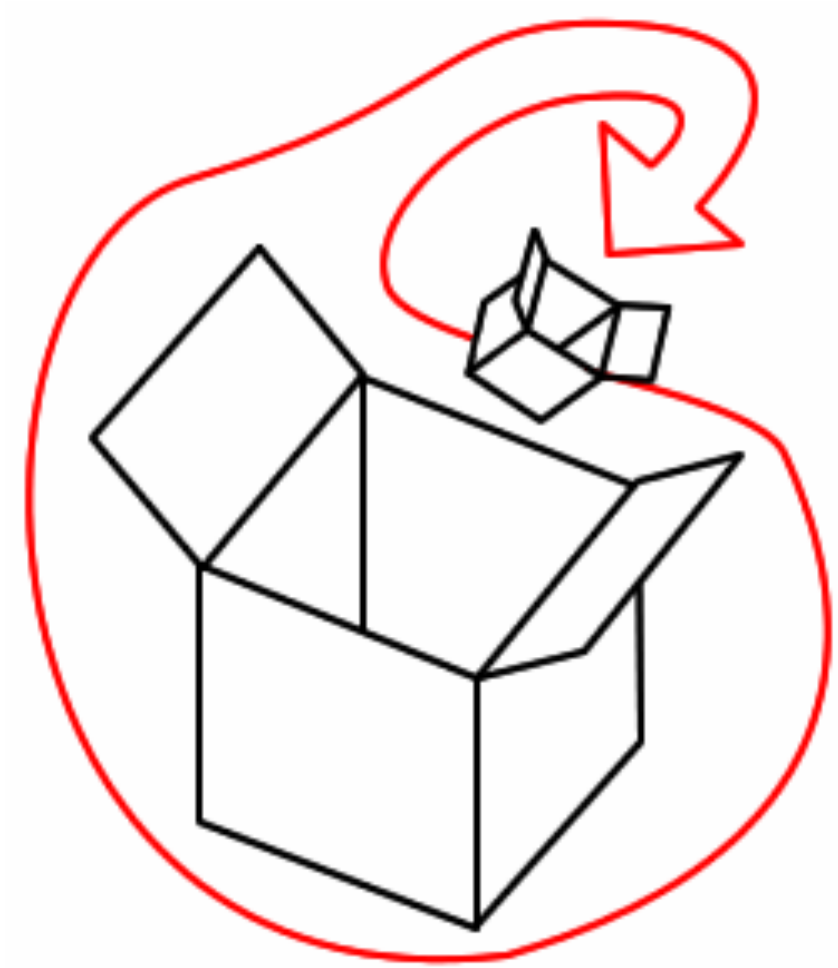
Russell's Paradox

Let *R* be the set of all sets that are not members of themselves. **Is *R* a member of itself**?

- If so, this contradicts with *R*'s definition
- If not, by definition, *R* should contain itself



Formalism in naïve set theory:

Let $R = \{x \mid x \notin x\}$, then $R \in R \iff R \notin R$

bar·ber
par·a·dox

The Barber Paradox

There is a town with a male barber who shaves all and only those men who do not shave themselves. **Who shaves the barber**?

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- If he does shave himself, according to the rule he will not shave himself.

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There is a town with a male barber who shaves all and only those men who do not shave themselves. **Who shaves the barber**?

Naïve set theory contains contradiction

- If the ba[illegible]ding to the rule he r[illegible]
- If he does shave himself, according to the rule he will not shave himself.

Types to the Rescue

Constructs a hierarchy of types.

Any object is built only from those of higher types, which prevents circular referencing.

1) a barber as a citizen of the town, who shaves himself

and

2) a barber as a professional, who shaves others

are of different types.

Type Theory

An alternative to set theory as a foundation for mathematics, in which each term has a type

Simply typed λ-calculus is one of the many forms of type theory, which consists of

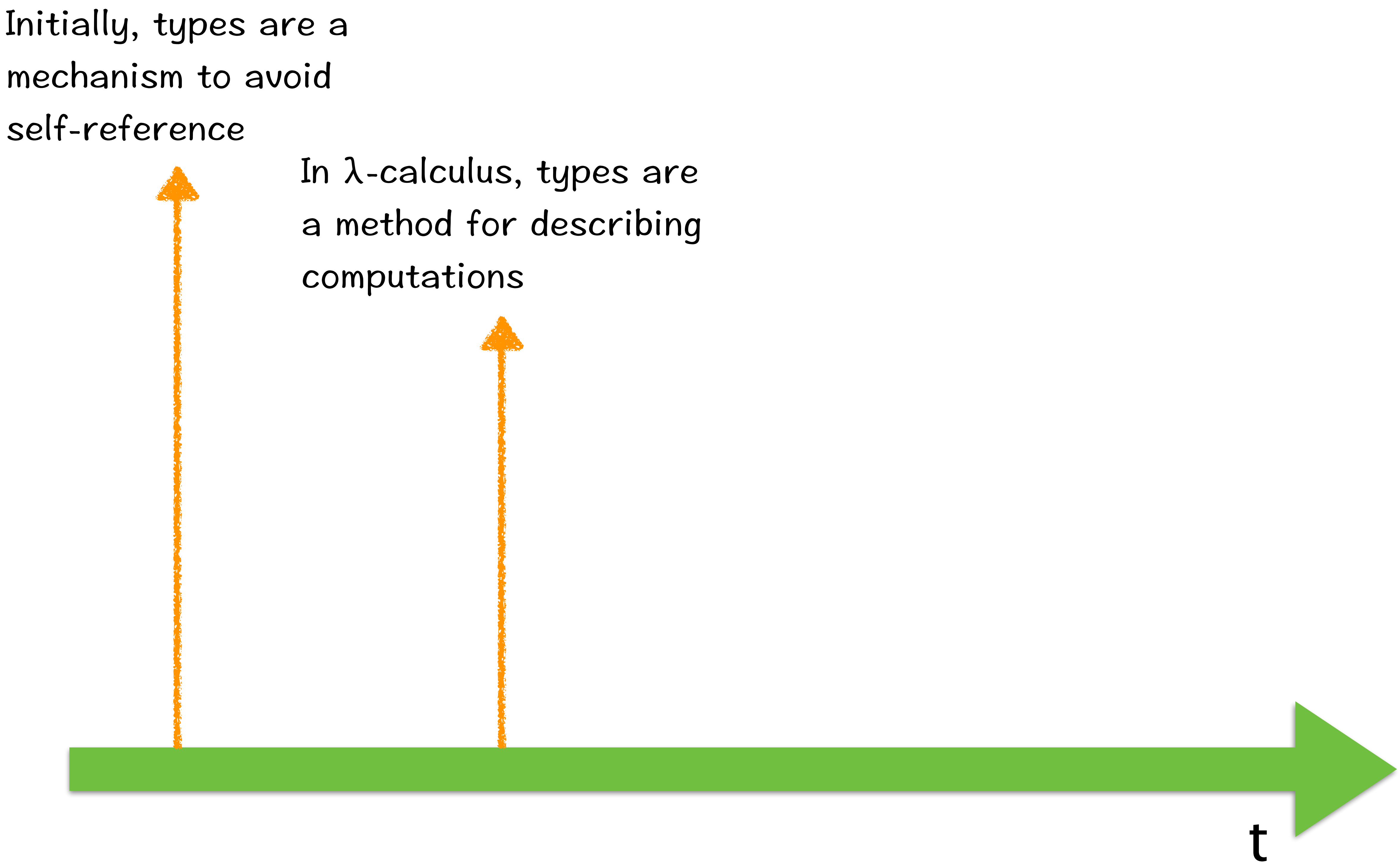
- Base types
- Only one type constructor, $\longrightarrow$, used to model the type of functions

The Evolution of Types

The Evolution of Types

Initially, types are a
mechanism to avoid
self-reference

The Evolution of Types



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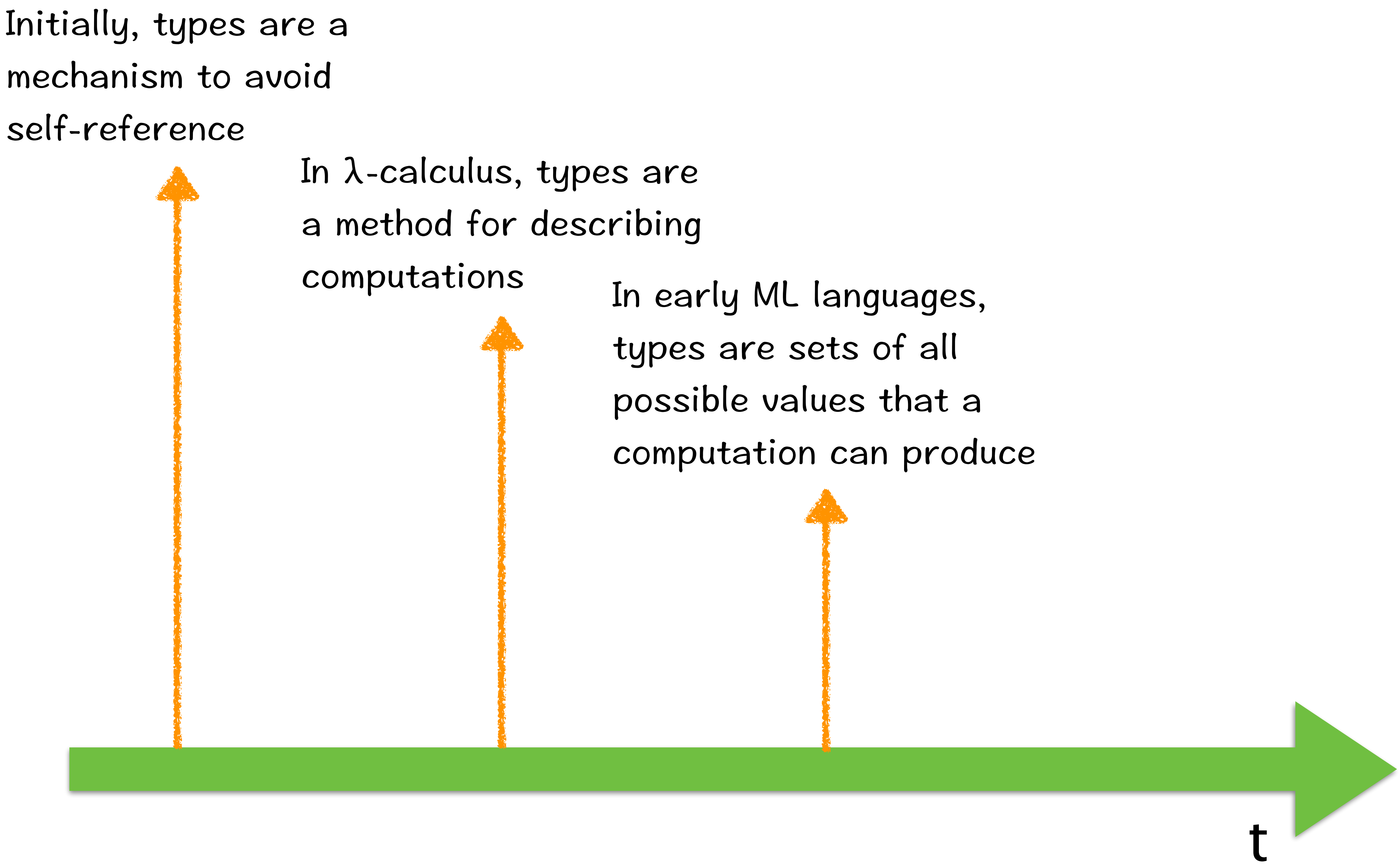

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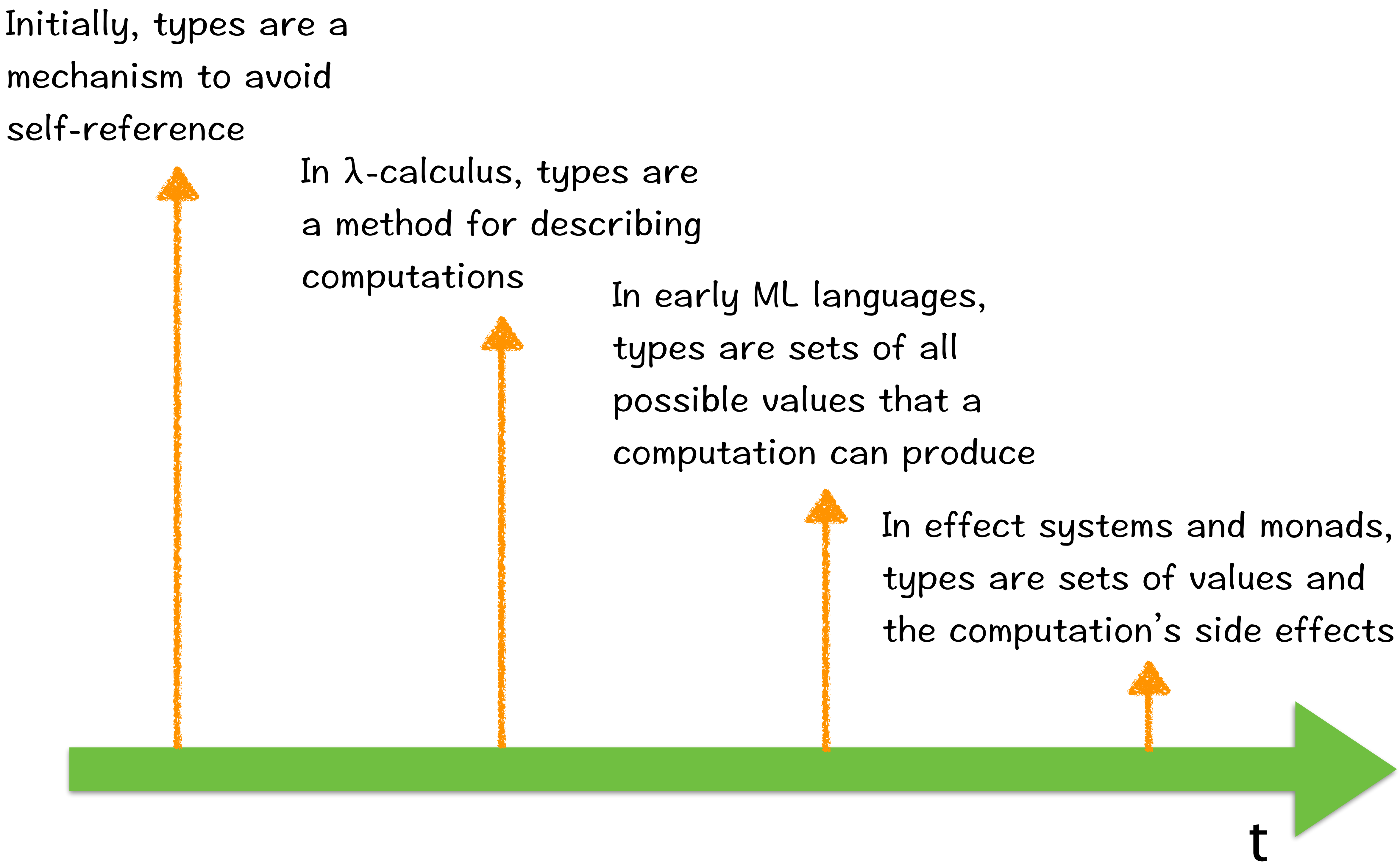

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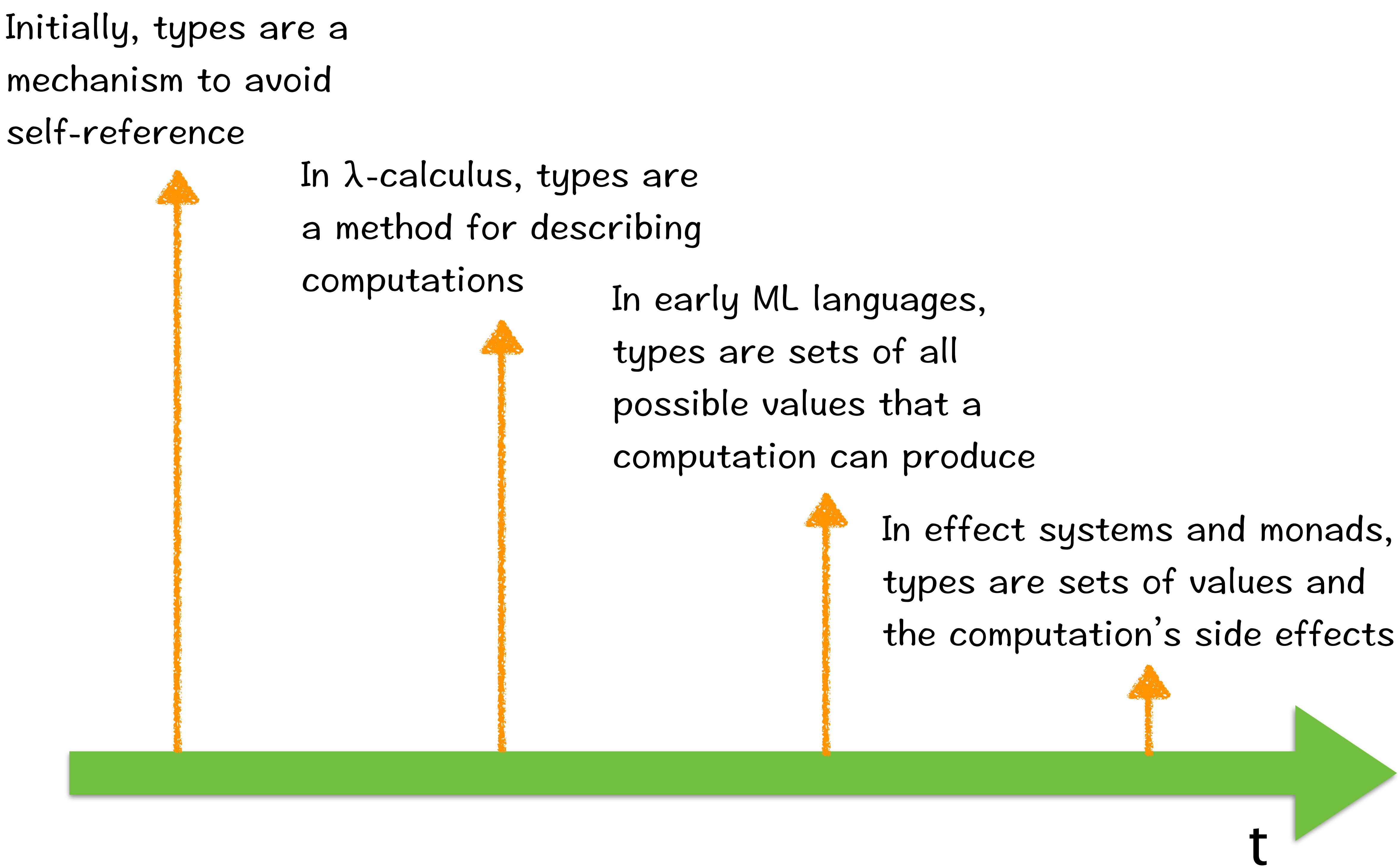
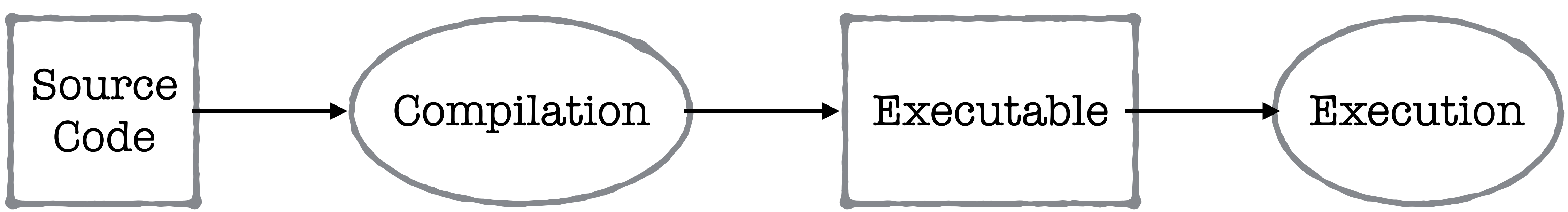


Type System

A tractable method that assigns types to syntactic phrases that compose a program, and automatically checks whether the usage of these phrases comply with their types

An over-approximation of the run-time behaviour of program terms

Static & Dynamic Type Checking



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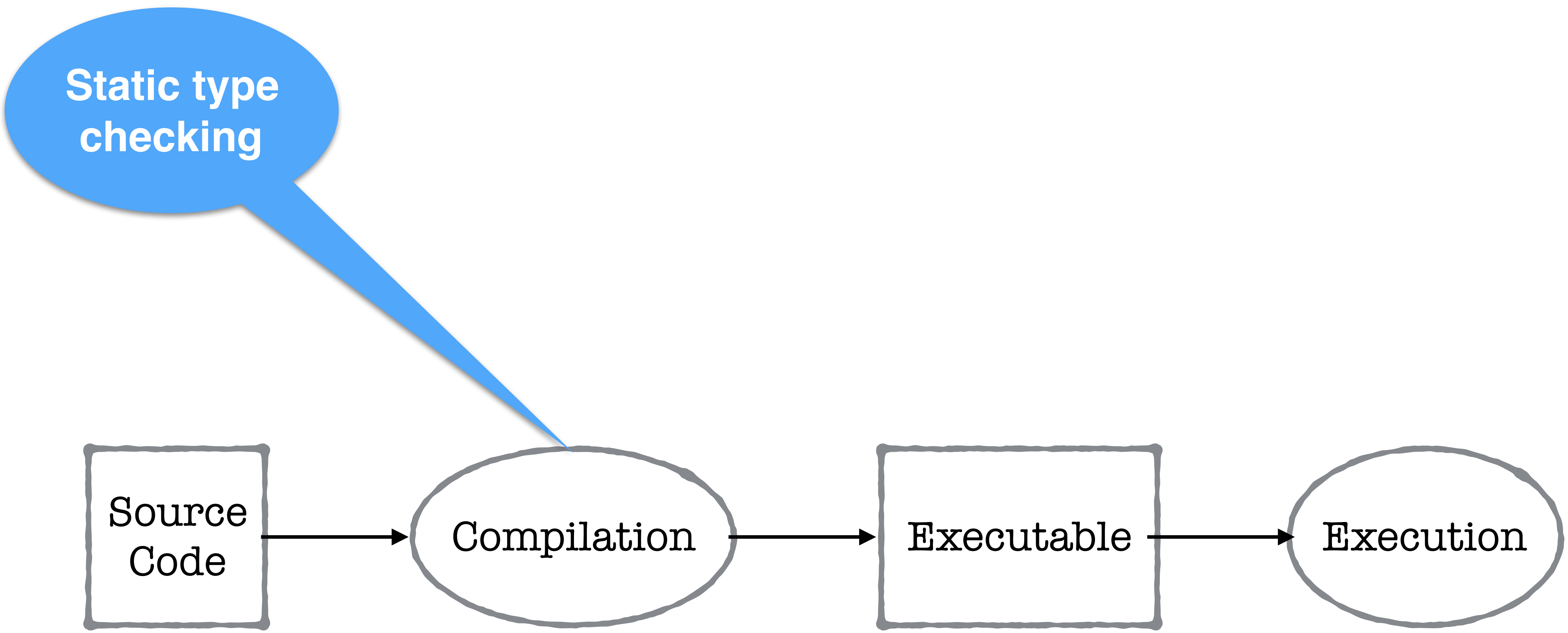

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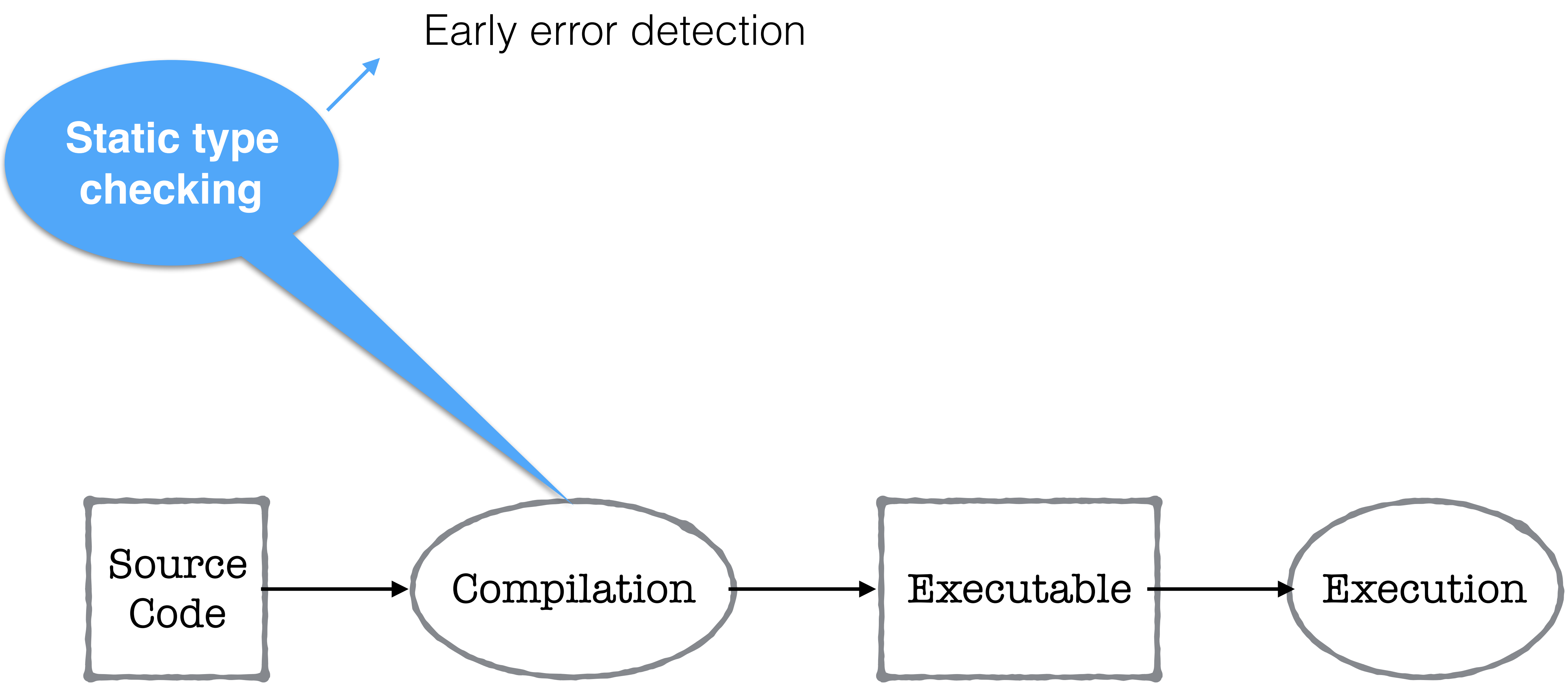

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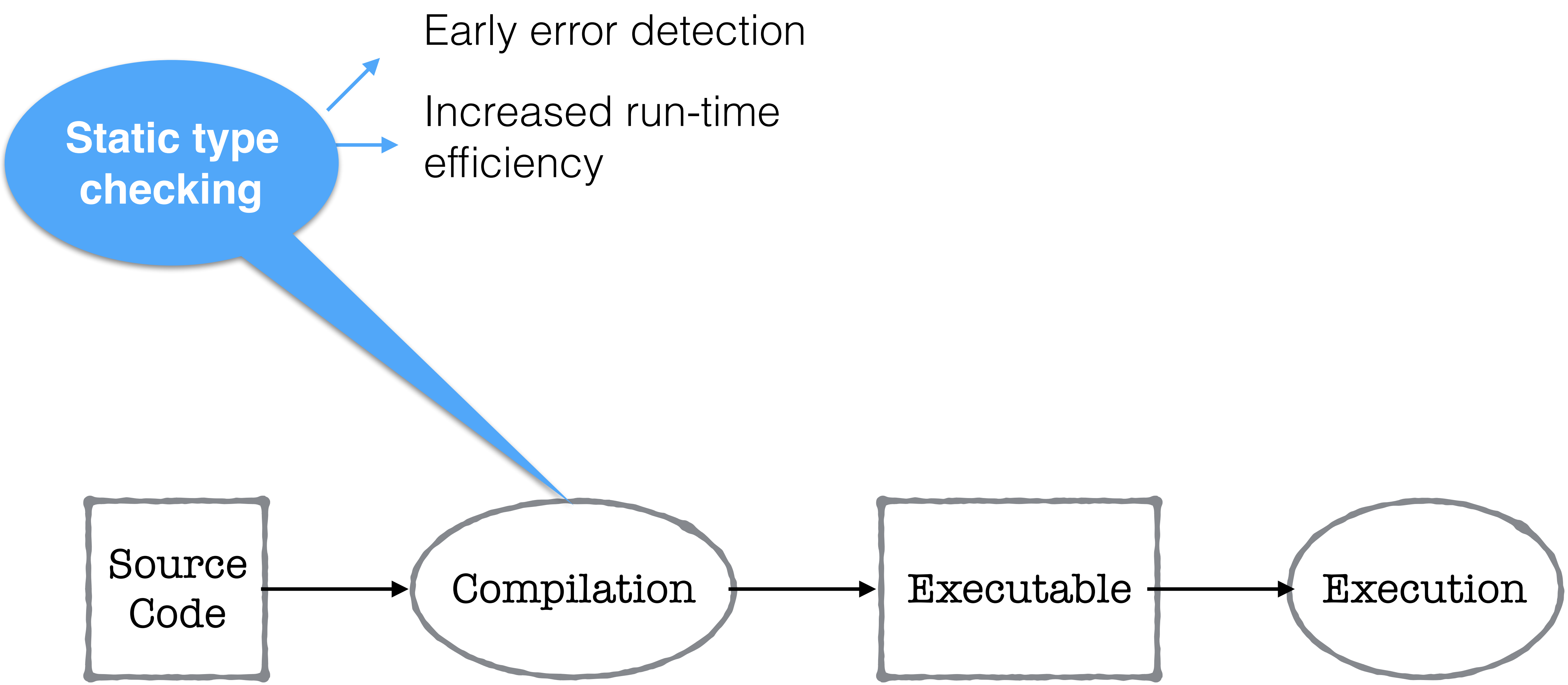

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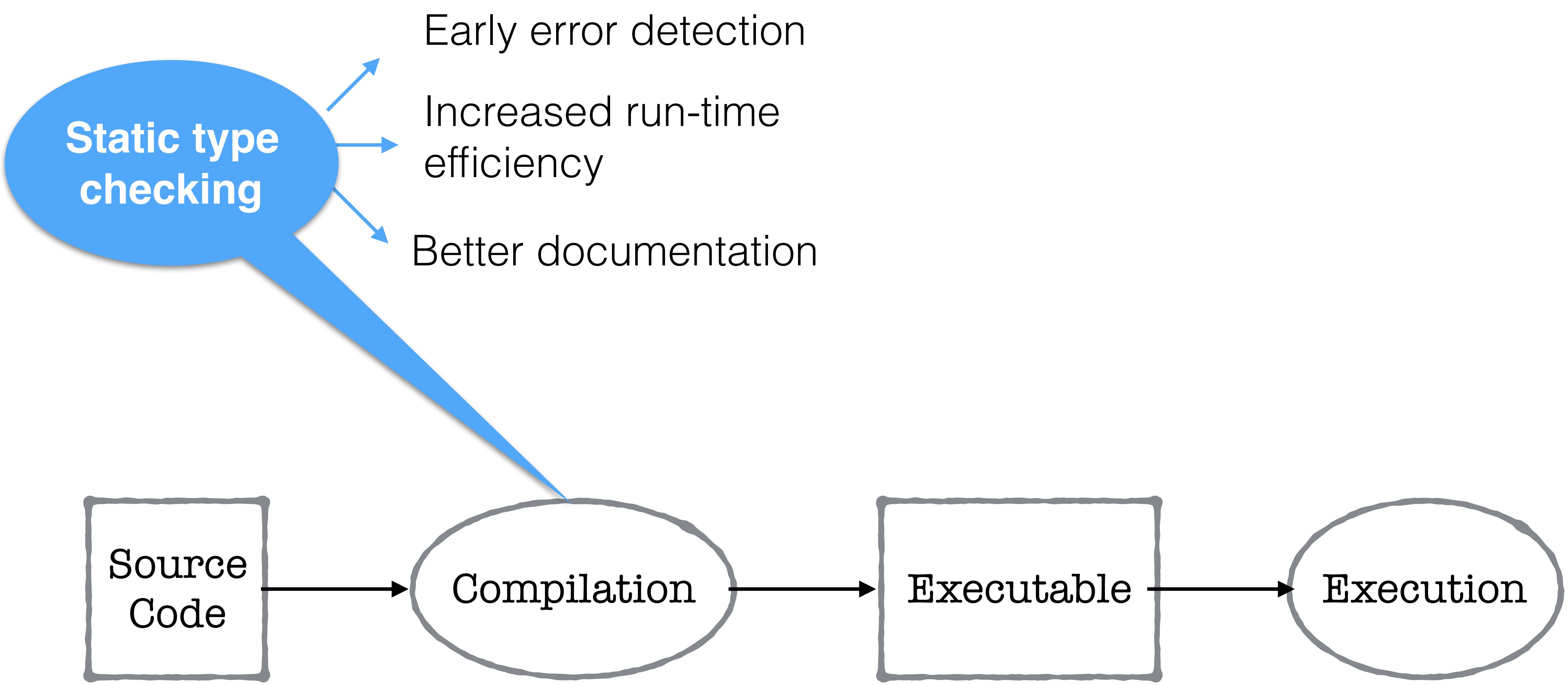

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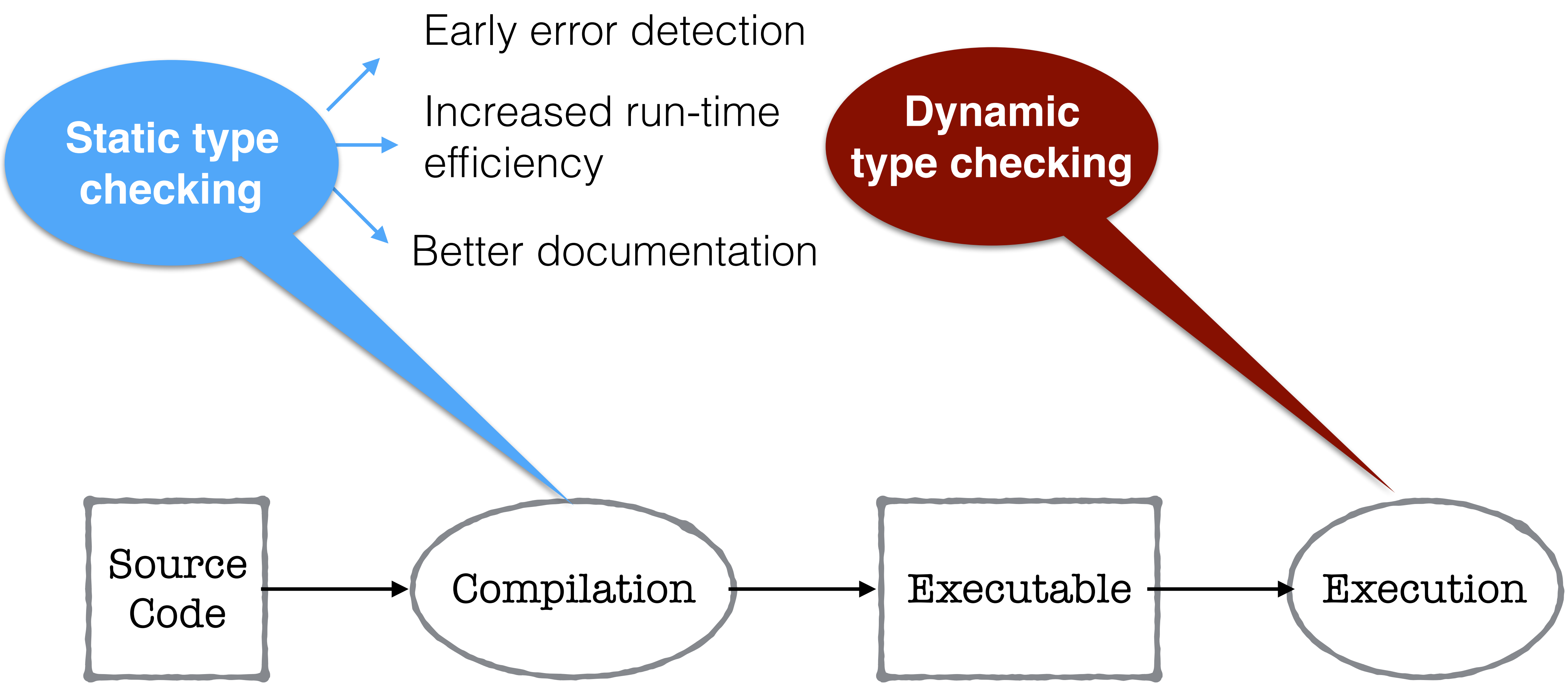

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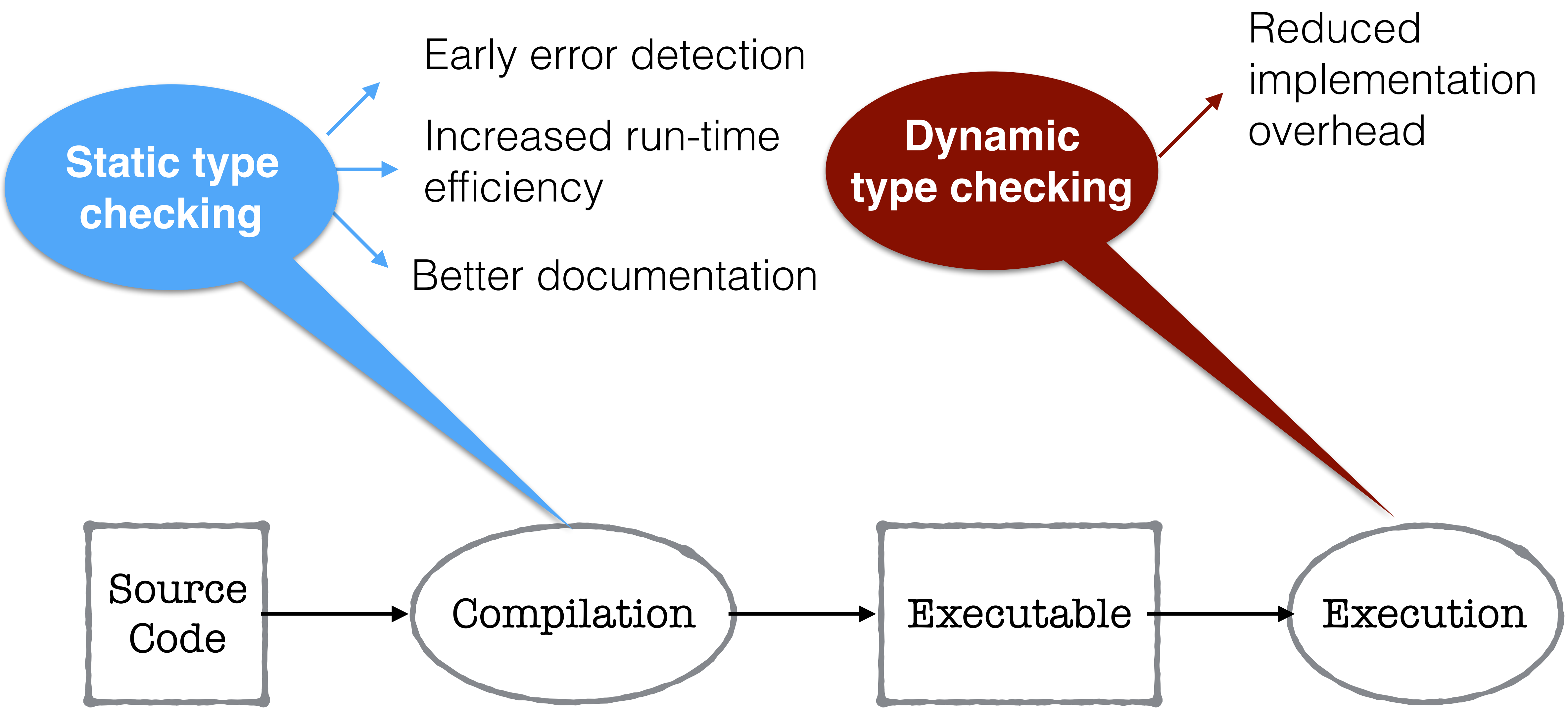

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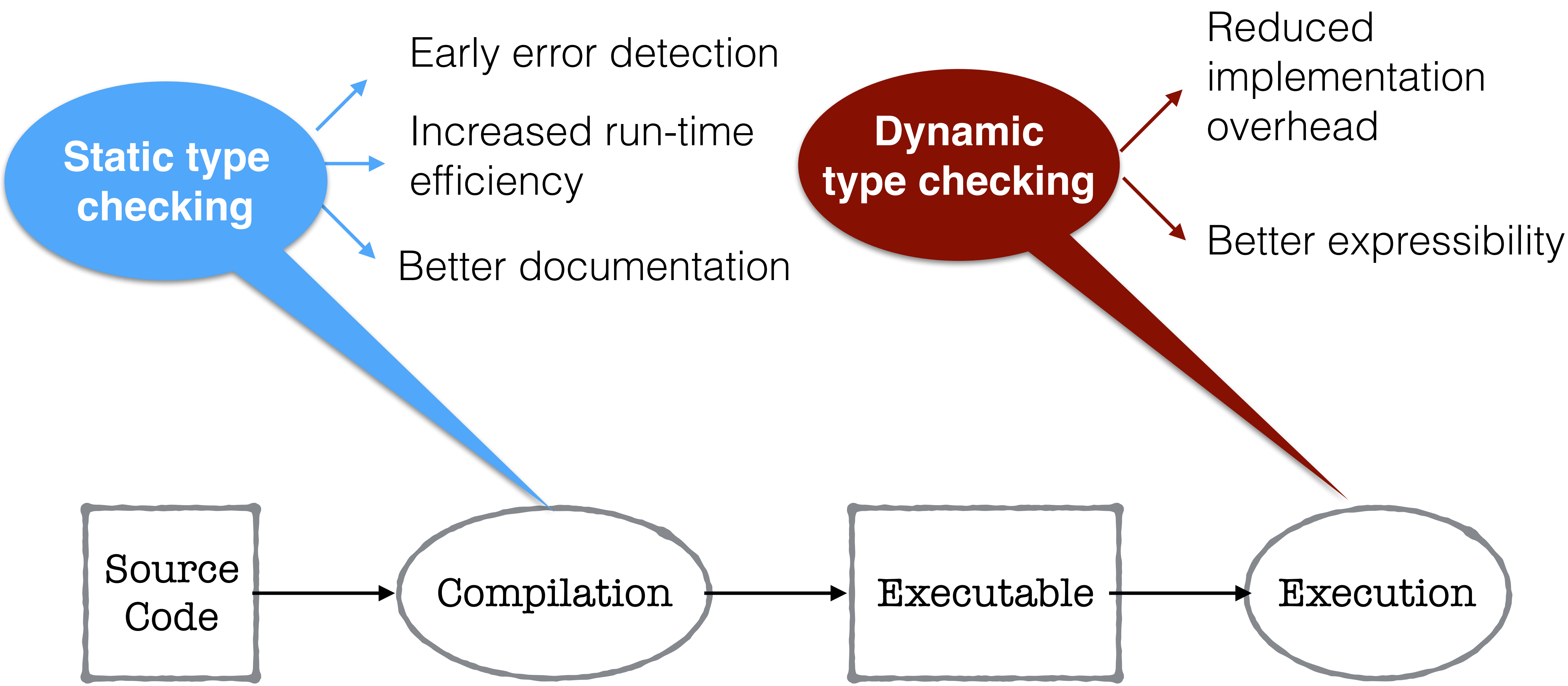

Why We Care

Generally, almost all real-world programming languages have type systems which offers multiple benefits.

Specifically for GI/GP, type systems have the promise to guide the search and avoid the construction of invalid individuals.

Java's Static Type Checking

Suppose we have:

```
class A {
    A me() {
        return this;
    }

    public void doA() {
        System.out.println("Do A");
    }
}

class B extends A {
    public void doB() {
        System.out.println("Do B");
    }
}

class C extends A{
    public void doC() {
        System.out.println("Do C");
    }
}
```

Java's Static Type Checking

```
new B().me().doB();

new B().me().doA();

((B) new B().me()).doB();

((C) new B().me()).doC();
```

Java's Static Type Checking

```
new B().me().doB();
```

Illegal. Compiler thinks new B().me() returns an object of class A, but at run-time, this returns an objects of class B.

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Legal.

```java
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Java's Static Type Checking

`new B().me().doB();`	Illegal. Compiler thinks new B().me() returns an object of class A, but at run-time, this returns an objects of class B.
`new B().me().doA();`	Legal.
`((B) new B().me()).doB();`	Legal.
`((C) new B().me()).doC();`	Legal. But throws cast exception at run-time.

Hindley Milner's Type System

One of the most famous type systems for the typed λ-calculus with parametric polymorphism:

- A fast (nearly linear time) algorithm that automatically infer types of the constructs from their usage

- A set of typing rules, e.g.

$$\frac{\Gamma \vdash e_0 : \tau \rightarrow \tau' \qquad \Gamma \vdash e_1 : \tau}{\Gamma \vdash e_0\ e_1 : \tau'} \quad [\mathrm{App}]$$

HM Example

Let us assume that we have a function myFunc of type:

$$\text{myFunc} : \text{ADT} \longrightarrow \text{int}$$

And we want to infer the type of a function someFunc

someFunc (x) + myFunc (x)

Step One

someFunc (x) + myFunc (x)

Step One

someFunc (x) + myFunc (x)

↓

x : α

Step Two

someFunc (x) + myFunc (x)

↓ ↓

$x : \alpha$ $\quad$ $ADT \longrightarrow int$

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α = ADT

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↓

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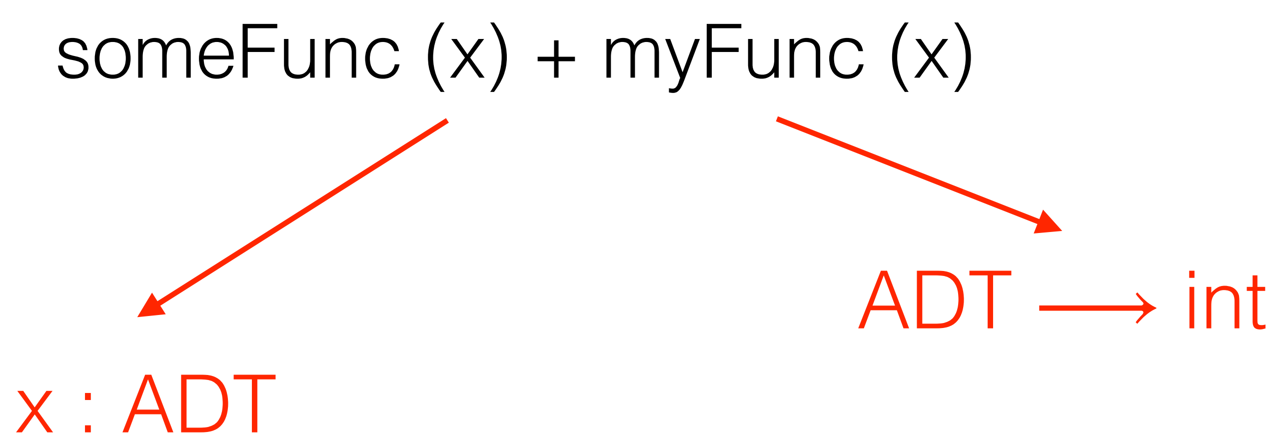
Step Three



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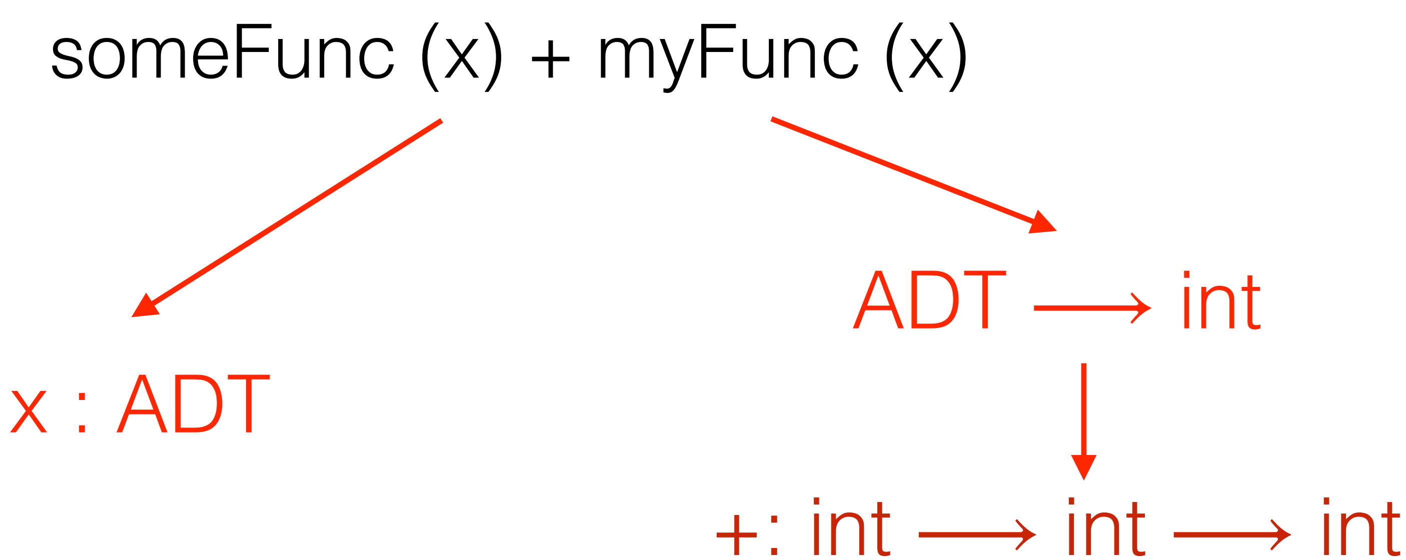

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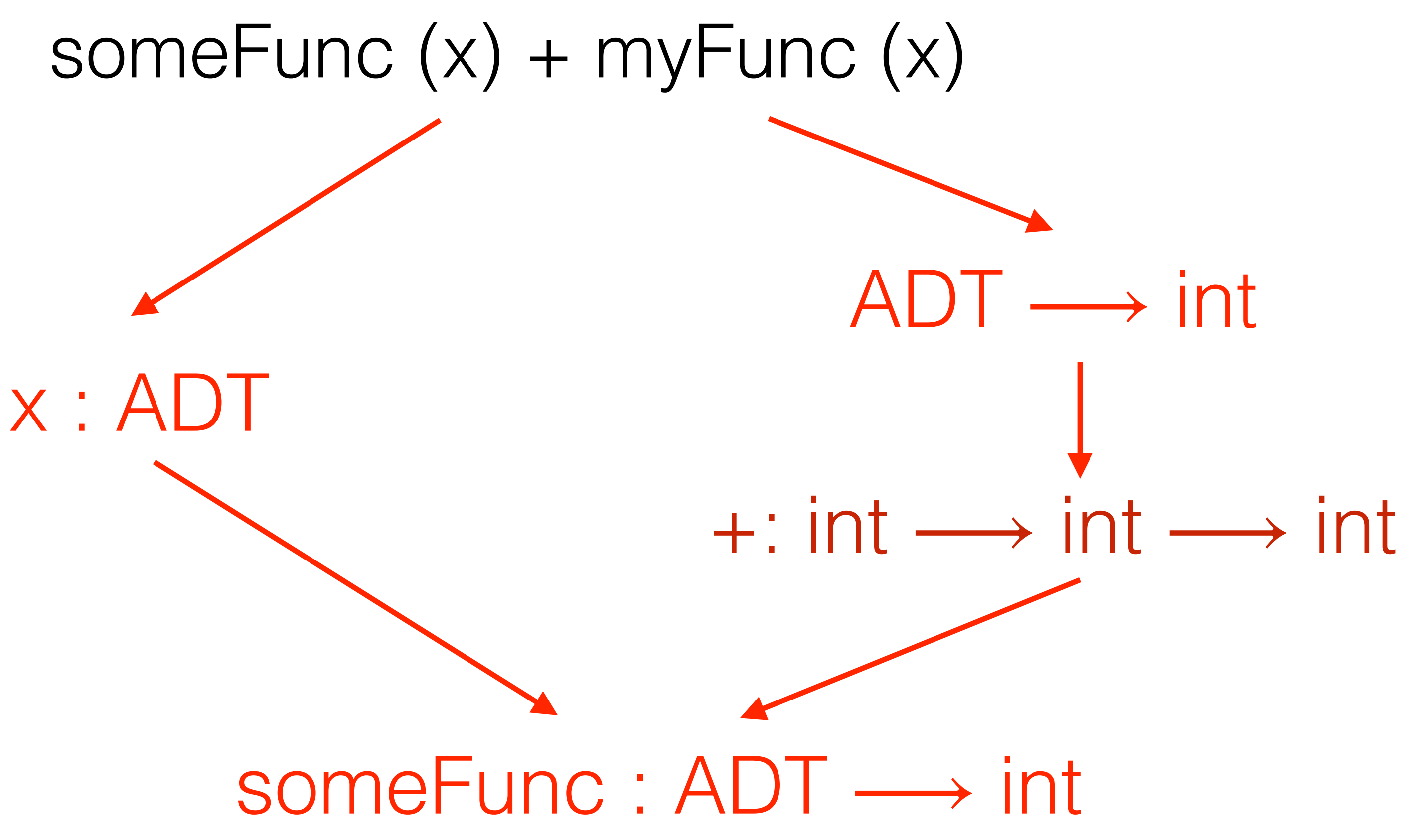

Polymorphism

The provision of
a single interface
to entities of
different types

Polymorphism {

Parametric

Ad Hoc

Inclusion

Parametric Polymorphism

Generic programming in programming languages

```
class List<T> {
    class Node<T> {
        T elem;
        Node<T> next;
    }
    Node<T> head;
    int length() { ... }
}

List<B> map(Func<A,B> f, List<A> xs) {
    ...
}
```

Rank-N polymorphic function is a function whose parameters are Rank-(N-1) polymorphic

Ad Hoc Polymorphism

Function overloading in programming languages

```
function Add( x, y : Integer ) : Integer;
begin
    Add := x + y
end;

function Add( s, t : String ) : String;
begin
    Add := Concat( s, t )
end;
```

Inclusion Polymorphism

Inheritance creates inclusion polymorphism (subtyping)

```
abstract class Animal {
    abstract String talk();
}

class Cat extends Animal {
    String talk() {
        return "Meow!";
    }
}

class Dog extends Animal {
    String talk() {
        return "Woof!";
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```

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```

Cat < Animal

Dog < Animal

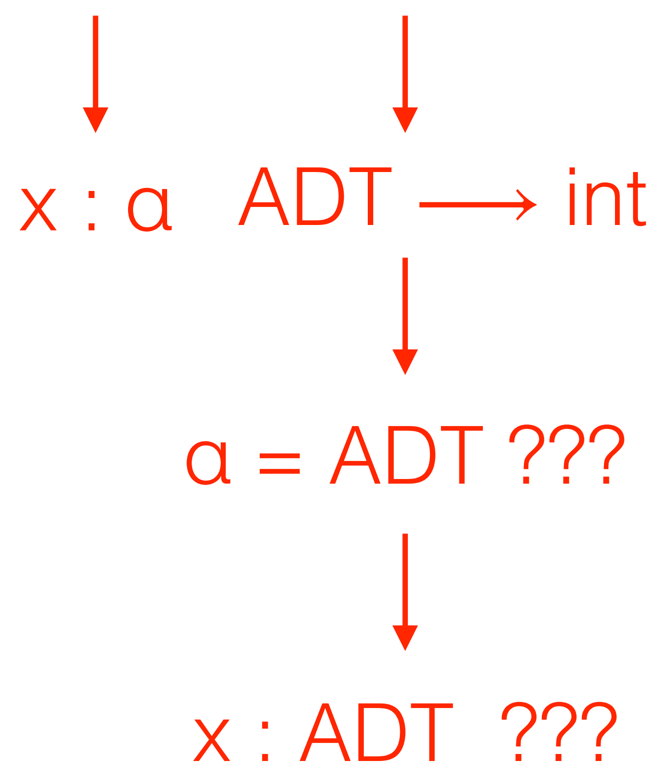
HM Limitations

- Limited to rank 1 parametric polymorphism
- Does not support ad hoc polymorphism
- No notion of subtyping

Limitation Example One

Suppose we have subtyping B < A, any function that takes arguments of type A is expected to takes arguments of type B as well.

someFunc (x) + myFunc (x)



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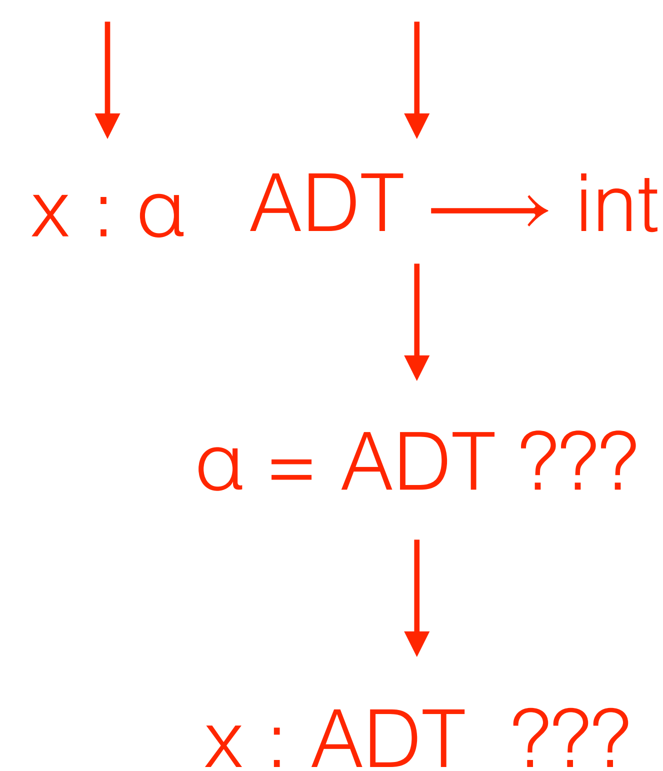

α could be any subtype of ADT

Limitation Example Two

In HM, an assumption set may contain at most one typing assumption for an construct

The operator < , for example, has types:

$\text{char} \longrightarrow \text{char} \longrightarrow \text{bool}$

$\text{int} \longrightarrow \text{int} \longrightarrow \text{bool}$

But it does not have the type:

$\forall \alpha . \alpha \longrightarrow \alpha \longrightarrow \text{bool}$

So any single typing is either too narrow or too wide

Intersection Types

Allow a term to have multiple types by introducing a type constructor ∧, a universal type ω used for untypable constructs, and the following typing rules:

$$\frac{M : (\sigma_1 \wedge \sigma_2)}{M : \sigma_1} \quad \frac{M : (\sigma_1 \wedge \sigma_2)}{M : \sigma_2} \qquad (\wedge E)$$

$$\frac{M : \sigma_1 \quad M : \sigma_2}{M : (\sigma_1 \wedge \sigma_2)} \qquad (\wedge I)$$

In practice, intersection types enable **function overloading**.

Union Types

The dual notion of intersection types, which introduces a type constructor $\vee$ and similar typing rules.

In C / C++, union types are the construct **union**

Example

Consider the following code snippet in C++:

```
typedef struct {
    char c;
    bool b;
} ADT;

typedef union {
    int i;
    ADT a;
} unionType;

void foo(unionType x, int y) {};
void foo(unionType x, float y) {};
```

The type of function **foo** would be:

$$((\text{int} \vee \text{ADT}) \longrightarrow \text{int} \longrightarrow \text{void}) \wedge ((\text{int} \vee \text{ADT}) \longrightarrow \text{float} \longrightarrow \text{void})$$

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Retype

A general tool that automatically replaces certain types, together with the corresponding operations if necessary, of a program with new ones.

Potential Applications

Reducing energy consumption

Precision tracking and improvement for FP programs

New mutation operators in GI/GP

Taint analysis

Symbolic execution

Auto-transplantation

Intersection Types in Retype

We use intersection types to cleanly model function overloading, because Retype may generate new overloads of an existing operator.

Consider the following code snippet:

```
int main() {
    int a, b;
    b = a + 2;
    a = b + ext_func(a);
    return 0;
}
```

<u>Assumption</u>: an external function ext_func of type int $\longrightarrow$ int

<u>Objective</u>: retype int to ADT

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Before retyping

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int main() {
    ADT a, b;
    b = a + 2;
    a = b + ext_func(a);
    return 0;
}
```

After retyping

+: ADT ⟶ ADT ⟶ ADT

+: ADT ⟶ **int** ⟶ ADT

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