Reinforcement Learning and Simulation-Based Search

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Outline

Markov Decision Process

Definition

A *Markov Decision Process* is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a state transition probability matrix, $\mathcal{P}^a_{ss'} = \mathbb{P}[s' \mid s, a]$
- $\mathcal{R}$ is a reward function, $\mathcal{R}^a_s = \mathbb{E}[r \mid s, a]$

Assume for this talk that all sequences terminate, $\gamma = 1$

Planning and Reinforcement Learning

- Planning:
  Given MDP $\mathcal{M}$, maximise expected future reward
- Reinforcement Learning:
  Given sample sequences from MDP

$$\{s_1, a_1^k, r_1^k, s_2^k, a_2^k, ..., s_{T^K}^k\}_{k=1}^K \sim \mathcal{M}$$

  Maximise expected future reward

Simulation-Based Search

- A *simulator* $\mathcal{M}$ is a generative model of an MDP
  - Given a state s_t and action a_t
  - The simulator can generate a next state s_{t+1} and reward r_{t+1}
- A simulator can be used to generate sequences of experience
- Starting from any "root" state s_1

$$\{s_1, a_1, r_1, s_2, a_2, ..., s_T\} \sim \mathcal{M}$$

- Simulation-based search applies reinforcement learning to simulated experience

Monte-Carlo Simulation

- Given a model $\mathcal{M}$ and a *simulation policy*

$$\pi(s, a) = Pr(a \mid s)$$

- Simulate K episodes from root state s_1

$$\{s_1, a_1^k, r_1^k, s_2^k, a_2^k, ..., s_{T^K}^k\}_{k=1}^K \sim \mathcal{M}, \pi$$

- Evaluate state by mean total reward (*Monte-Carlo evaluation*)

$$V(s_1) = \frac{1}{K} \sum_{k=1}^{K} \sum_{t=1}^{T^K} r_t^k \xrightarrow{P} \mathbb{E}\left[\sum_{t=1}^{T^K} r_t^k \;\middle|\; s_1\right]$$

Simple Monte-Carlo Search

- Given a model $\mathcal{M}$ and a simulation policy π
- For each action $a \in \mathcal{A}$
  - Simulate K episodes from root state s_t

$$\{s_1, a, a_1^k, r_1^k, s_2^k, a_2^k, ..., s_T^k\}_{k=1}^K \sim \mathcal{M}, \pi$$

  - Evaluate actions by mean total reward

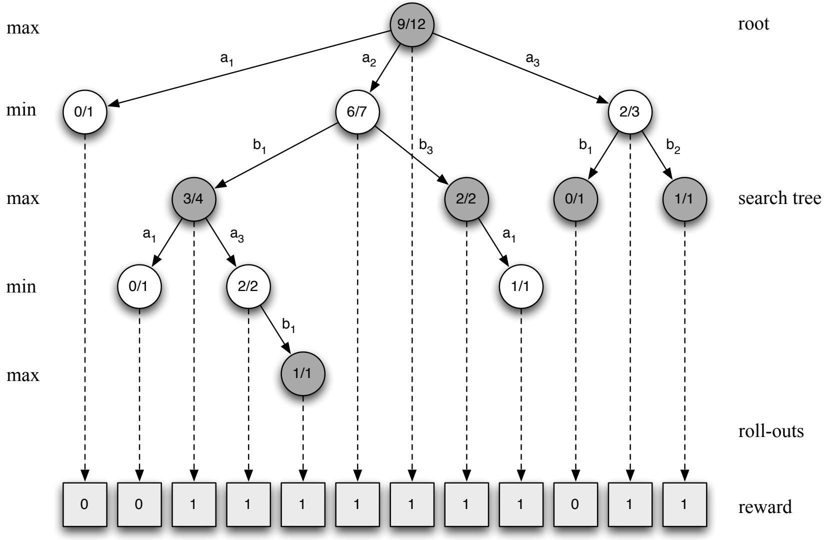
$$Q(s_1, a) = \frac{1}{K} \sum_{k=1}^{K} \sum_{t=1}^{T^K} r_t^k \xrightarrow{P} \mathbb{E}\left[\sum_{t=1}^{T^K} r_t^k \,\middle|\, s_1, a \right]$$

- Select real action with maximum value

$$a_t = \operatorname*{argmax}_{a \in \mathcal{A}} Q(s_t, a)$$

Monte-Carlo Tree Search

- Simulate sequences starting from root state s_1
- Build a search tree containing all visited states
- Repeat (each simulation)
  - Evaluate states $V(s)$ by mean total reward of all sequences through node s
  - Improve simulation policy by picking child s' with max $V(s')$
- Converges on the optimal search tree, $V(s) \rightarrow V^*(s)$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Advantages of MC Tree Search

- Highly selective best-first search
- Focused on the future
- Uses sampling to break curse of dimensionality
- Works for "black-box" simulators (only requires samples)
- Computationally efficient, anytime, parallelisable

Disadvantages of MC Tree Search

- Monte-Carlo estimates have high variance
- No generalisation between related states

Temporal-Difference Search

- Simulate sequences starting from root state s_1
- Build a search tree containing all visited states
- Repeat (each simulation)
  - Evaluate states $V(s)$ by temporal-difference learning
  - Improve simulation policy by picking child s' with max $V(s')$
- Converges on the optimal search tree, $V(s) \rightarrow V^*(s)$

Linear Temporal-Difference Search

- Simulate sequences starting from root state s_1
- Build a linear function approximator $V(s) = \phi(s)^\top \theta$ over all visited states
- Repeat (each simulation)
  - Evaluate states $V(s)$ by linear temporal-difference learning
  - Improve simulation policy by picking child s' with max $V(s')$

Demo

Planning Under Uncertainty

- Consider a history h_t of actions, observations and rewards

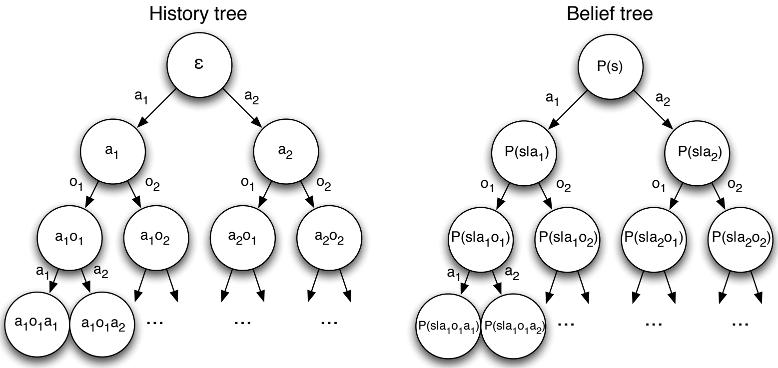
$$h = a_1, o_1, r_1, ..., a_t, o_t, r_t$$

- What if the state s is unknown?
- i.e. we only have some beliefs $b(s) = P(s \mid h_t)$
- What if the MDP dynamics $\mathcal{P}$ are unknown?
- i.e. we only have some beliefs $b(\mathcal{P}) = p(\mathcal{P} \mid h_t)$
- What if the MDP reward function $\mathcal{R}$ is unknown?
- i.e. we only have some beliefs $b(\mathcal{R}) = p(\mathcal{R} \mid h_t)$

Belief State MDP

- Plan in augmented state space over beliefs
- Each action now transitions to a new belief state
- This defines an enormous MDP over belief states

Histories and Belief States



Belief State Planning

- We can apply simulation-based search to the belief state MDP
- Since these methods are effective in very large state spaces
- Unfortunately updating belief states is slow
- Belief state planners cannot scale up to realistic problems

Root Sampling

- Each simulation, pick one world from root beliefs: sample state/transitions/reward function
- Run simulation as if that world is real
- Build plan in history space (fast!)
- Evaluate histories $V(h)$ e.g. by Monte-Carlo evaluation
- Improve simulation policy e.g. by greedy action selection $a_t = \underset{a}{\operatorname{argmax}}\, V(h_t a)$
- *Never* updates beliefs during search
- But still converges on the optimal search tree w.r.t. beliefs, $V(h) \to V^*(h)$
- Intuitively, it averages over different worlds, tree provides filter

Demo