Approximate Program Transformation

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Outline

Timing Attacks

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P.C. Kocher: *Cryptanalysis of Diffie-Hellman, RSA, DSS, and other cryptosystems using timing attacks*, CRYPTO '95.

A simple Algorithm for ME

The simple modular exponentiation algorithm below computes $R = y^x \mod n$, where x is w bits long:

```
Let s0 = 1.
For k = 0 upto w-1:
If (bit k of x) is 1 then
Let Rk = (sk * y) mod n.
Else
Let Rk = sk.
Let sk+1 = Rk*Rk mod n.
EndFor.
Return (Rw-1).
```

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- making software run in fixed time is hard;
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Program Obfuscation

In order to prevent timing attacks we can 'garble' a program so as to produce another program with the same I/O behaviour as the original one but internally different.

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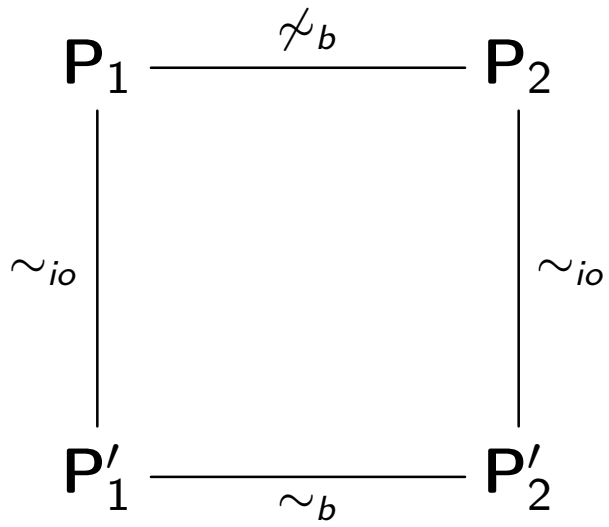

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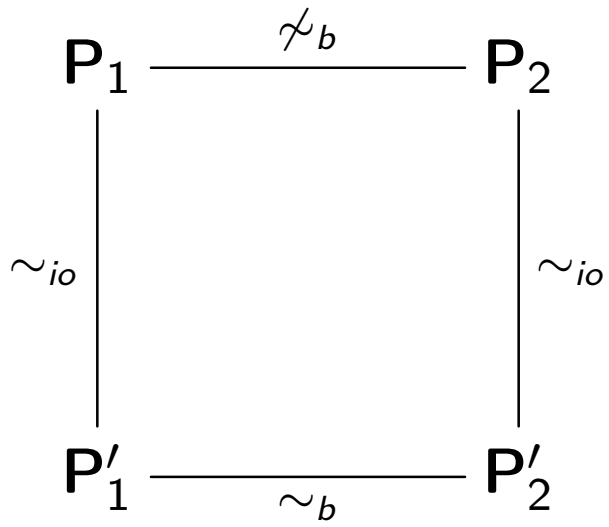

An adversarial entity that look at an obfuscated version P' of a program P is unable to understand the internal working of P.

Outline

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A model for the semantics of probabilistic languages is given via Probabilistic Transition Systems (PTS).

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A Probabilistic Transition System is a tuple $(S, A, \longrightarrow, \pi_0)$:

- S is a non-empty, set of states,
- A is a non-empty, finite set of actions,
- $\longrightarrow \subseteq S \times Dist(A \times S)$ is a transition relation,
- $\pi_0 \in Dist(S)$ is an initial distribution on S.

Matrix Representation

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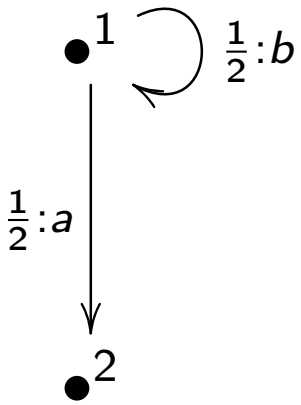
The direct sum of a set $\{\mathbf{M}_i\}_{i=1}^{k}$ of $n_i \times m_i$ matrices, is defined by the $(\sum_{i=1}^{k} n_i) \times (\sum_{i=1}^{k} m_i)$ matrix:

$$\mathbf{M} = \bigoplus_{i} \mathbf{M}_i = \begin{pmatrix} \mathbf{M}_1 & 0 & 0 & \dots & 0 \\ 0 & \mathbf{M}_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \mathbf{M}_k \end{pmatrix}$$

Encoding a PTS into a DTMC



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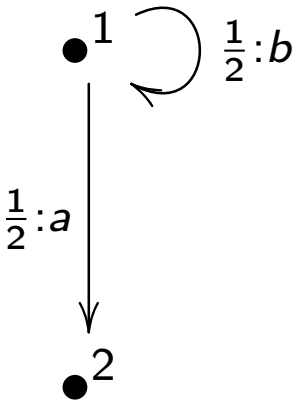

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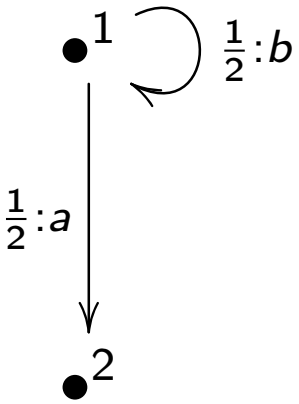

$$\mathbf{M}(A) = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 0 \end{pmatrix}$$

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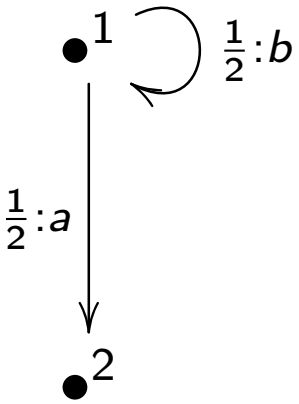

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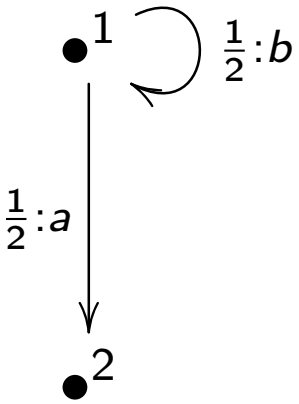

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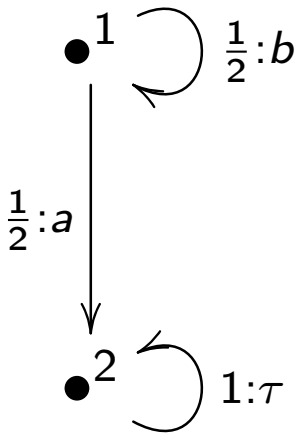

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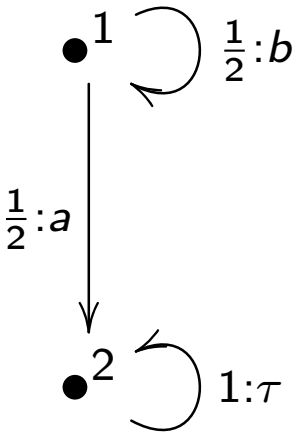

$$\mathbf{M}(A) = \mathbf{M}_a(A) \oplus \mathbf{M}_b(A) \oplus \mathbf{M}_\tau(A)$$

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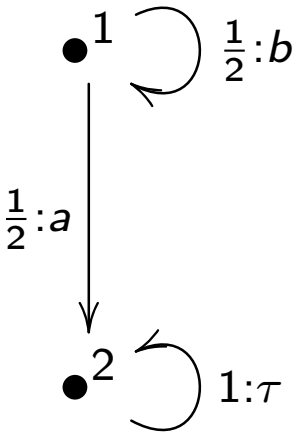

$$\mathbf{M}(A) = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Outline

Classical Bisimulation

A bisimulation is a binary relation $\sim_b$ on states of a labelled transition system satisfying for all $\alpha \in A$:

$$p \sim_b q \text{ and } p \xrightarrow{\alpha} p' \Rightarrow \exists q' : q \xrightarrow{\alpha} q' \text{ and } p' \sim_b q',$$
$$p \sim_b q \text{ and } q \xrightarrow{\alpha} q' \Rightarrow \exists p' : p \xrightarrow{\alpha} p' \text{ and } q' \sim_b p'.$$

Probabilistic Bisimulation

A probabilistic bisimulation is an equivalence relation $\sim_b$ on states of a PTS satisfying for all $\alpha \in A$:

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$$\Rightarrow$$
$$q \longrightarrow_\alpha \varrho \quad \text{and} \quad \pi \sim_b \varrho,$$

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where

$$\pi \sim_b \varrho \quad \text{iff} \quad \forall [s] \in S/_{\sim_b} : \pi([s]) = \varrho([s]).$$

Probabilistic Bisimulation as PAI

Given the operator representation $\mathbf{M}(X)$ and $\mathbf{M}(Y)$ of two PTS's X and Y, then X and Y are (probabilistically) bisimilar iff there exist classification matrices $\mathbf{K}_X$ and $\mathbf{K}_Y$ such that:

$$\mathbf{K}_X^\dagger \mathbf{M}(X)\mathbf{K}_X = \mathbf{K}_Y^\dagger \mathbf{M}(Y)\mathbf{K}_Y$$

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$$(\bigoplus_{\alpha} \mathbf{K}_X^\dagger)\mathbf{M}(X)(\bigoplus_{\alpha} \mathbf{K}_X) = (\bigoplus_{\alpha} \mathbf{K}_Y^\dagger)\mathbf{M}(Y)(\bigoplus_{\alpha} \mathbf{K}_Y)$$

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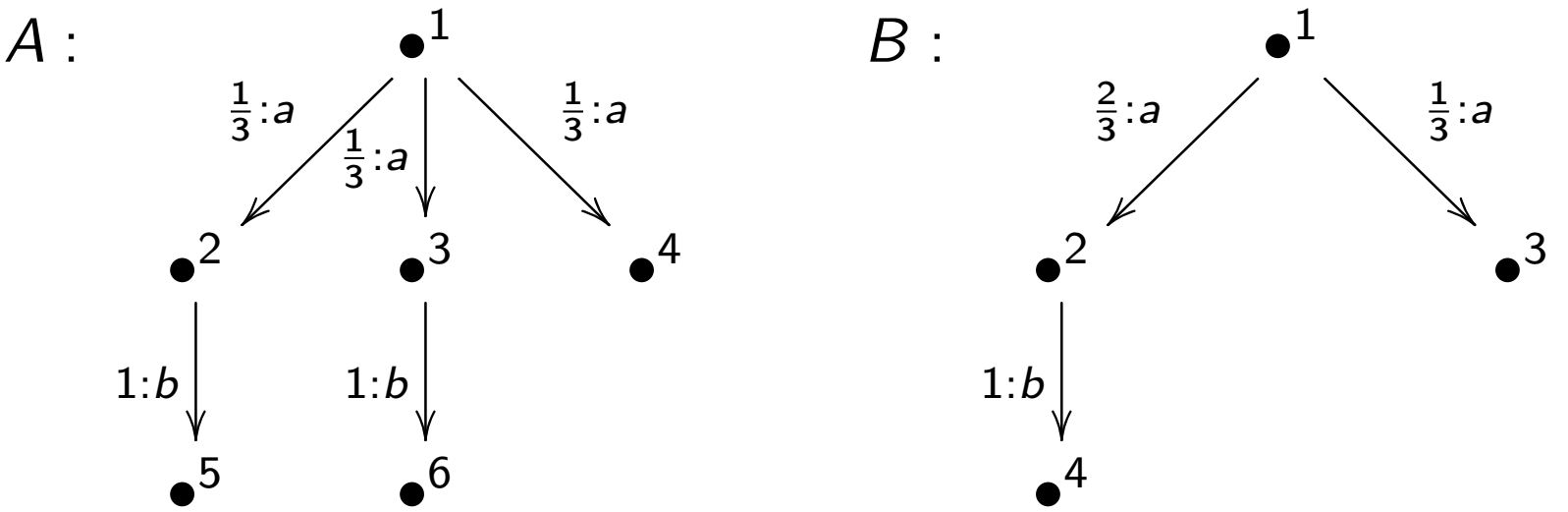
We call an $n \times m$-matrix $\mathbf{K}$ a classification matrix iff $\mathbf{K}$ represents a surjective function $\kappa : \{1, \ldots, n\} \to \{1, \ldots, m\}$

$$\mathbf{K}_{ij} = \begin{cases} 1 & \text{if } j = \kappa(i) \\ 0 & \text{otherwise.} \end{cases}$$

Example



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$$\mathbf{M}(A) = \mathbf{M}_a(A) \oplus \mathbf{M}_b(A) =$$

$$= \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example

$$\mathbf{M}(B) = \mathbf{M}_a(B) \oplus \mathbf{M}_b(B) =$$

$$= \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Example

$$\mathbf{K}_A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{K}_A^{\dagger} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

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$$\mathbf{K}_A^\dagger \mathbf{M}(A) \mathbf{K}_A = \mathbf{M}(B)$$

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$$\begin{aligned} \mathsf{K}_A^\dagger \mathsf{M}_a(A) \mathsf{K}_A &= \mathsf{M}_a(B) \\ \mathsf{K}_A^\dagger \mathsf{M}_b(A) \mathsf{K}_A &= \mathsf{M}_b(B) \end{aligned}$$

Outline

Security Types

Security levels	$s ::= \mathsf{L} \mid \mathsf{H}$
Base types	$\overline{\tau} ::= \mathbf{Int} \mid \mathbf{Bool}$
Security types	$\tau ::= \overline{\tau}_s$

Low Equivalence

We associate to every variable x a security level

$$\Gamma(x) \in \bar{\tau}_{\{L,H\}}$$

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Identify environments which agree on the low variables:

$$E_1 =_L E_2 \quad \text{iff} \quad \forall l \text{ with } \Gamma(l) = \bar{\tau}_L : E_1(l) = E_2(l)$$

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Identify configurations which agree on the low variables:

$$\langle E_1 \mid C \rangle =_L \langle E_2 \mid C \rangle \text{ iff } \forall l \text{ with } \Gamma(l) = \bar{\tau}_L : E_1(l) = E_2(l)$$

Lifted Bisimilarity

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An equivalence relation $\sim \subseteq S \times S$ on S can be lifted to an (equivalence) relation $\sim^* \subseteq \mathbf{Dist}(S) \times \mathbf{Dist}(S)$ on distributions on S via

$$\mu \sim^* \nu \text{ iff } \forall [s] \in S/_\sim : \mu([s]_\sim) = \nu([s]_\sim).$$

Probabilistic Time Bisimilarity

Define the transition relation $\Longrightarrow \in (\mathbf{Conf}, \mathbf{JDist})$ between configurations to joint distributions $\chi \in \mathbf{JDist} = \mathbf{Dist}(\mathbb{T} \times \mathbf{Conf})$, that is distributions on time $t \in \mathbb{T}$ and states $s \in \mathbf{Conf}$ such that $\chi(c', t) = p$ iff $c \stackrel{p:t}{\Longrightarrow} c'$.

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A probabilistic time bisimilarity $\sim_{PT} = \sim$ is the largest symmetric relation on configurations such that whenever $c_1 \sim c_2$, then

$$c_1 \Longrightarrow \chi_1 \text{ implies } \exists \chi_2 \text{ such that } c_2 \Longrightarrow \chi_2 \text{ and } \chi_1 \sim_L^* \chi_2$$

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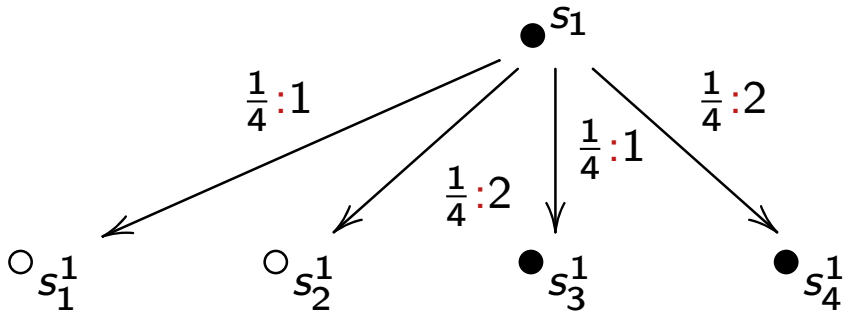
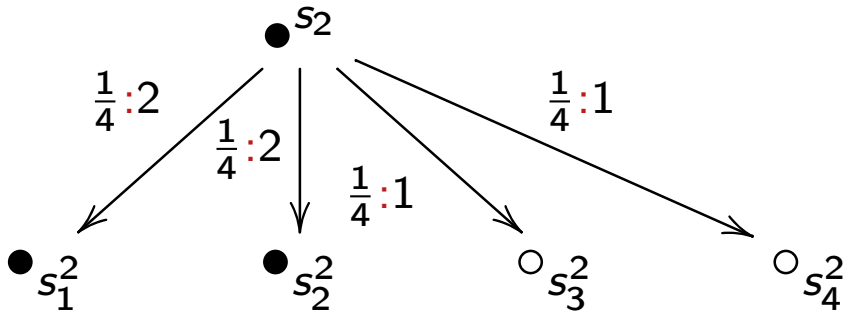
$$\text{with } \langle E_1 \mid C \rangle \sim_L \langle E_2 \mid C \rangle \text{ iff } \forall l \text{ with } \Gamma(l) = \bar{\tau}_L : E_1(l) = E_2(l)$$

$$\sim_L^* = (\sim_L)^* \neq (\sim^*)_L$$





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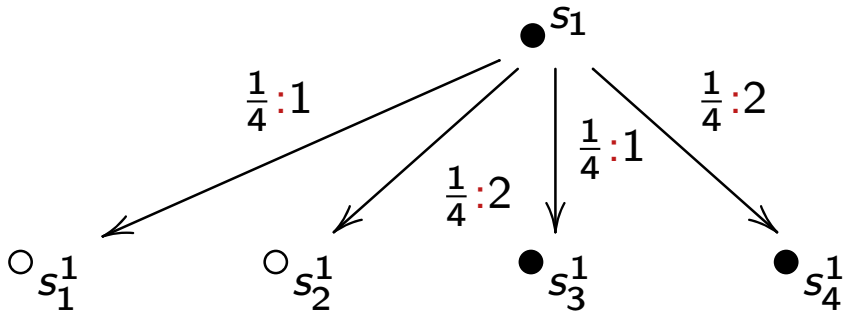

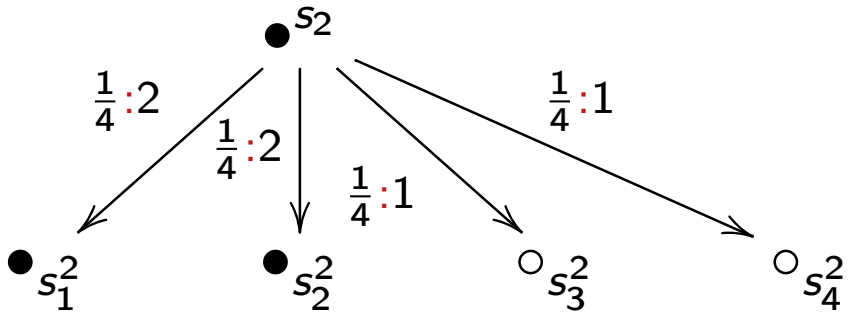

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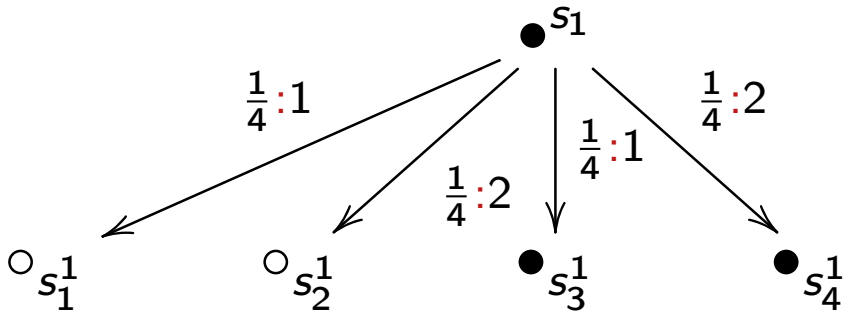

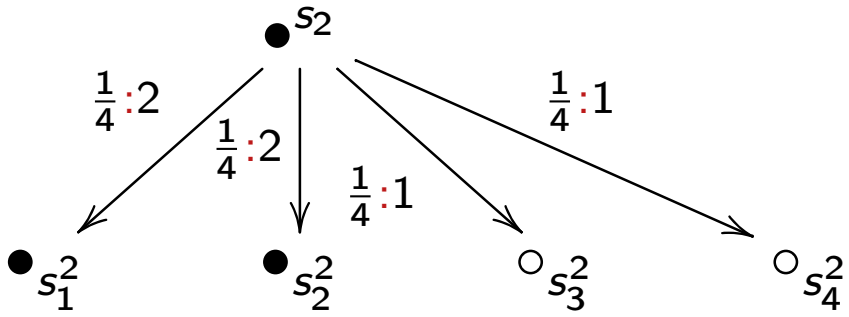

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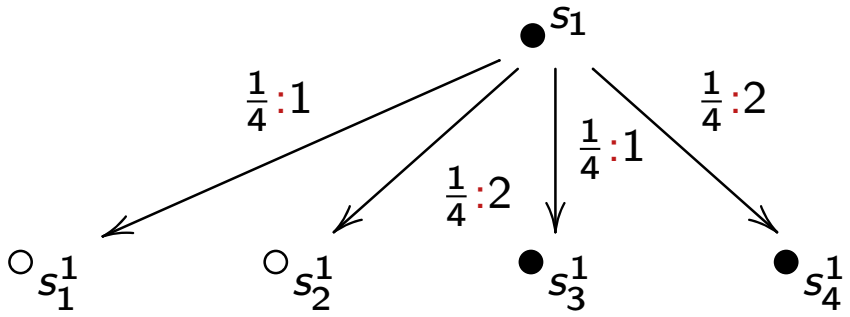

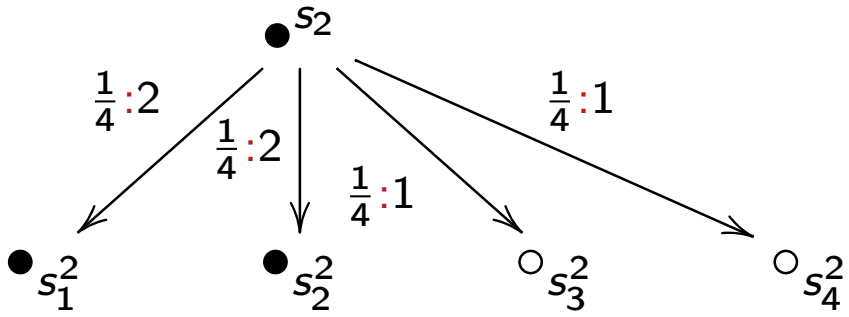

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Bisimilarity and Security

Security of code C corresponds to bisimilar behaviour for all low-equivalent initial configurations.

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- Can we make T_1 and T_2 bisimilar?

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Checking whether two systems are bisimilar or not is important in many other fields, e.g.

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where for $X \subseteq \Sigma$, $pre(X) = \{s \in \Sigma \mid s \rightarrow x \text{ for some } x \in X\}$

Are T_1 and T_2 bisimilar?

In order to check whether two execution trees T_1 and T_2 in our PTS model are bisimilar, apply a similar refinement technique

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1: procedure Lumping(T)
2:     P ← {S}
3:     while ¬Stable(P, T) do
4:         choose B ∈ P
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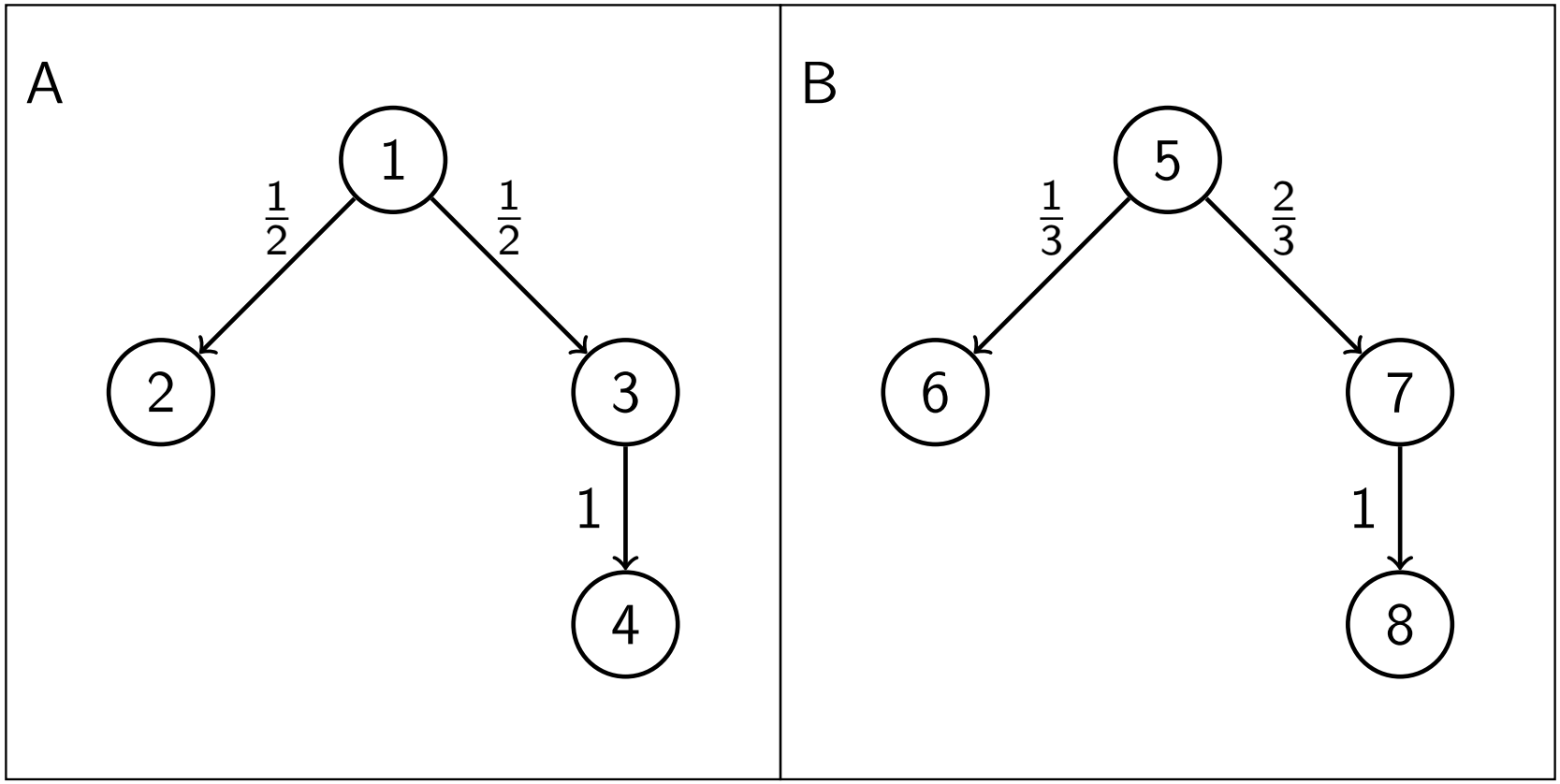
Lumping



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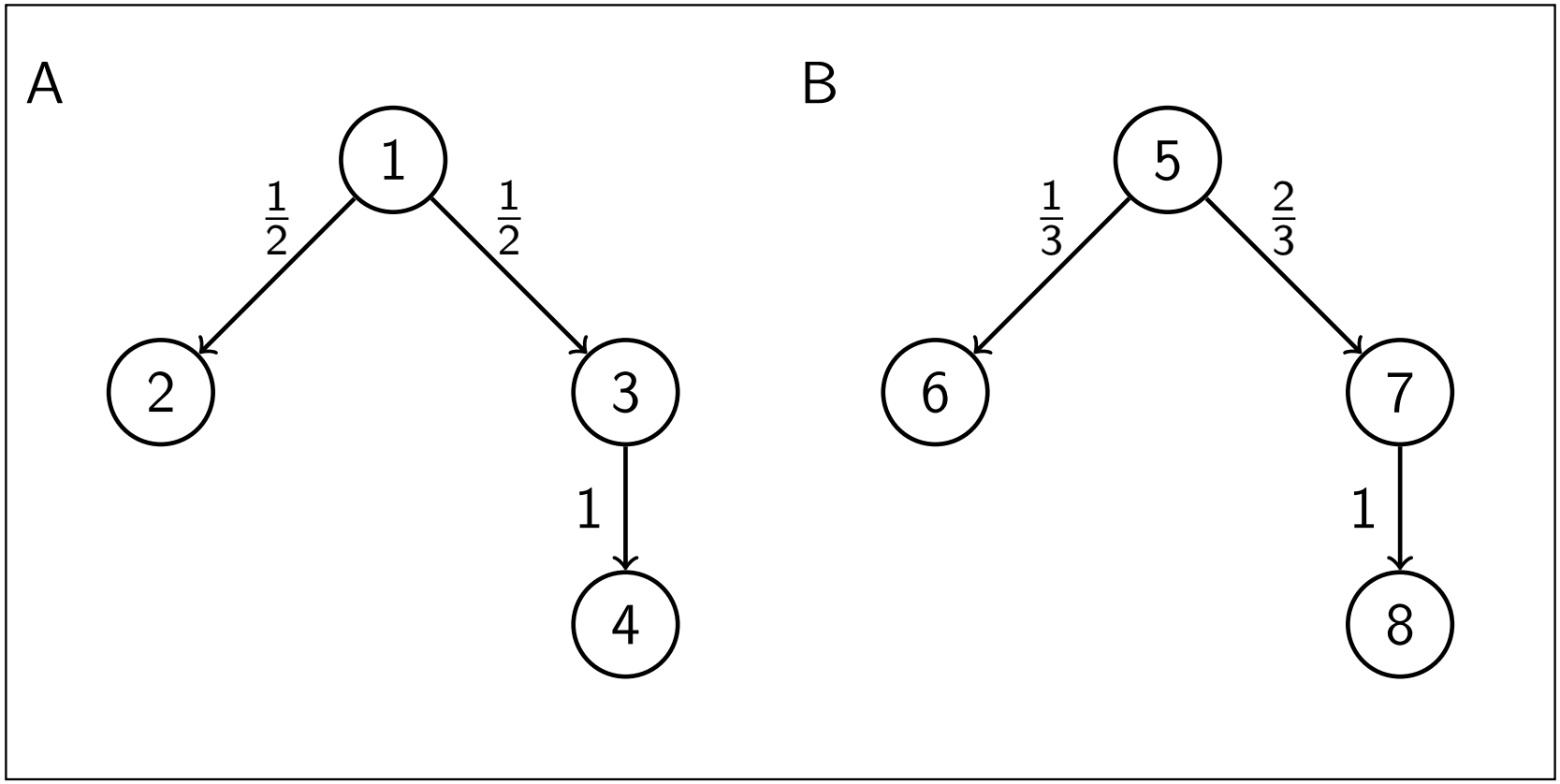

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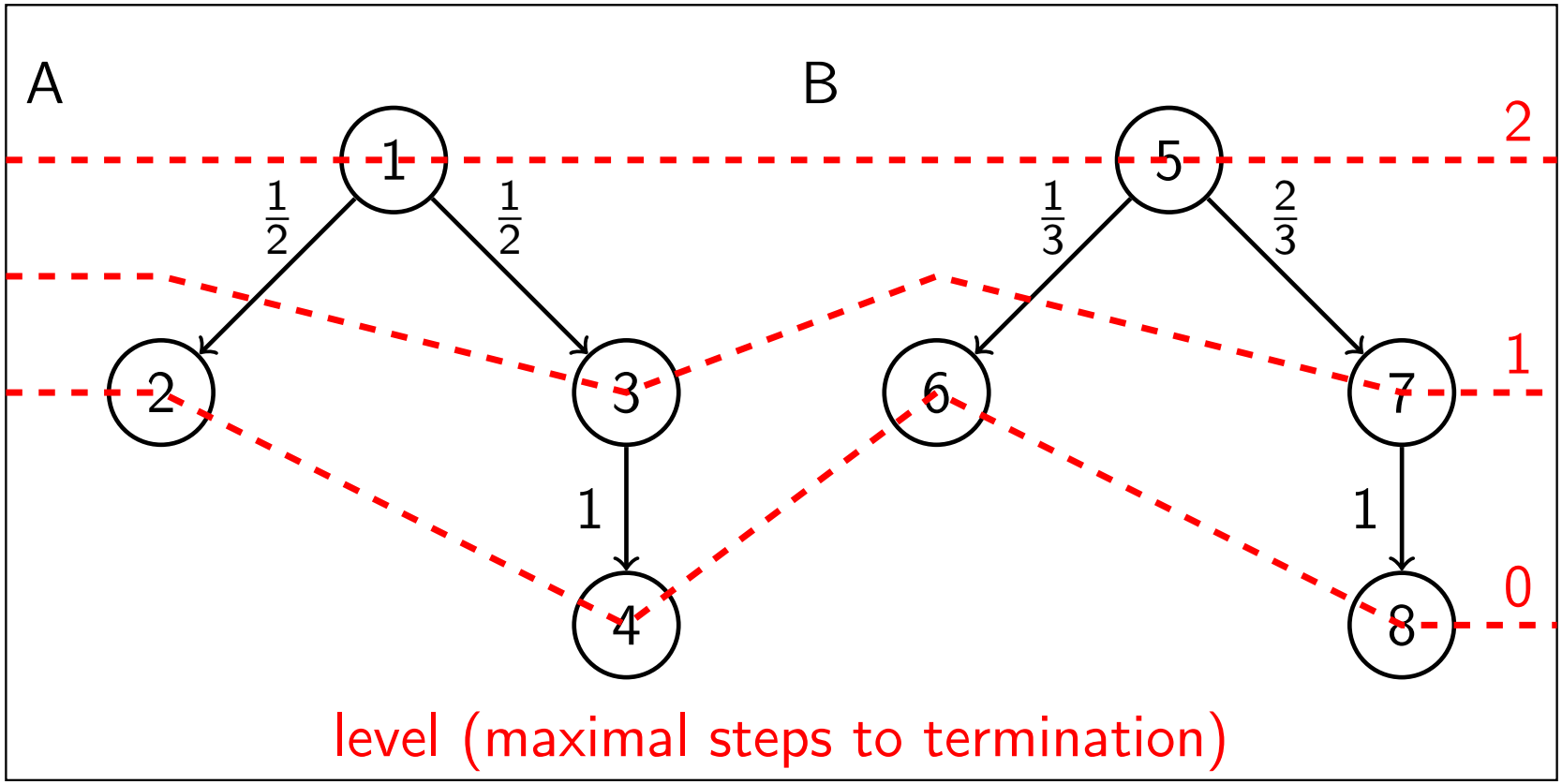
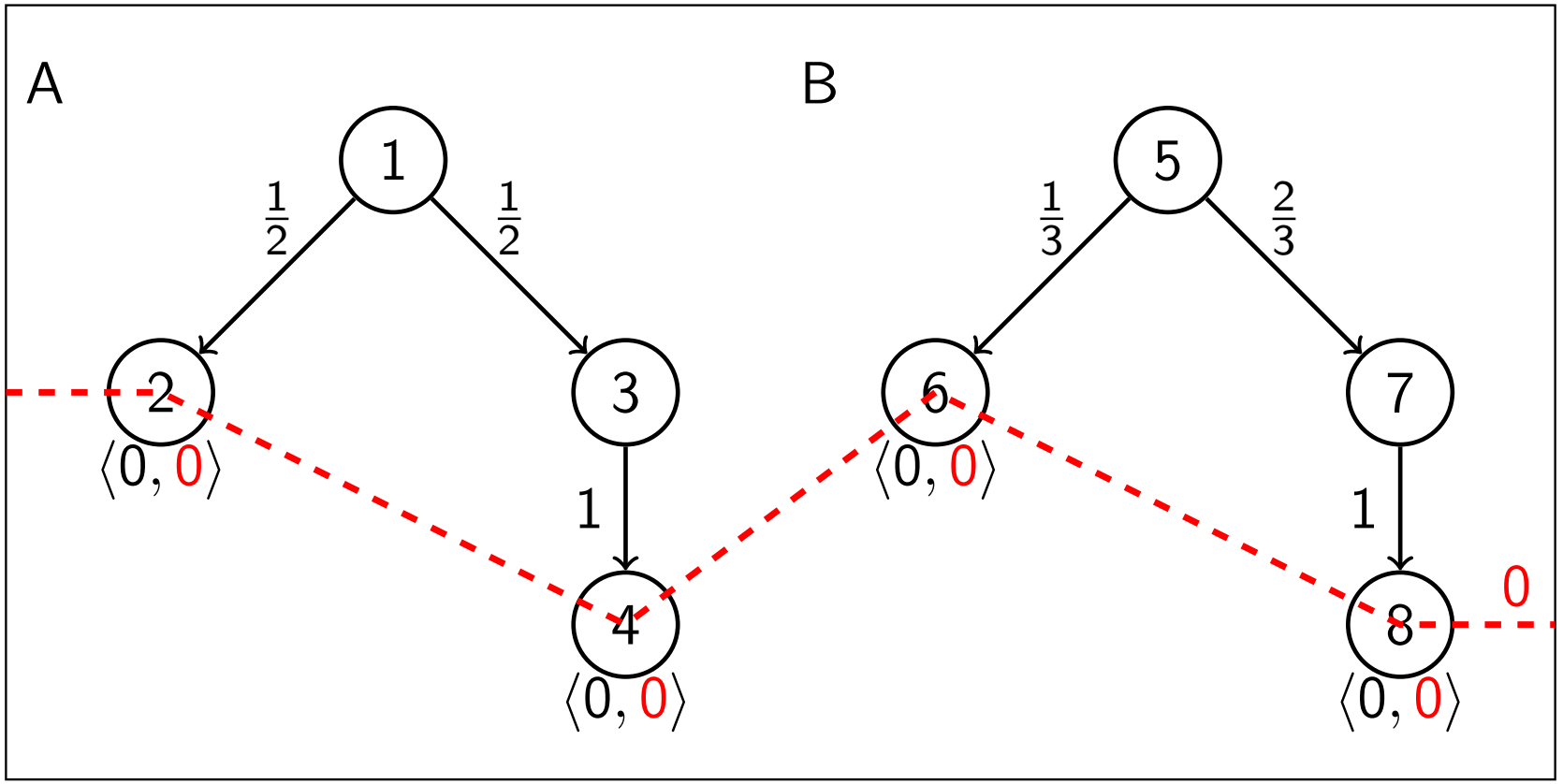

A
B
1
2
3
4
5
6
7
8
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{3}$
$\frac{2}{3}$
1
1
2
1
0
level (maximal steps to termination)

Lumping



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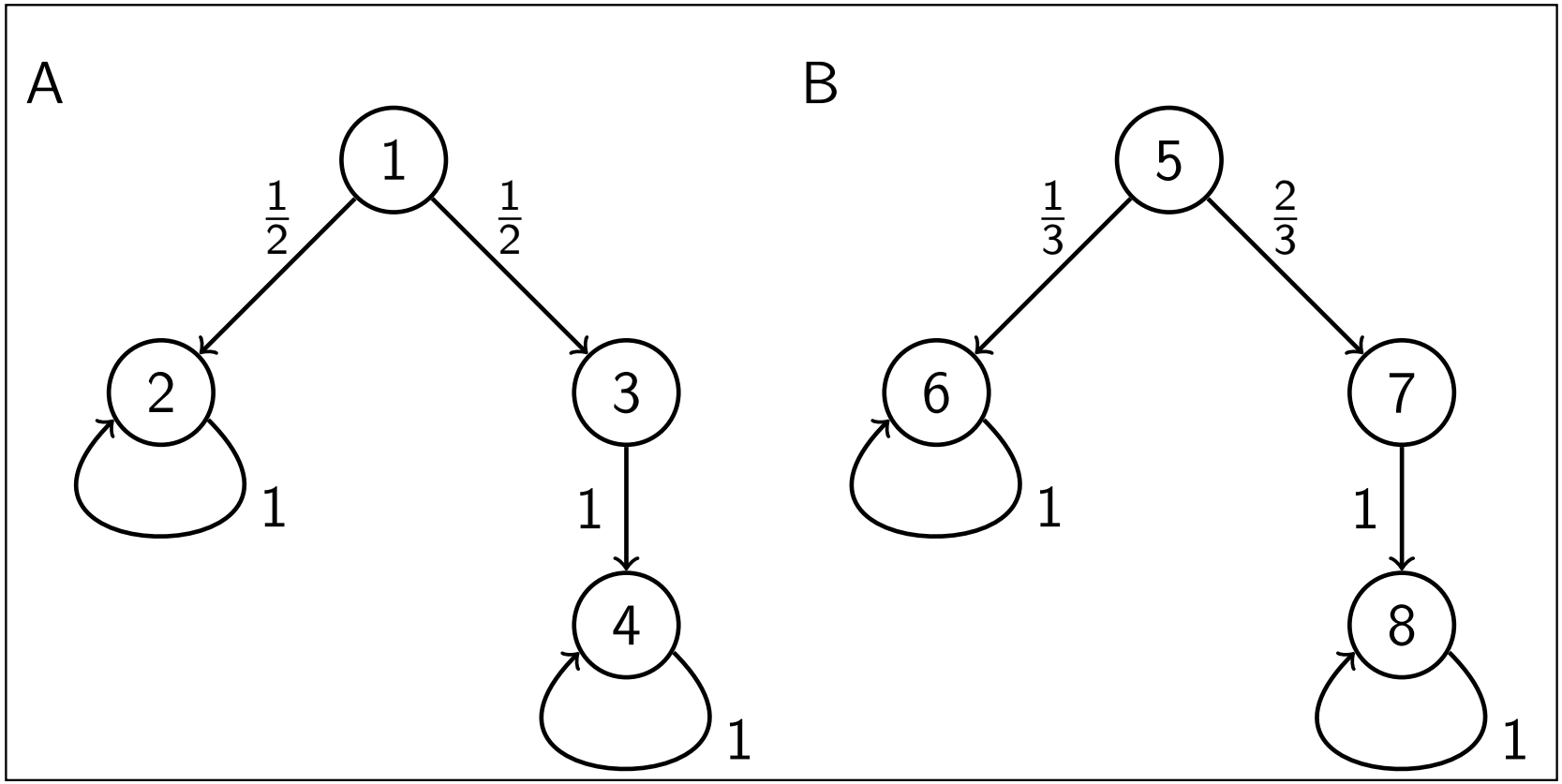
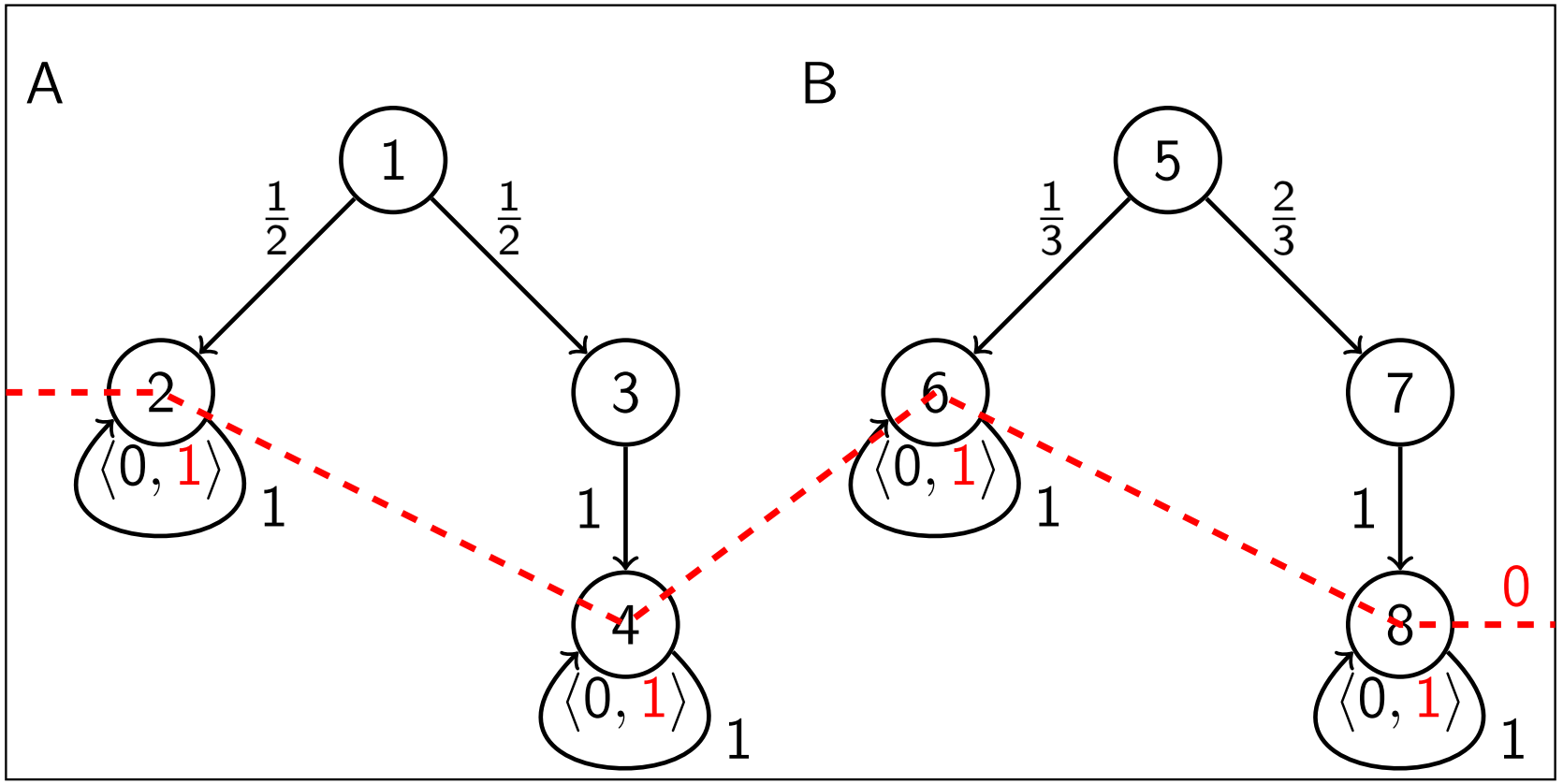


Lumping



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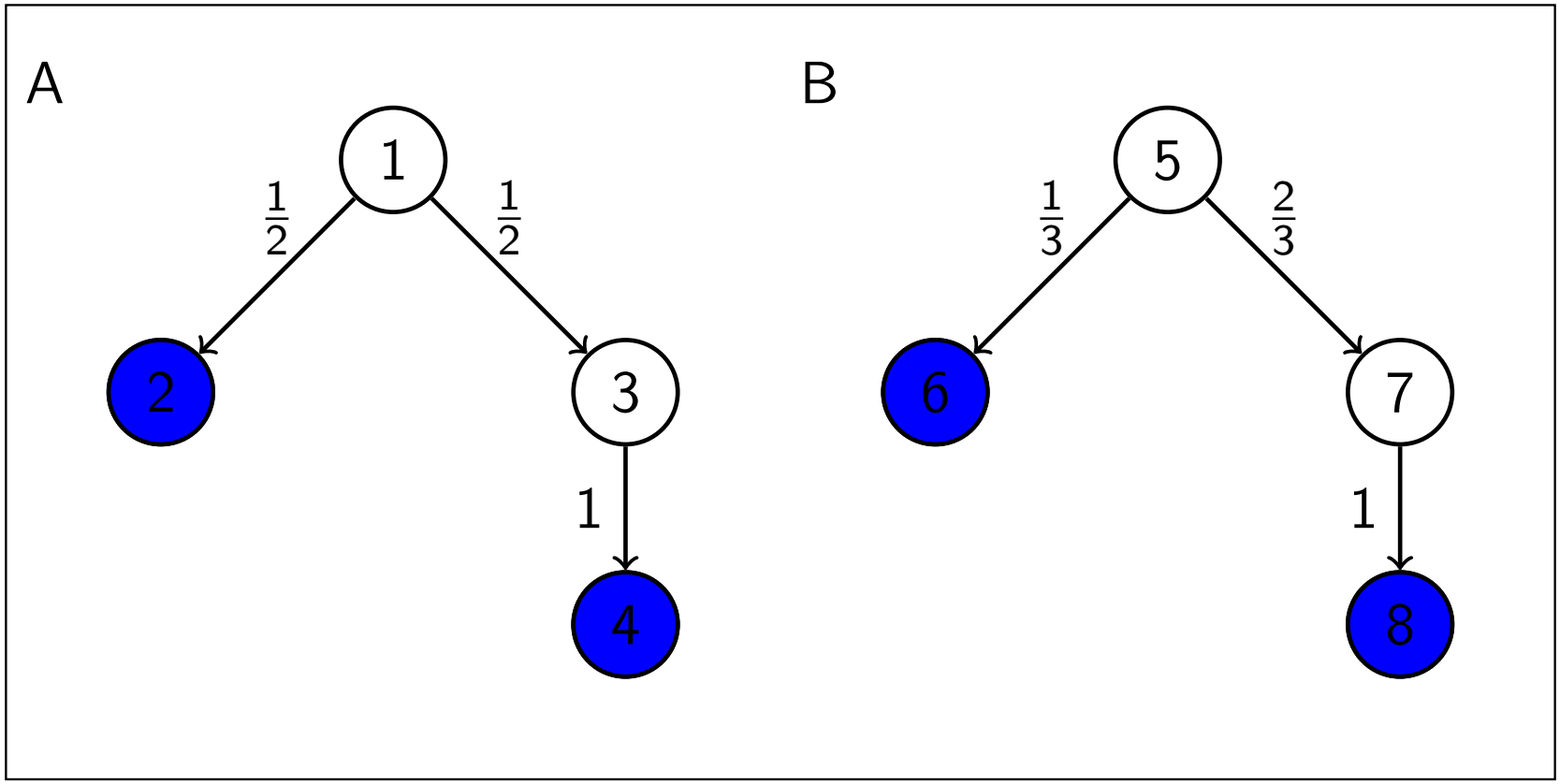
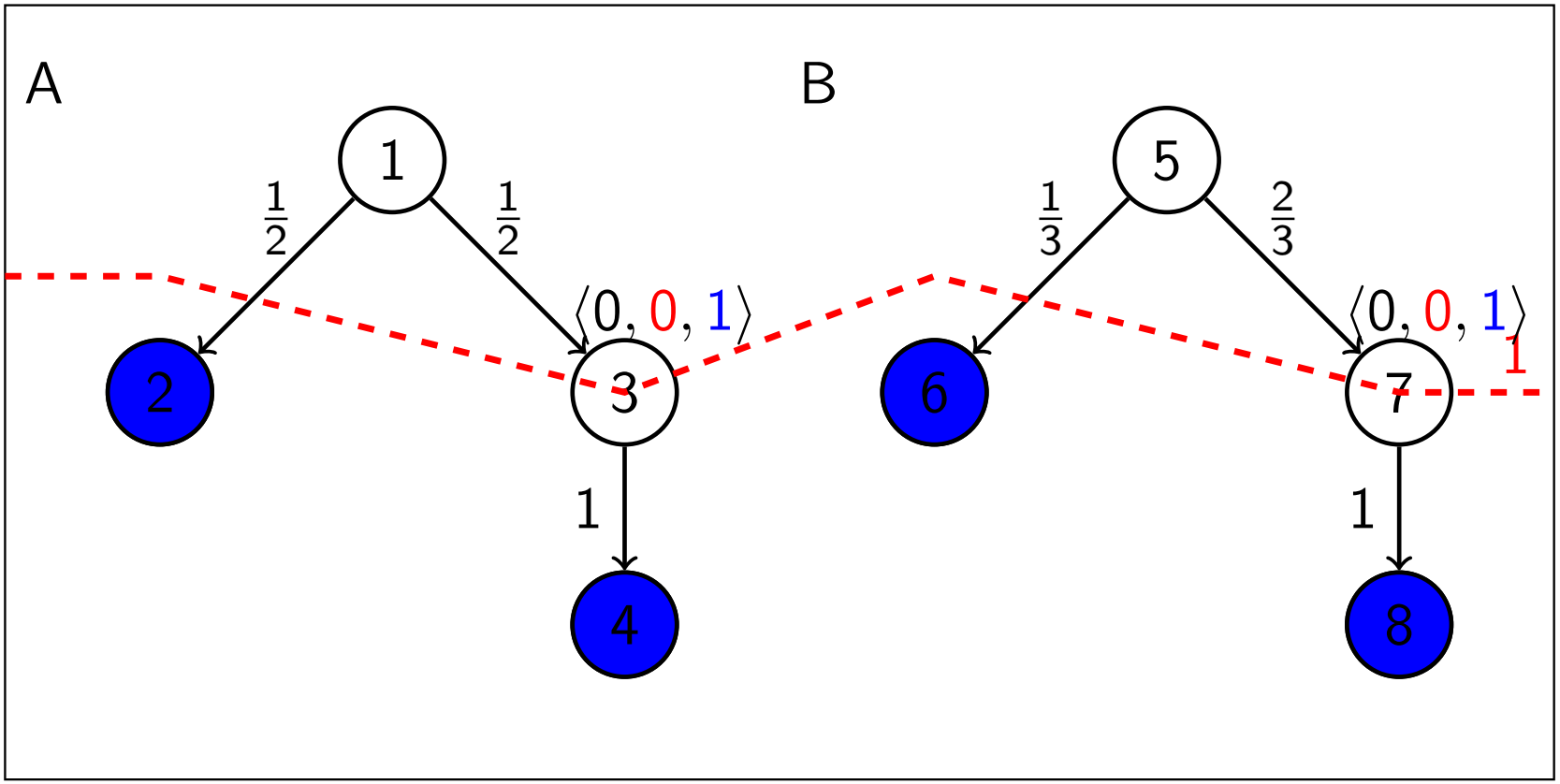


Lumping



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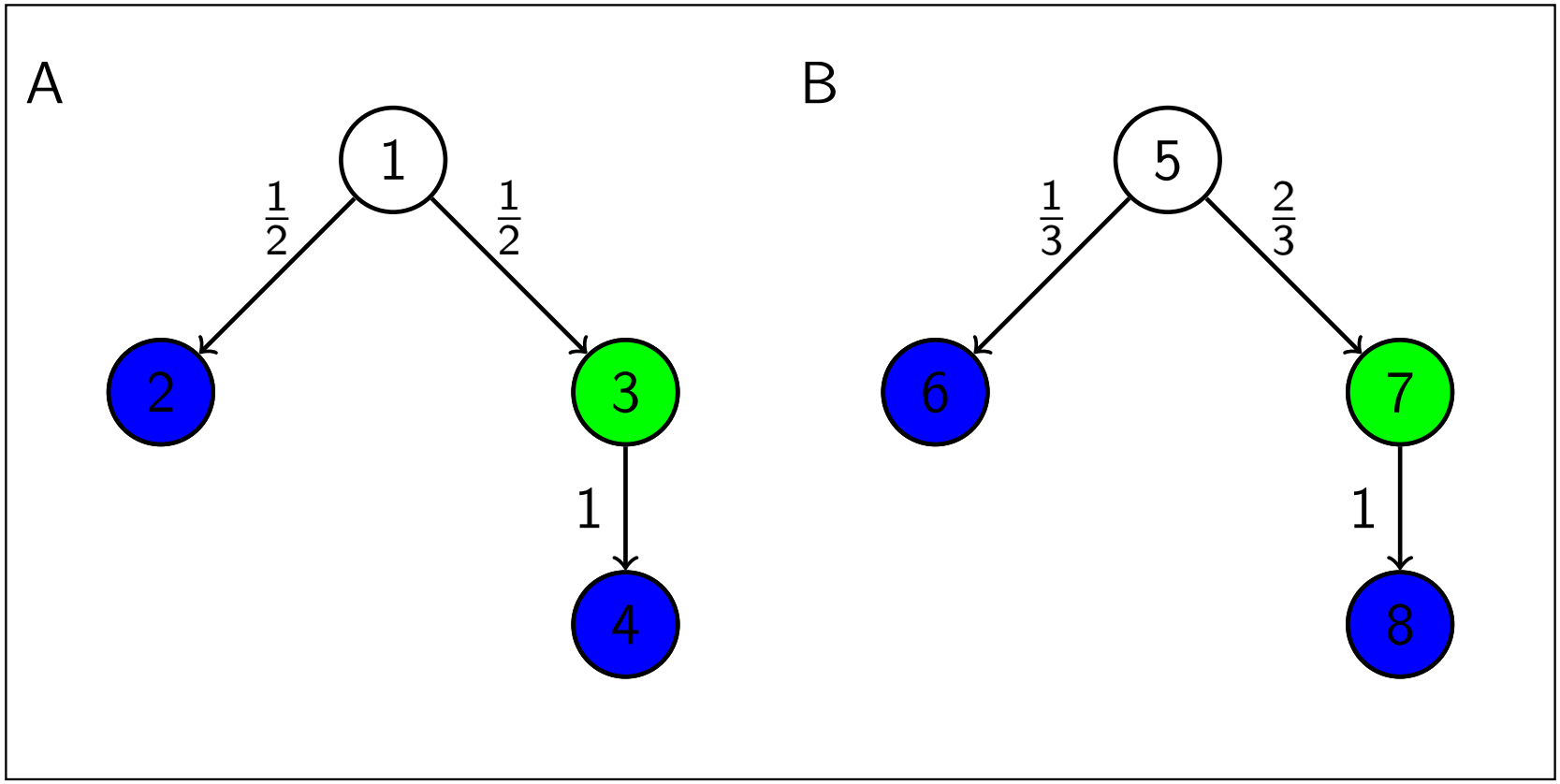
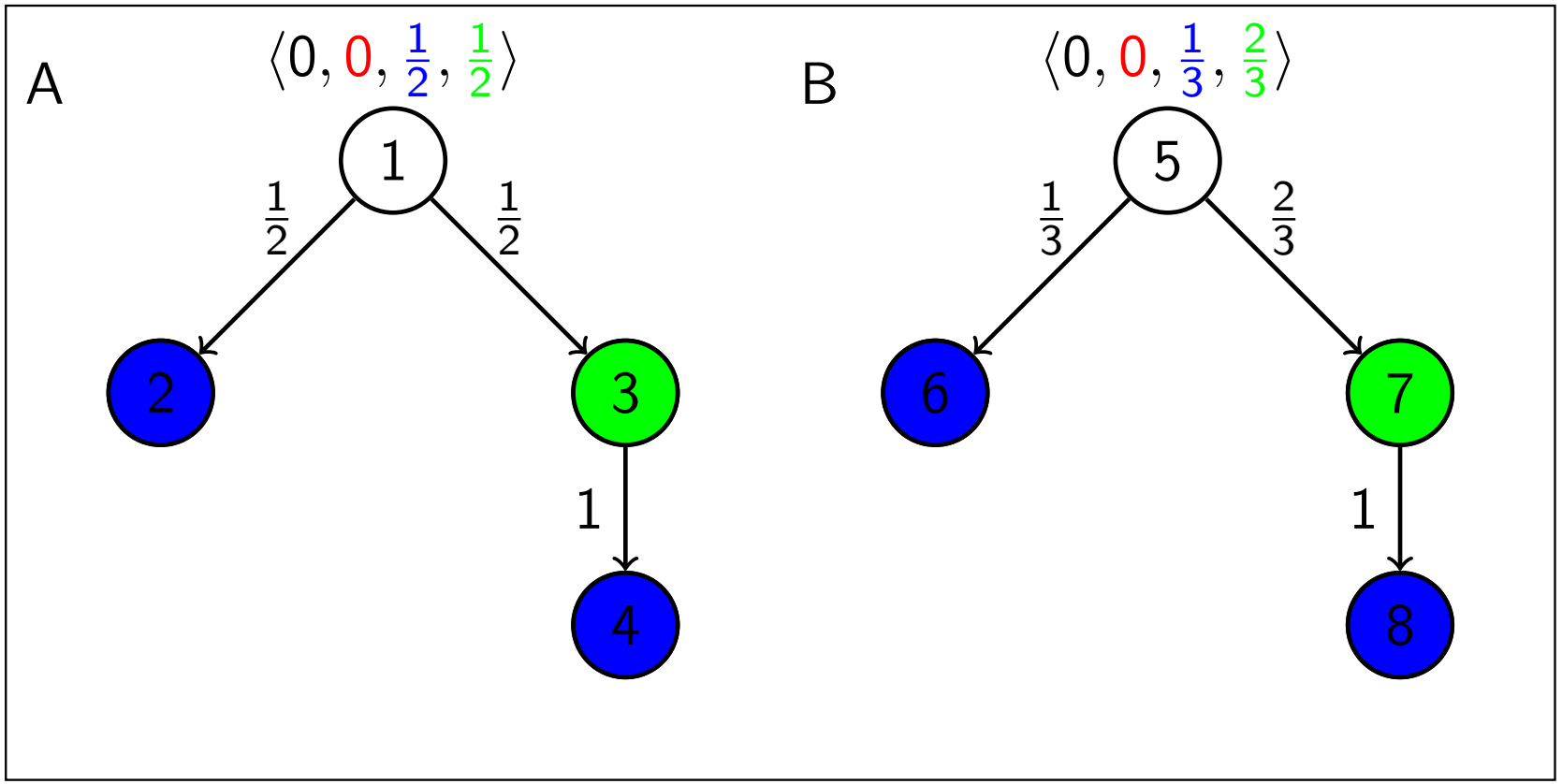
Lumping



Lumping



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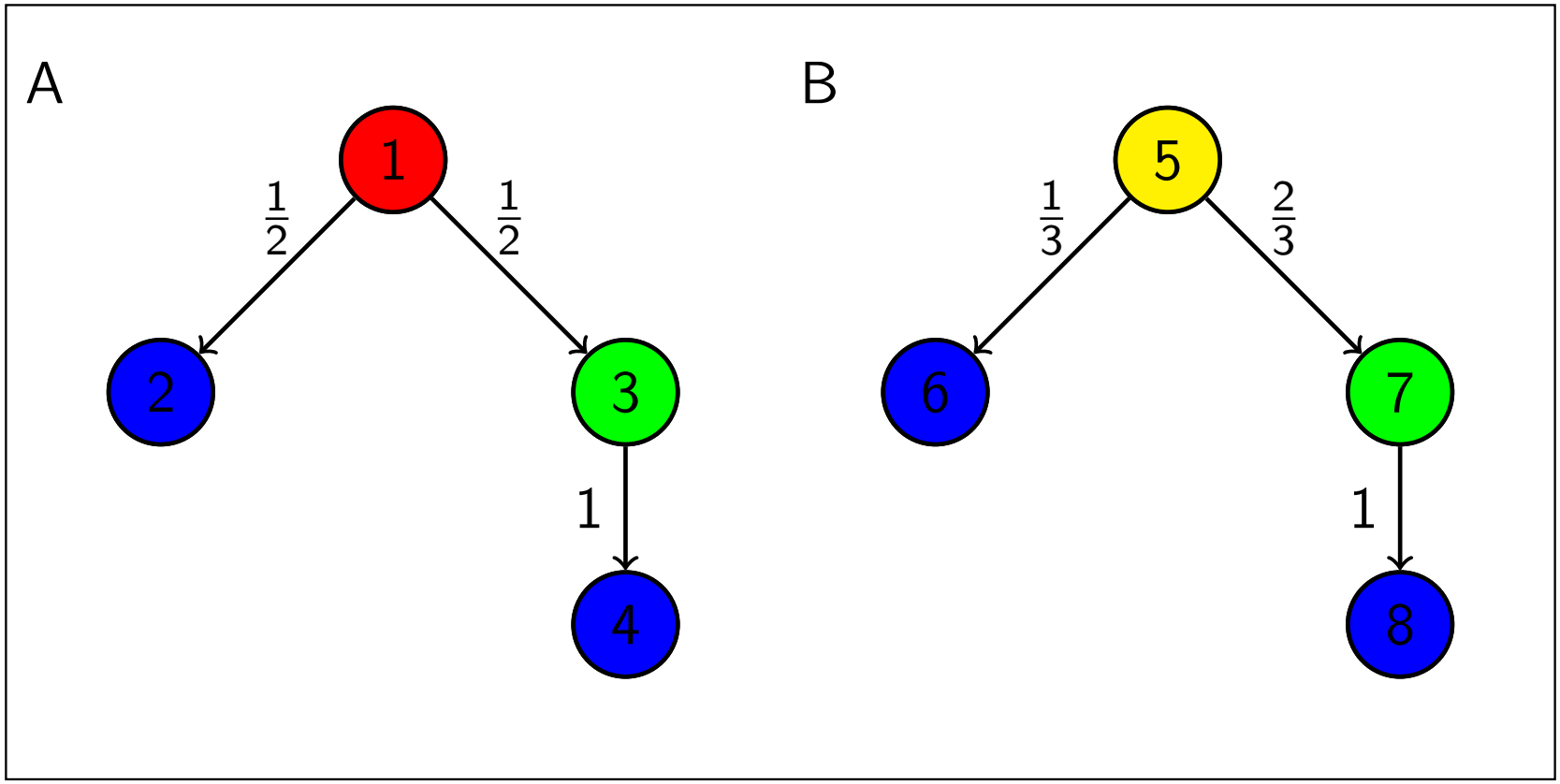

Outline

Program Transformation

If two programs are not indistinguishable we make them so by transforming their internal behaviour.

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In order to "fix" the bisimulation we introduce depleted actions/statement/transitions, e.g. statements which have no effect (e.g. tick) labels but just wait for time passing.

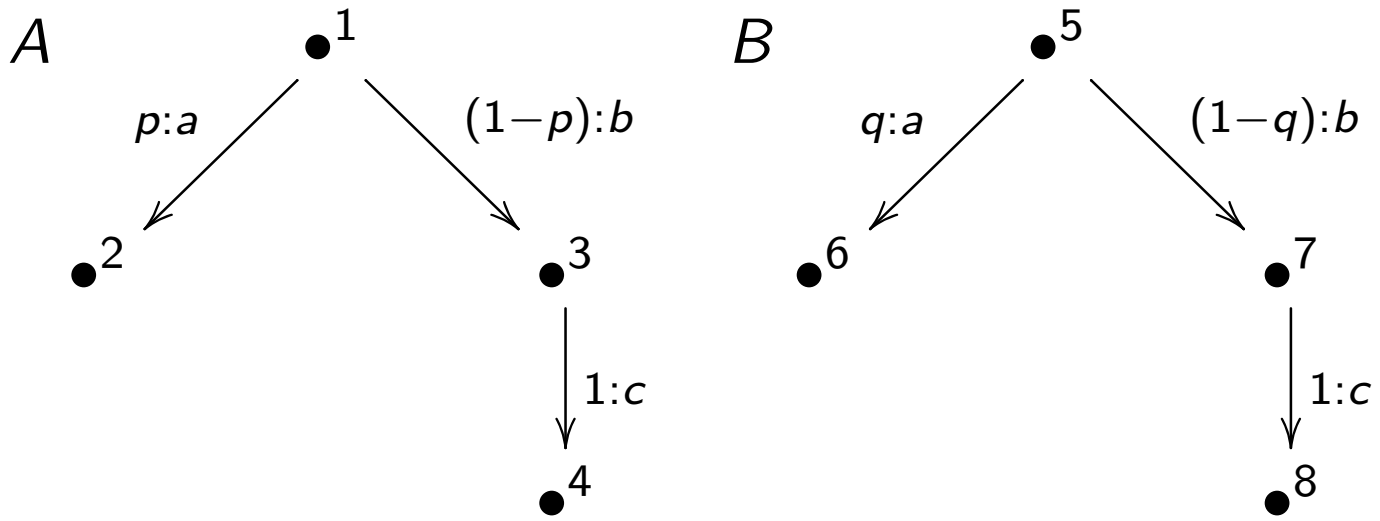
Concretly we

- introduce missing states, and
- redistribute distributions

Transformation Algorithm

```
1:  procedure Padding(T1, T2)
2:      P ← {S}
3:      while ¬Stable(P, T1 ⊕ T2) do
4:          choose B ∈ P
5:          P ← Splitting(B, P)
6:          if ClassMismatch then
7:              Fixing(T1, T2)
8:          end if
9:      end while
10: end procedure
```

A simple Example



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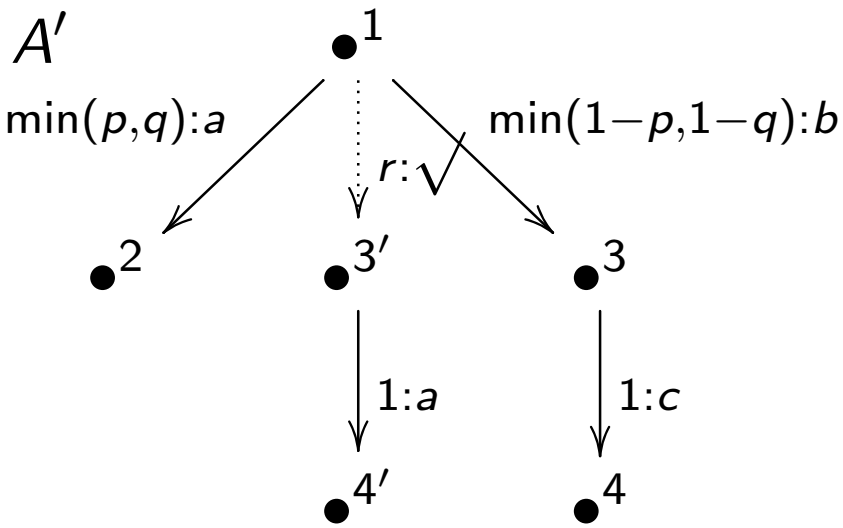

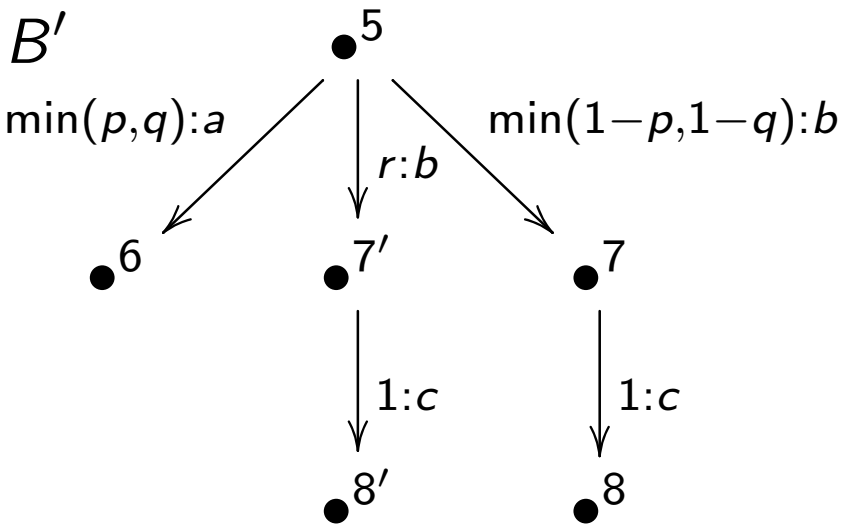

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i := 1;
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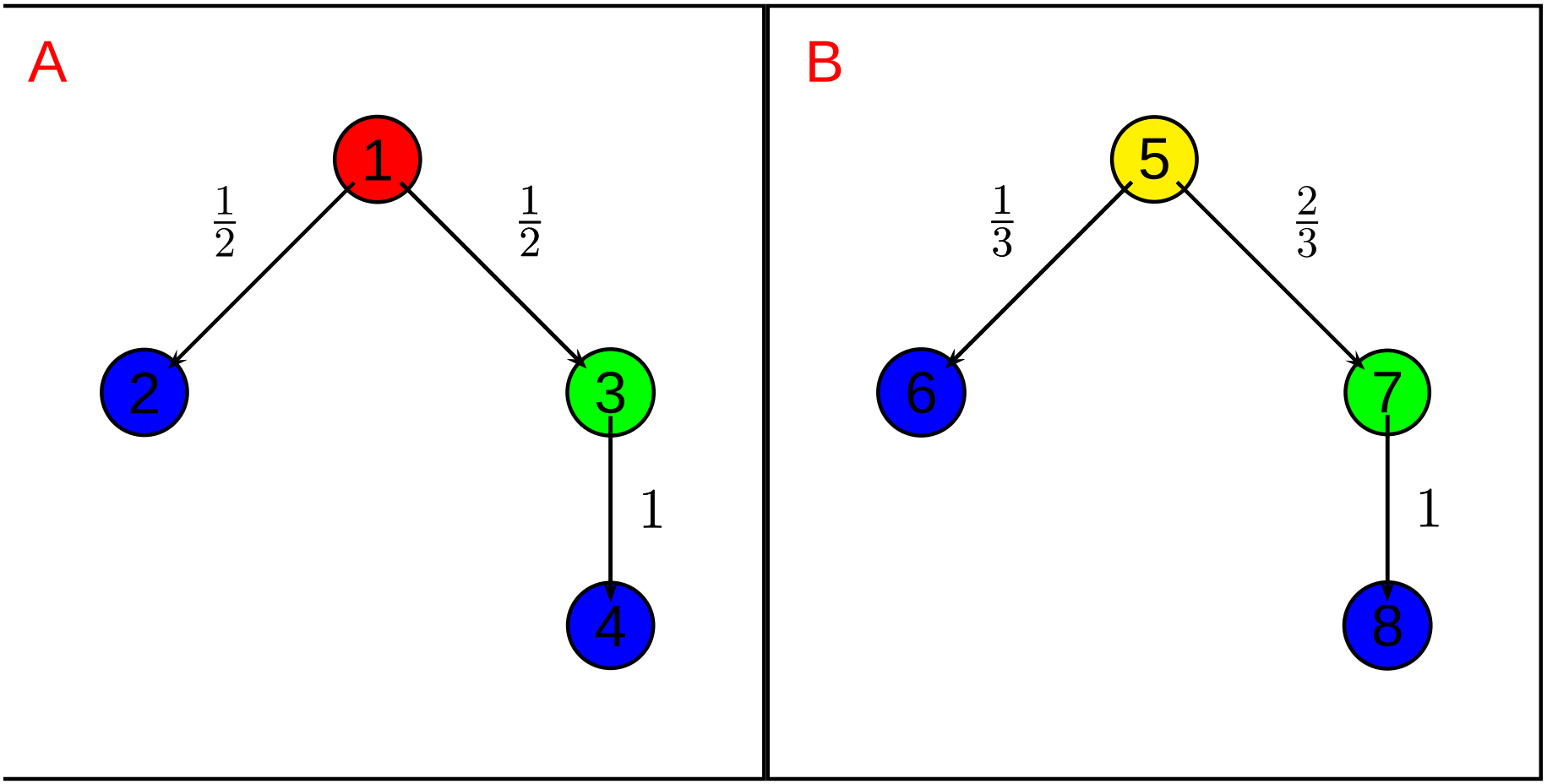

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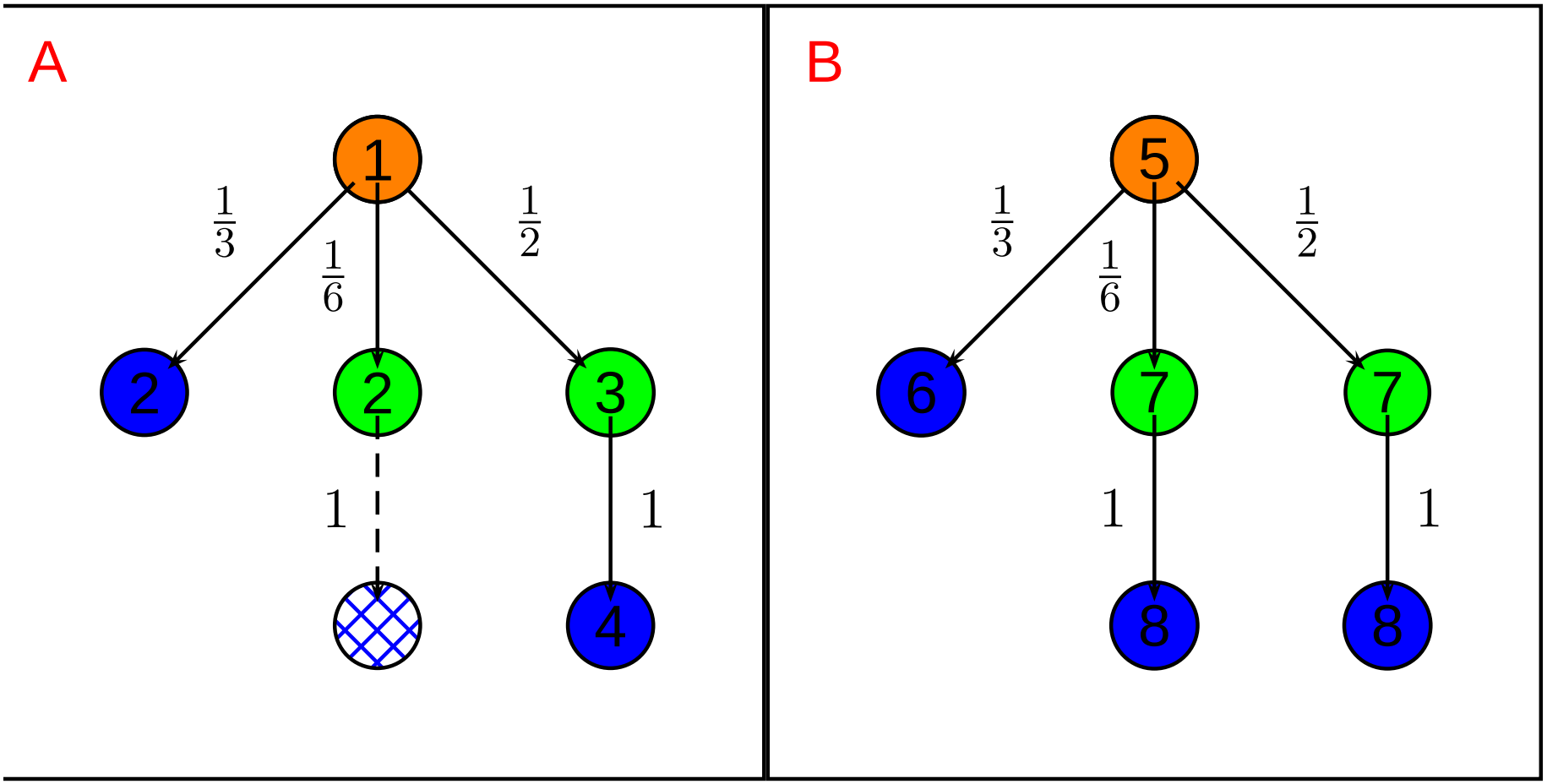

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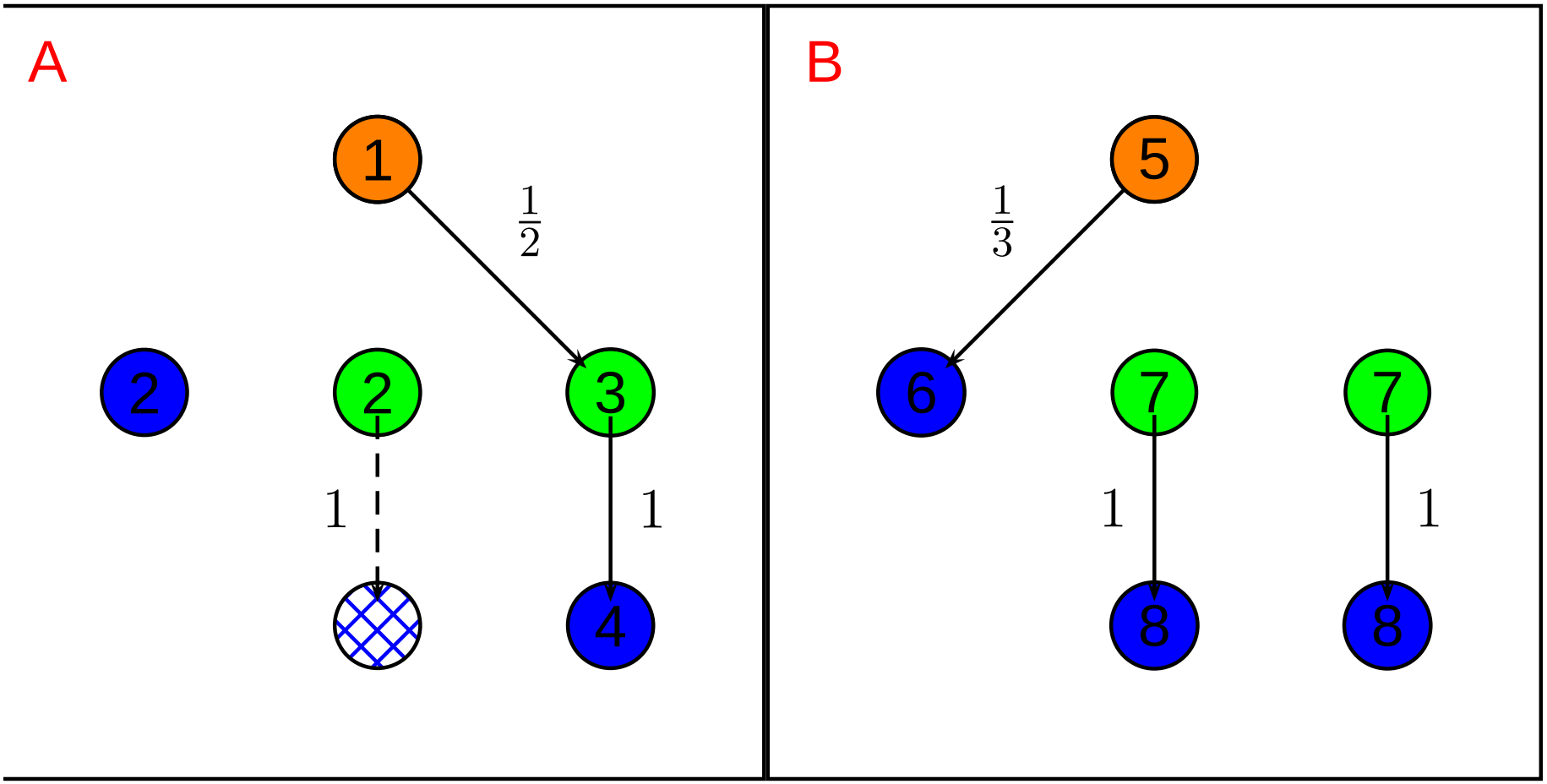

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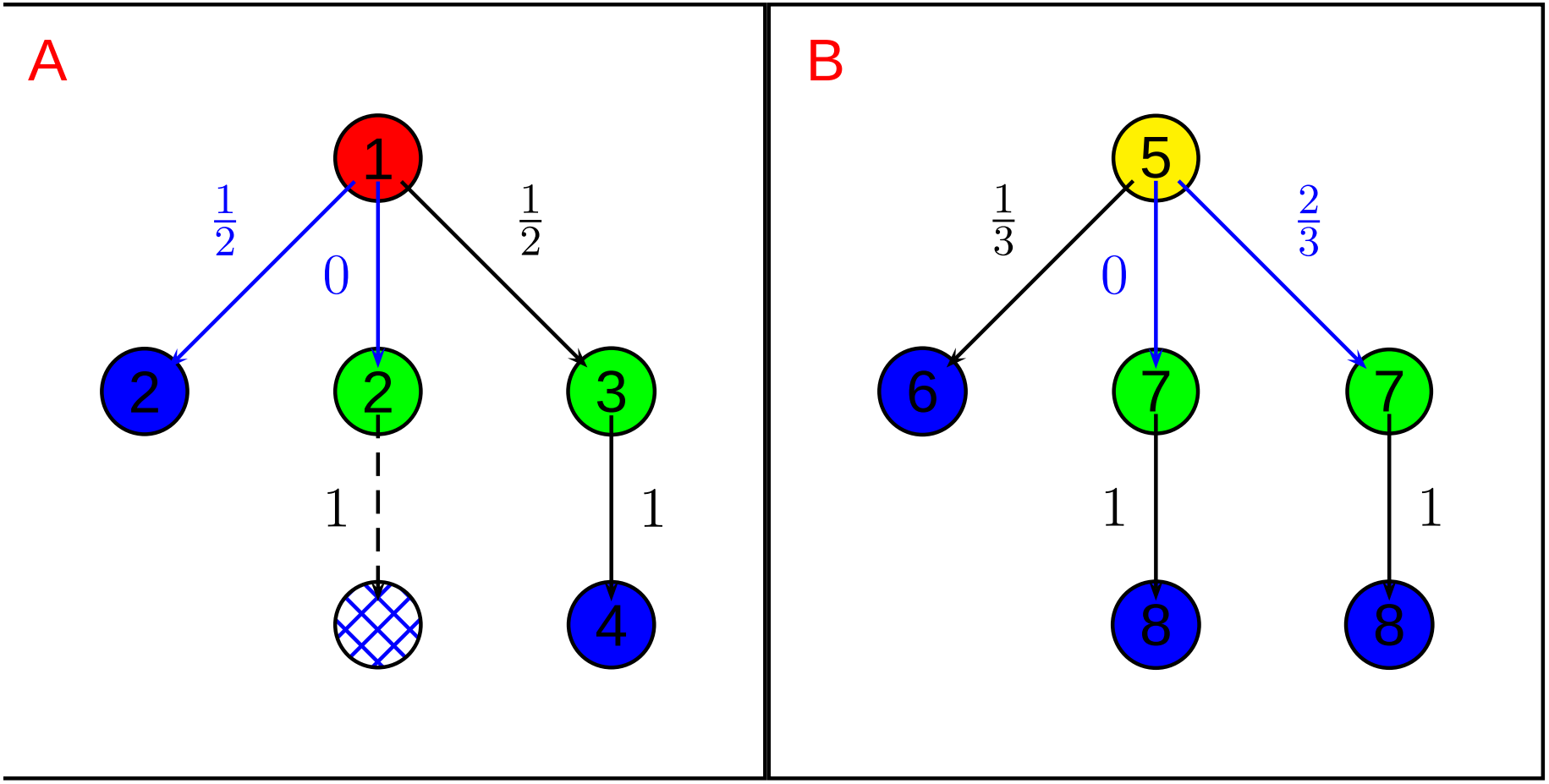

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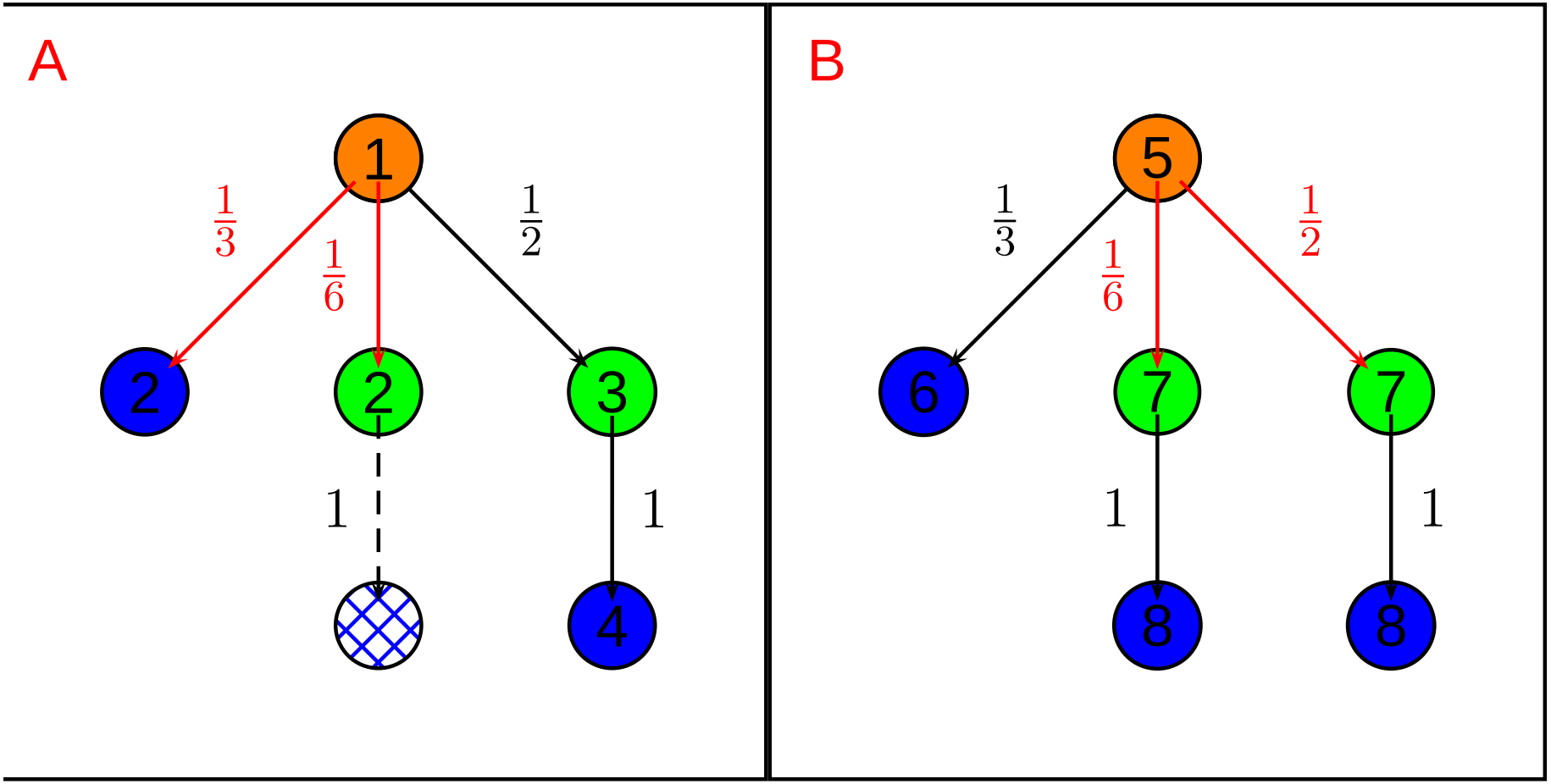

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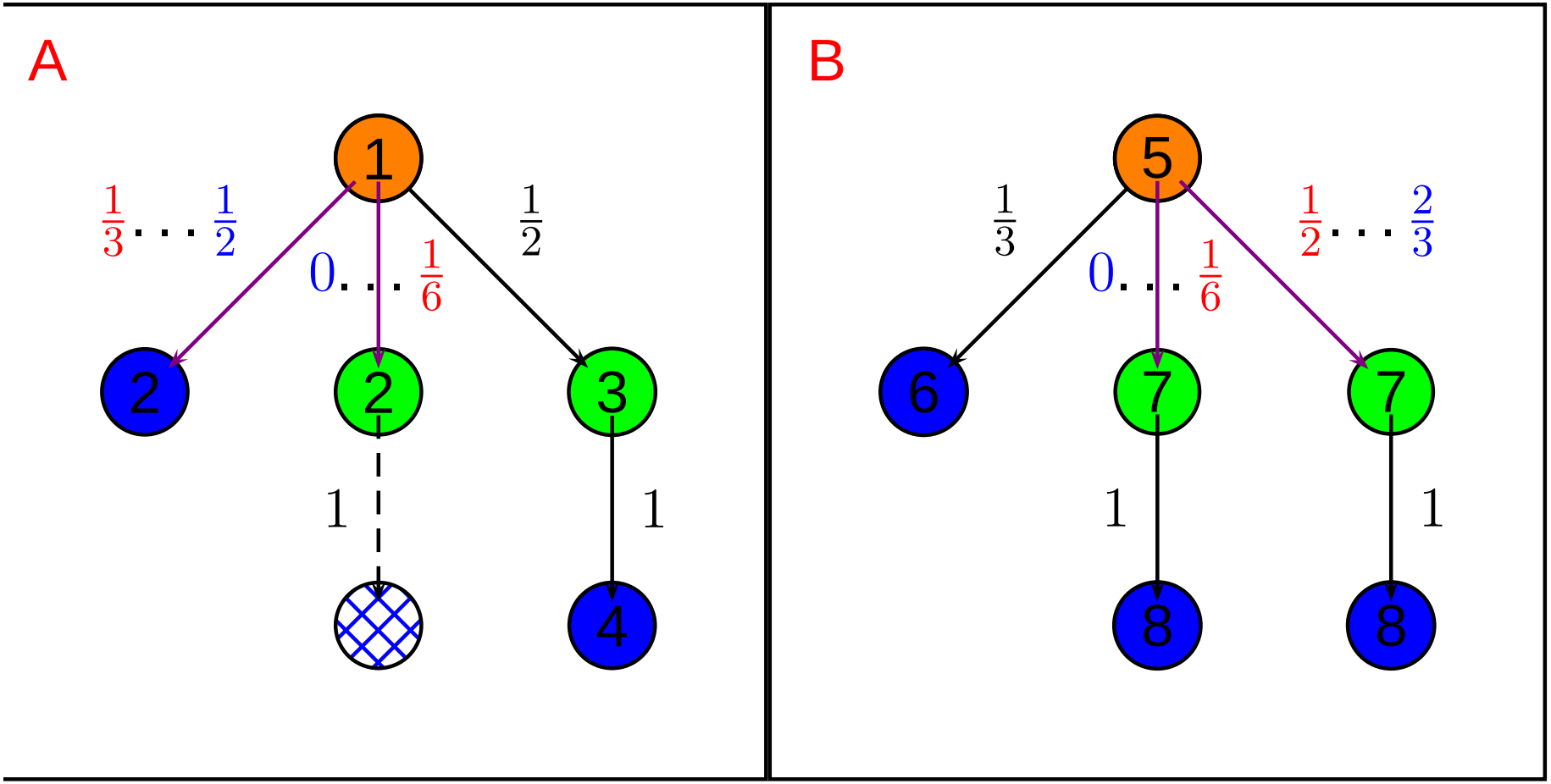

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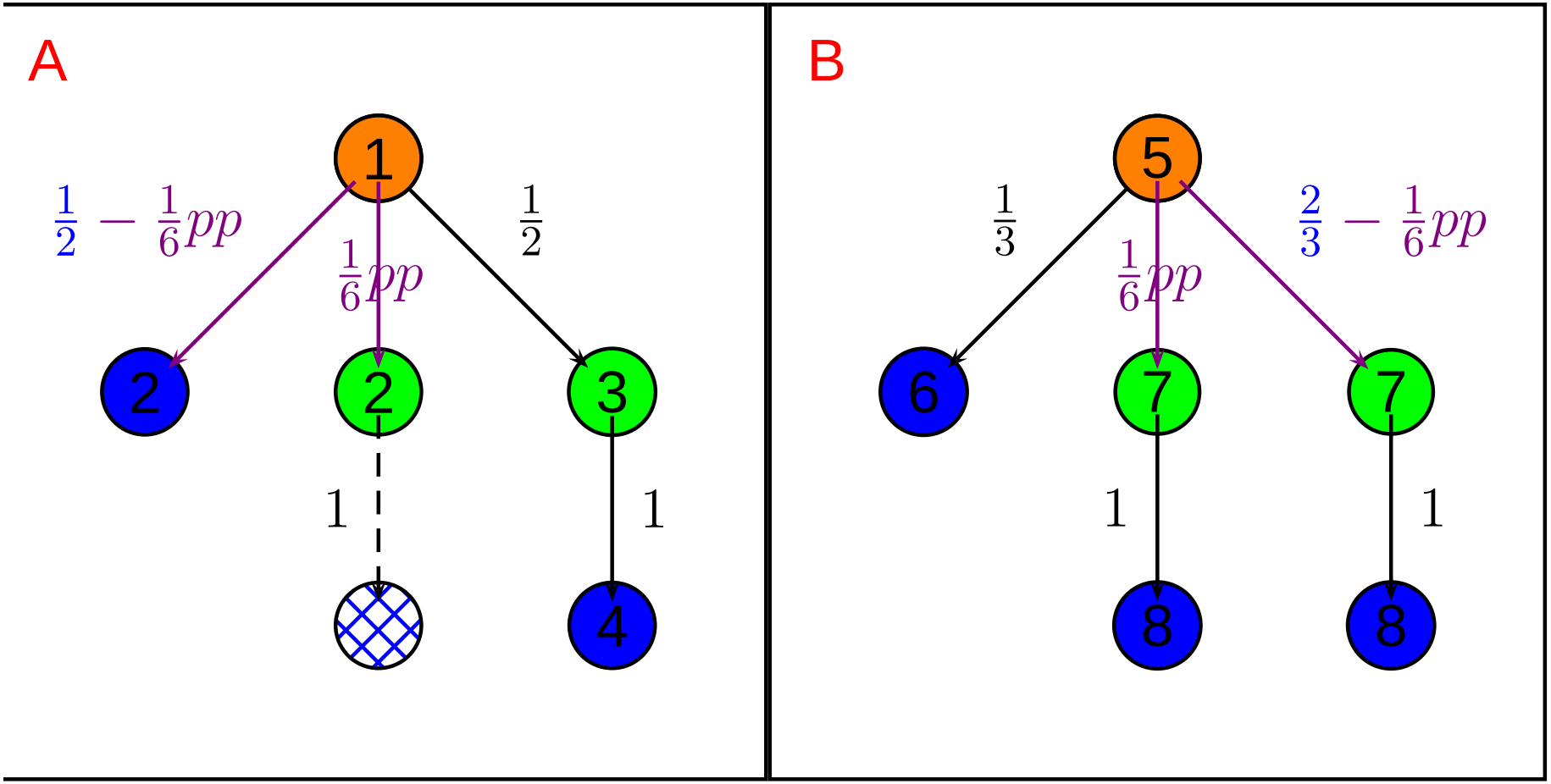

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```
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Probabilistic Fixing

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i := 1;
while i<=3 do
  if k[i]==1 then
     choose p:  s := s; [skip]
     or 1-p:  s := s
     ro
  else
     choose p:  skip
     or 1-p:  [s := s]; skip
     ro
  fi;
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Computing Non-Bisimilarity

1: **procedure** QLumping(T_1, T_2)
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3: **while** $\neg$Stable(P, $T_1 \oplus T_2$) **do**
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7: **for** $s_2 \in L_2$ **do**
8: $\beta \leftarrow \min(\beta, \|\chi_{s_1} - \chi_{s_2}\|_\infty)$
9: **end for**
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Our central theses are:

- it can make sense to not fix a time leak completely but instead – given the additional security costs – to settle for a partial solution;
- the analysis of such a trade-off situation (security vs. costs) can be done in a formal way.

An Example

```
i := 1;
while i<=3
do
 if k[i]==1
 then
   s := s;
 else
   skip;
 fi;
 i := i+1;
od;
```

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i := 1;
while i<=3
do
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   or     q: s := s
   ro
 else
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   or     q: skip
   ro
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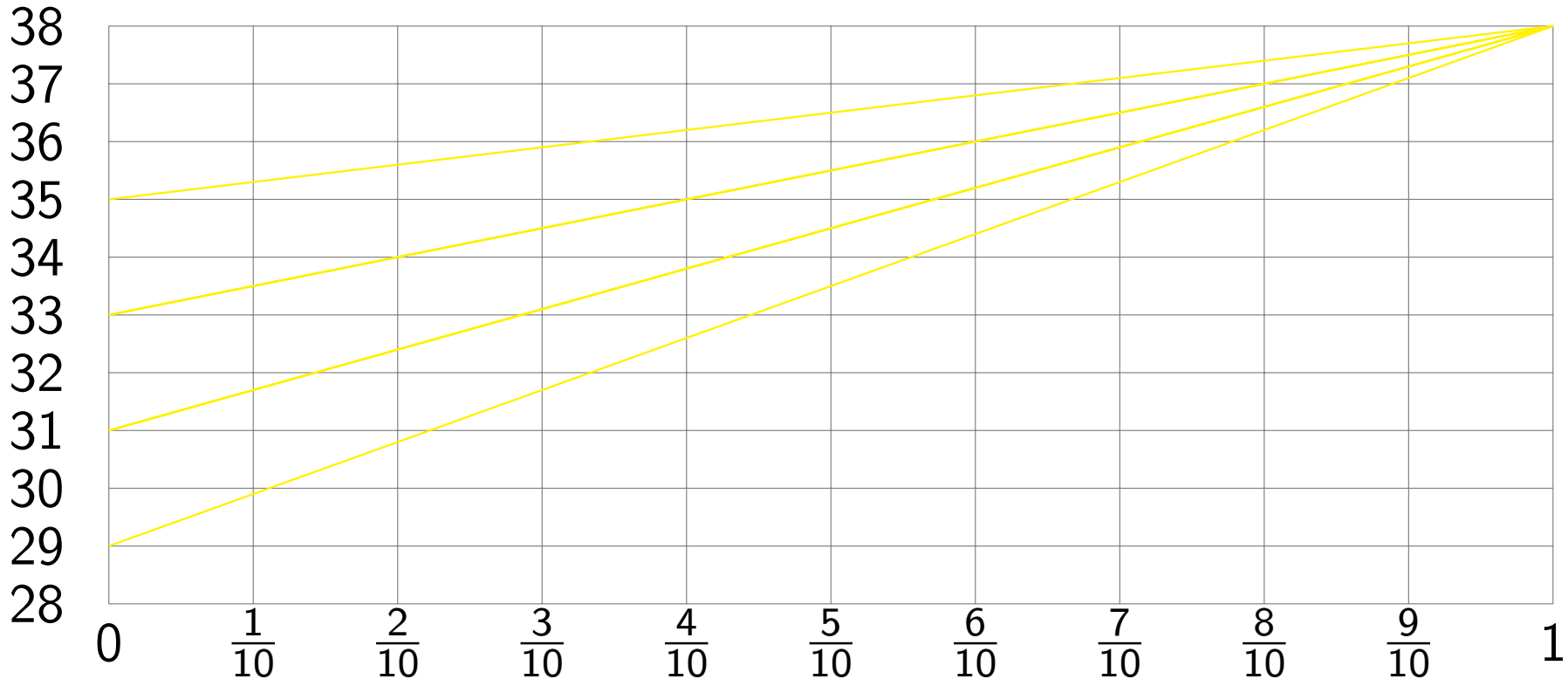
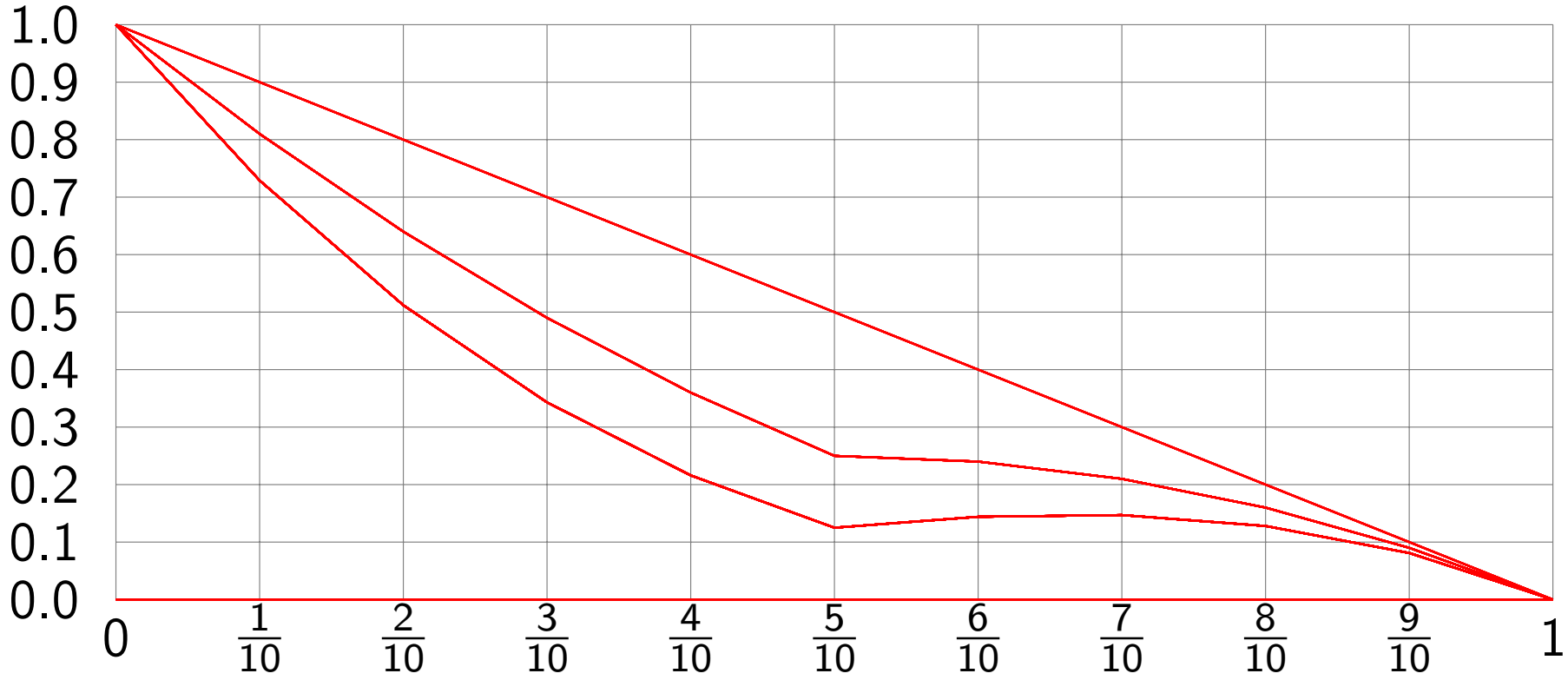
We implemented this example using execution times: $t_{asn} = 3$, $t_{br} = 2$, $t_{skip} = 1$ and $t_{ch} = 0$.

Concrete Experiments



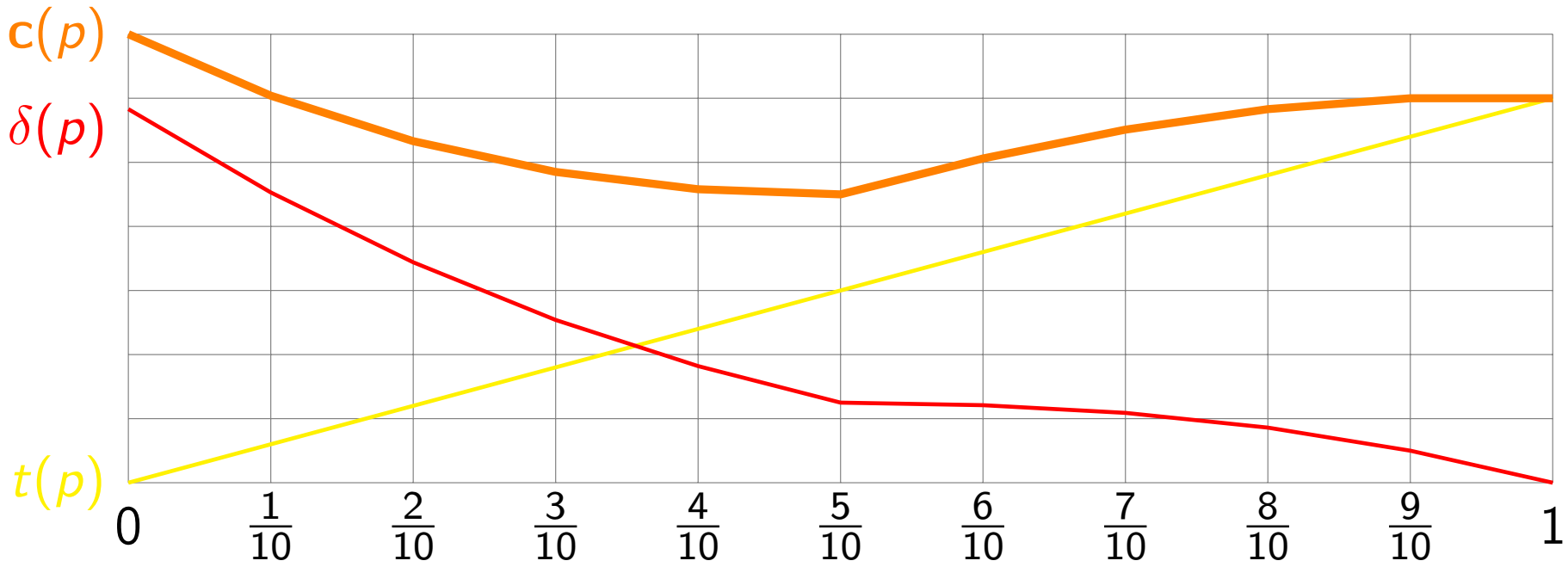
Average Running Time $t(p)$ for $k = 000, k = 100, k = 110, k = 111$.

Concrete Experiments



Security measured via $\varepsilon(p)$

Concrete Experiments



Cost Function: $c(p) = 6\varepsilon(p) + t(p)$