



# Optimisation in a Process Engineering Context

Eva Sorensen  
Chemical Engineering



# Product and Process Systems Engineering



# UCL

David Bogle, Vivek Dua, Eric Fraga, Lazaros Papageorgiou,  
Eva Sorensen and Michail Stamatakis



## Our key areas of research:

## Competence Areas

Product and Process Design

Operations and Control

Modelling and Model  
Solution Tools

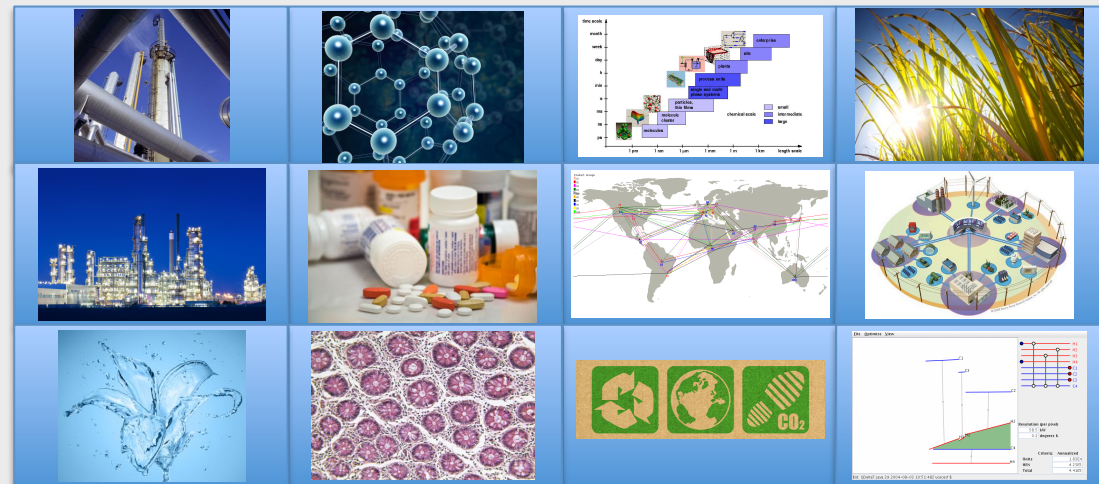
## Application Domains

Chemical  
Manufacturing  
Systems

Biological  
Systems  
Engineering

Supply  
Chains of  
the Future

Energy  
Systems  
Engineering

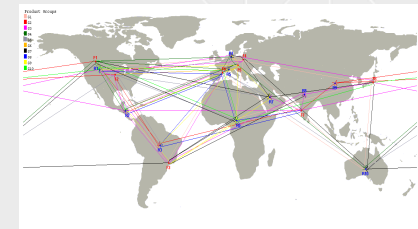
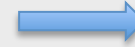
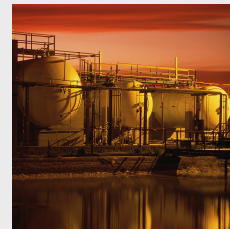
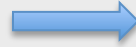
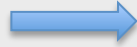
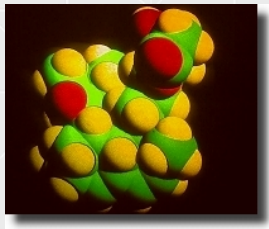


# Process Engineering



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- ***Process Systems Engineering** is concerned with the improvement of decision making processes for the creation and operation of the chemical supply chain.*



- *It deals with the discovery, design, manufacture and distribution of **chemical products** in the context of many conflicting goals.*

*Grossmann and Westerberg, Carnegie Mellon University*



# "Chemical products"



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# “Chemical products”



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## Application Domains:

- Chemical Manufacturing Systems
- *Molecular Systems Engineering*
- Biological Systems Engineering
- Supply Chains of the Future
- Energy Systems Engineering



# Process Engineering pipeline

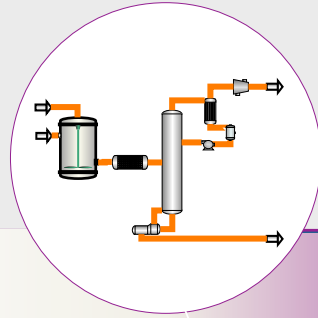


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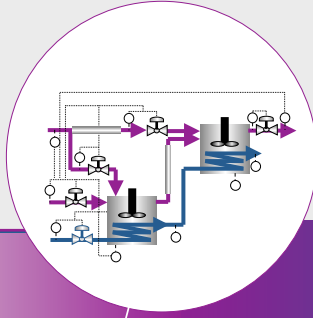
**Product & Process Development**



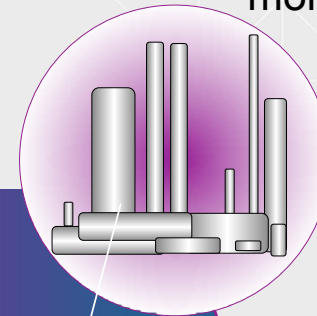
**Process flowsheeting**



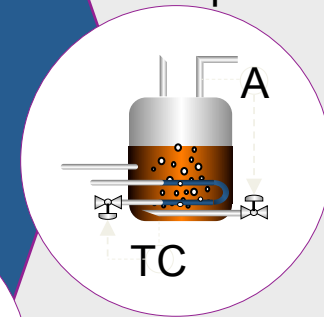
**Detailed design of process plant**



**Model-based automation applications – performance monitoring and decision support**



**Troubleshooting with detailed predictive models**

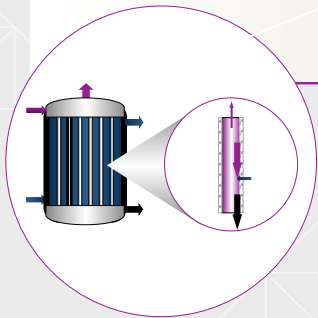


CONCEPT

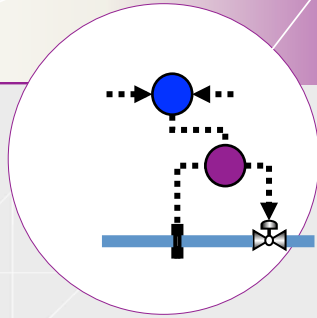
DESIGN

OPERATION

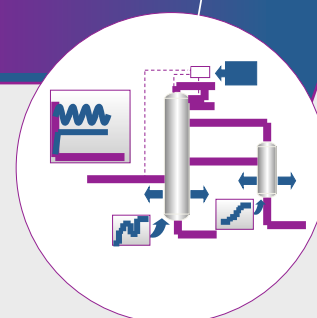
**Detailed design of complex units**



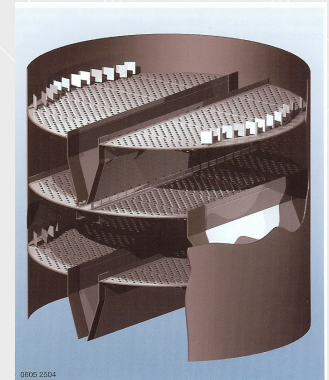
**Simultaneous design of equipment and control**



**Design of operating procedures**



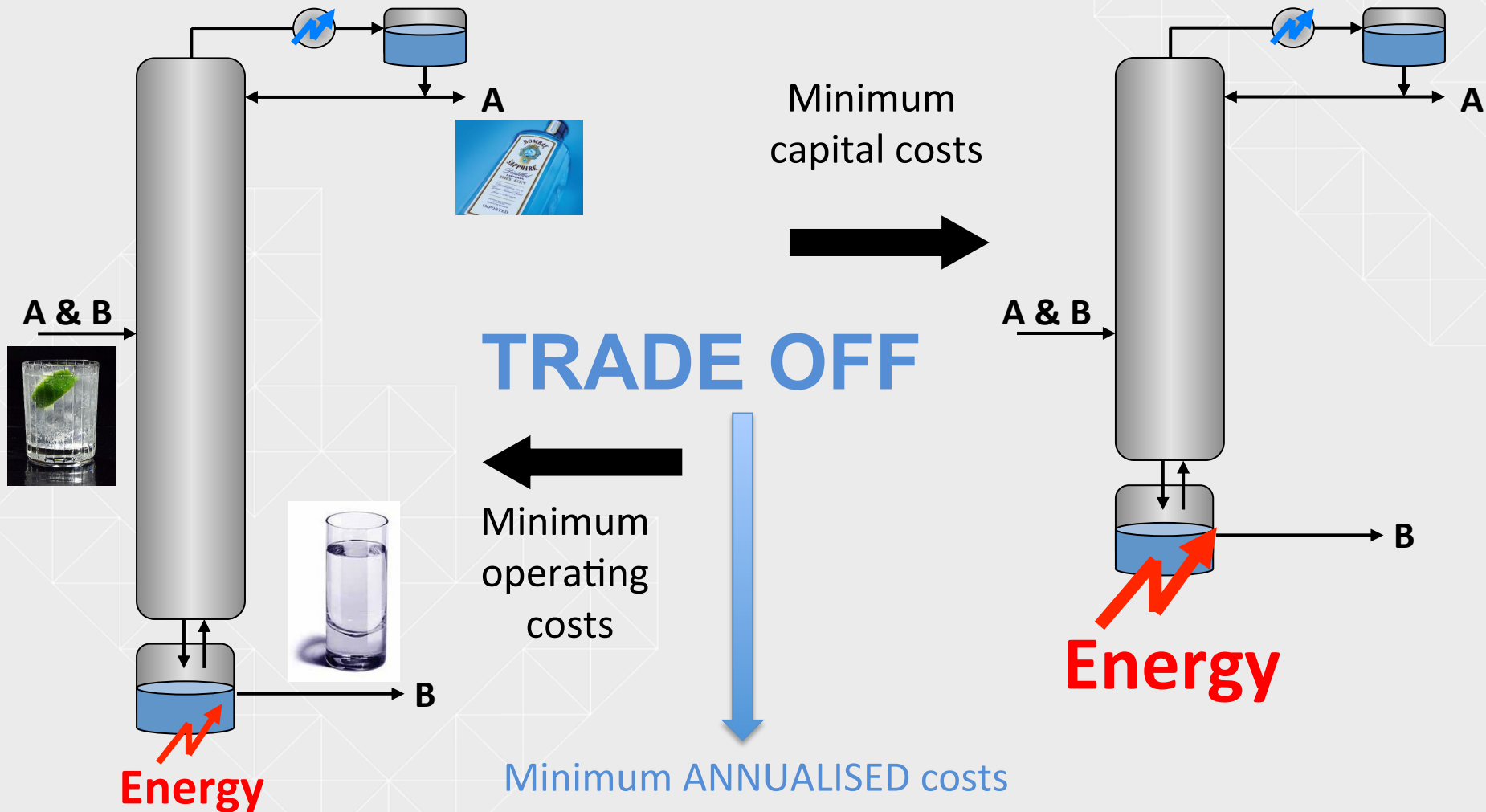
# Distillation column design



# Distillation column design *contd.*



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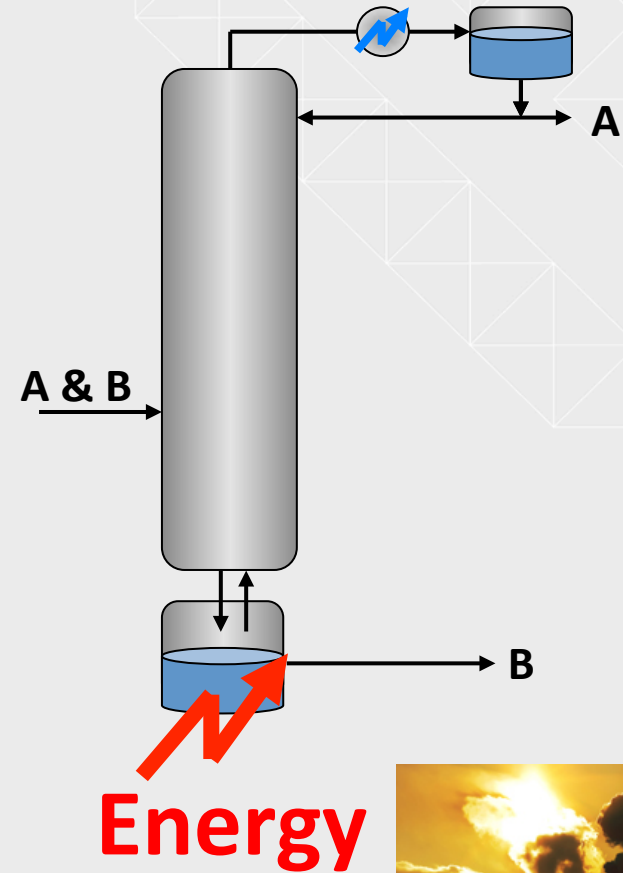
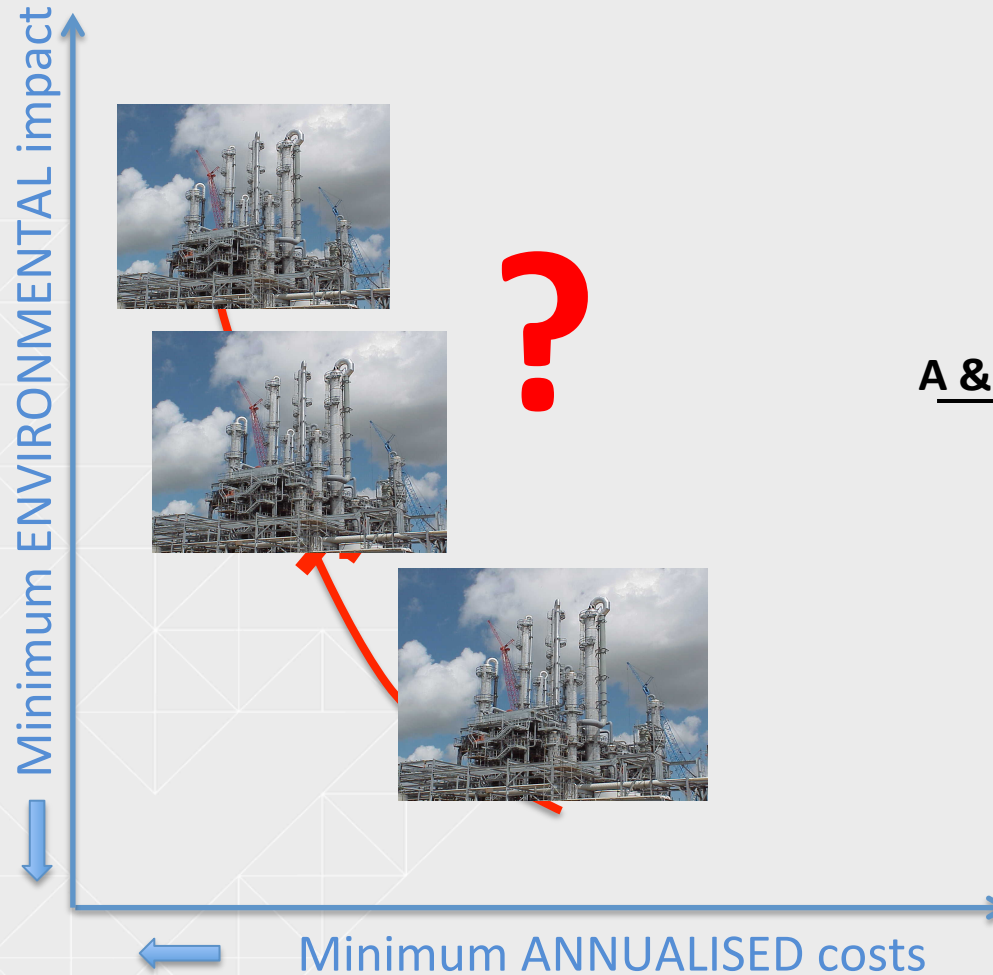




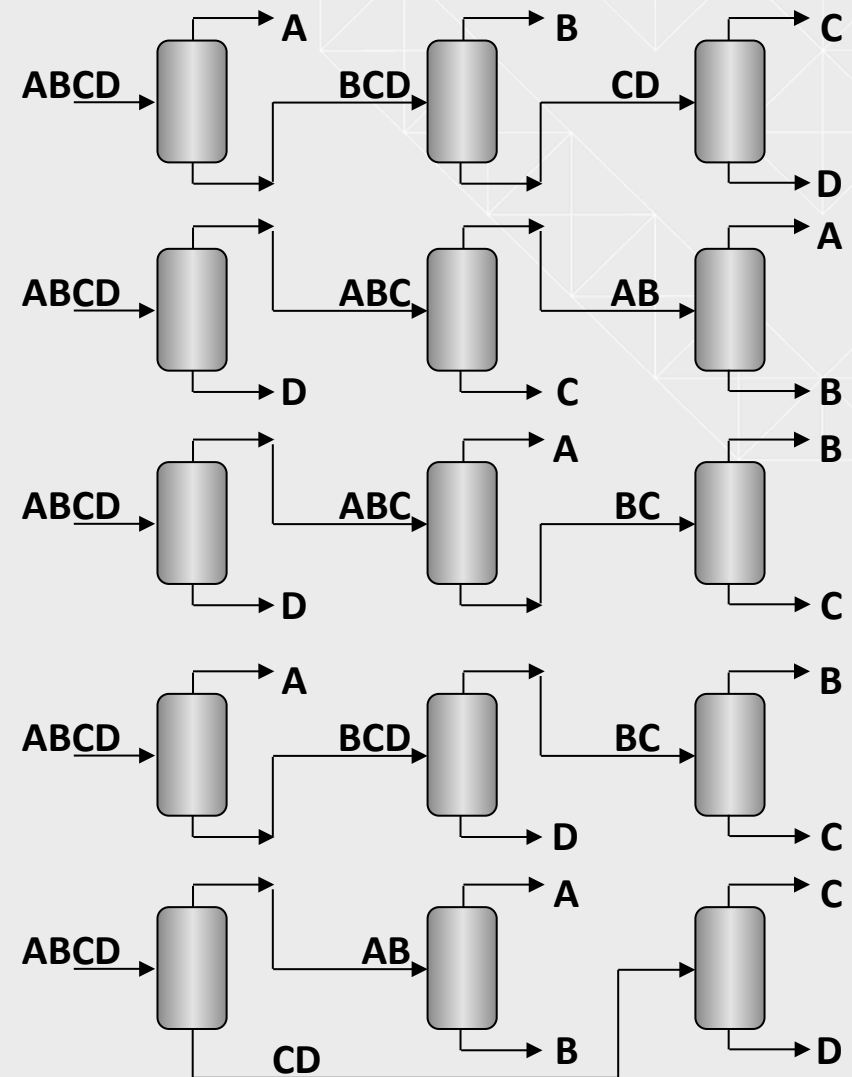
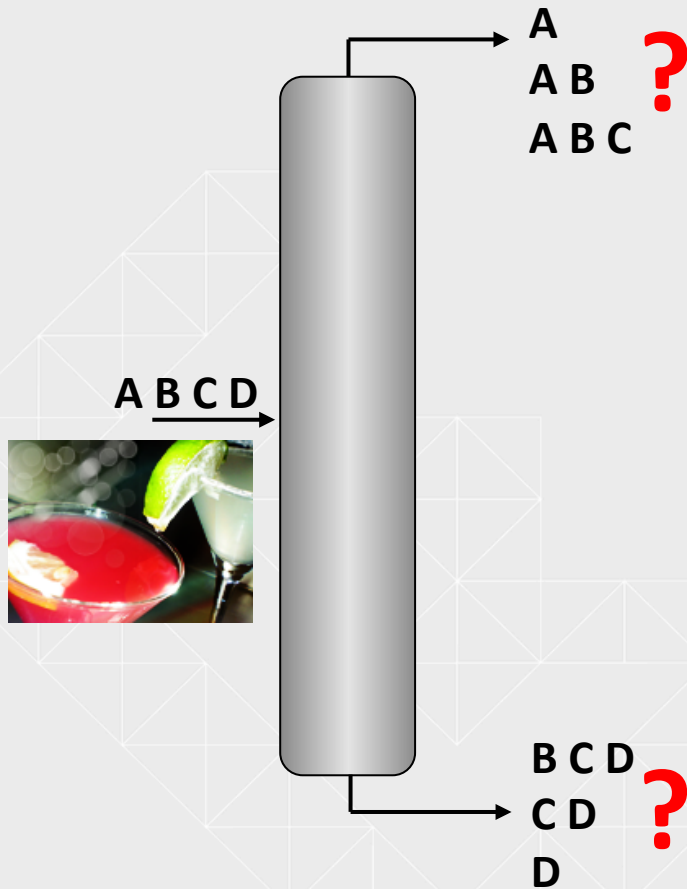
# Distillation column design *contd.*



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# Column scheduling



## The use of **validated predictive** models for...

*Quantified  
uncertainty in the  
model predictions*

**Quantification of  
technological risk involved  
in model-based decisions**

**Effective targeting of  
experimental R&D towards  
minimisation of this risk**



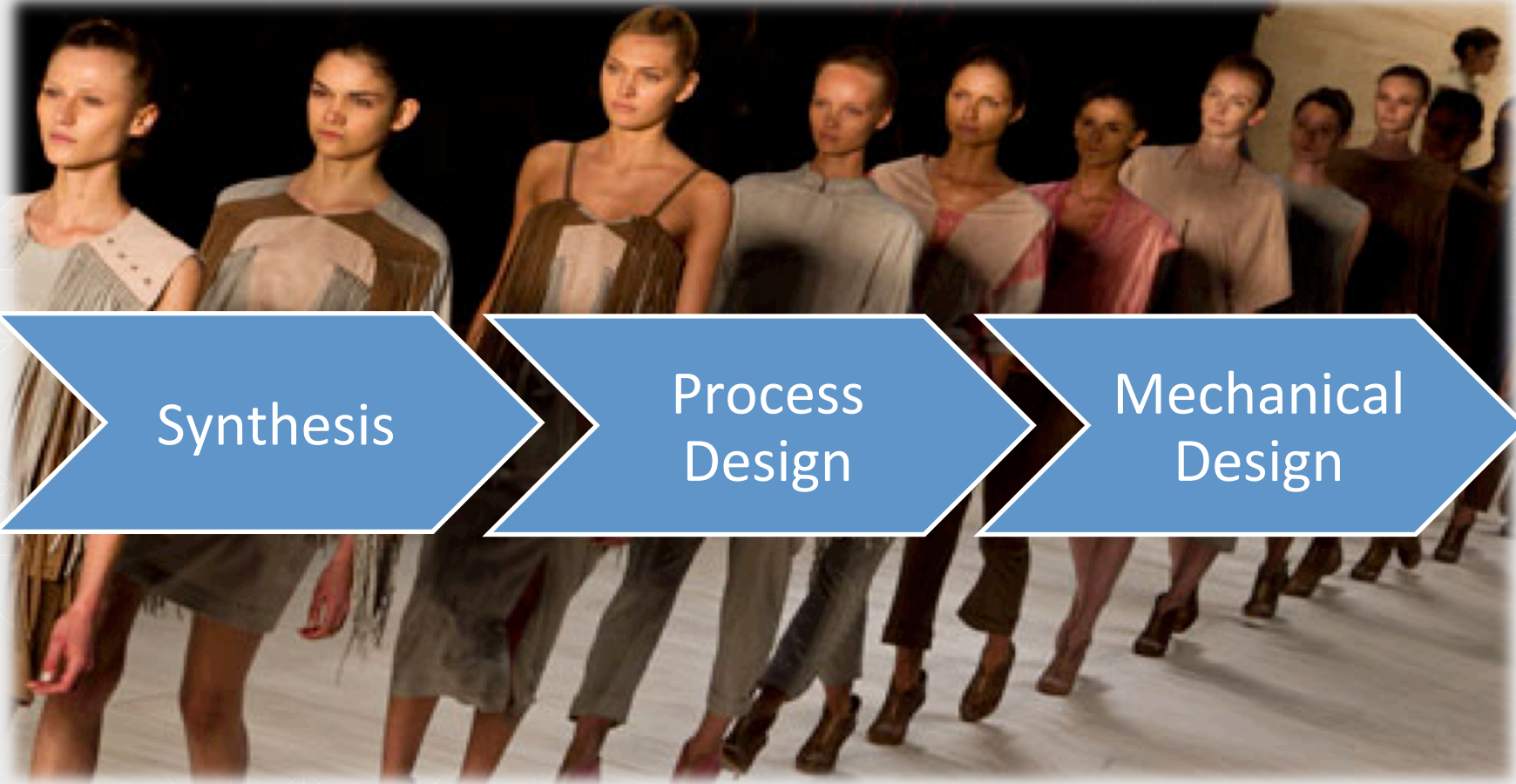
*Quantitative prediction of the  
effects of design & operating  
decisions on KPIs, within the  
accuracy necessary to achieve  
the business objectives*

**Optimisation of process  
design & operation by  
comprehensive exploration  
of alternatives**

# Model?



Catwalk!



Synthesis

Process  
Design

Mechanical  
Design

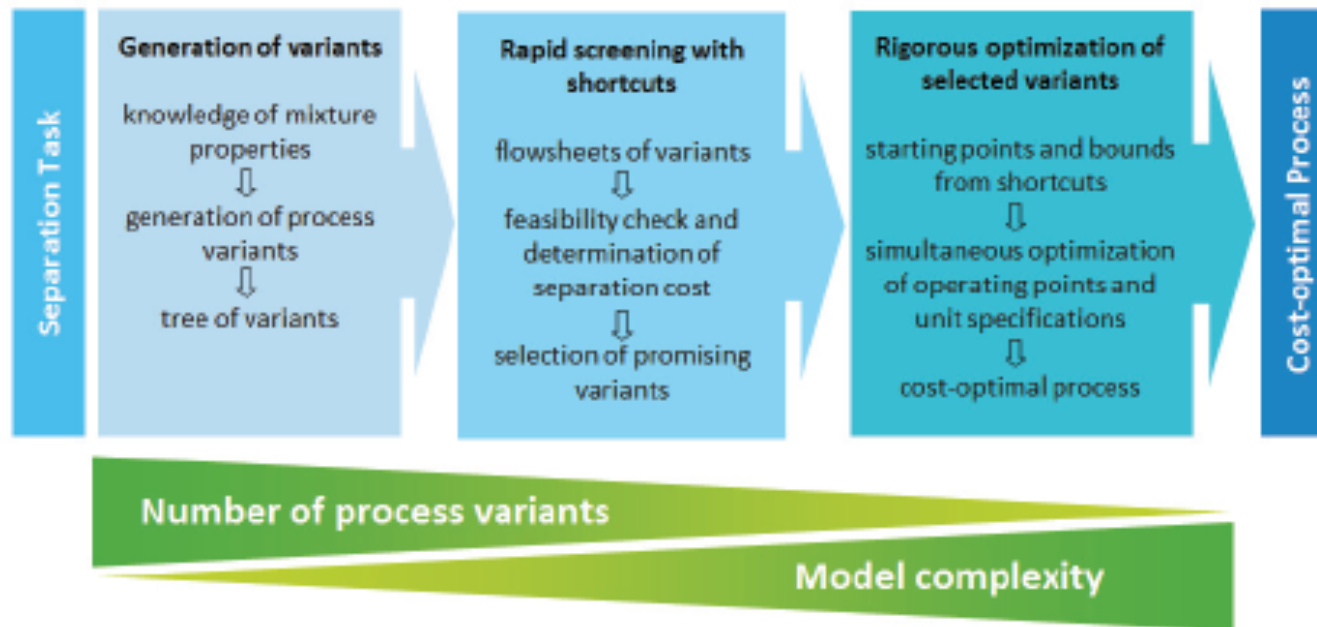
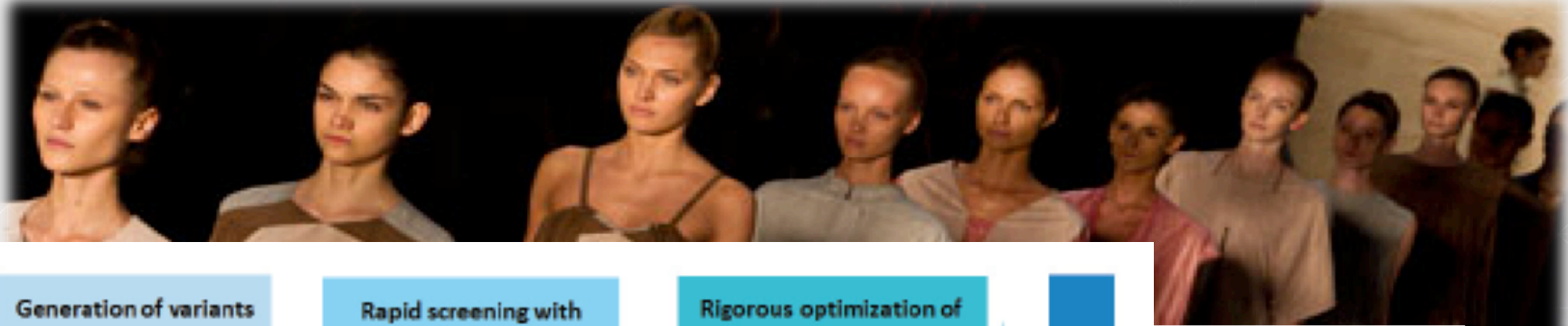




# Model?



## Catwalk!



Mechanical  
Design



# Model?



## Mathematical mass and energy balances

Mass  
Composition  
Temperature  
Pressure

### MODEL:

$$\text{Accumulated} = \text{In} - \text{Out} + \text{Generated} - \text{Consumed}$$

Mass  
Composition  
Temperature  
Pressure

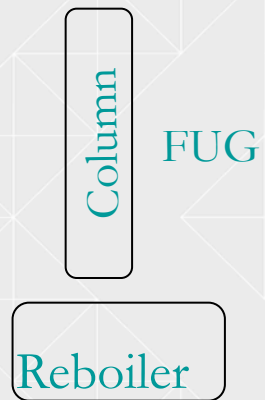
Mass    Composition    Temperature    Pressure



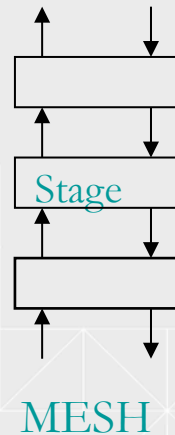
# Model details - distillation



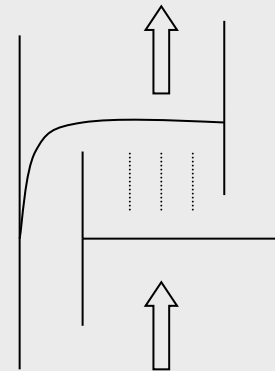
## Simplified Models



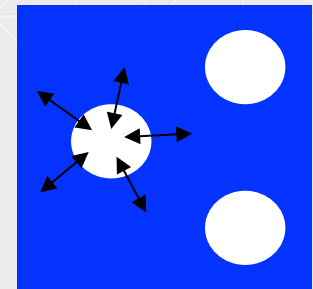
## Rigorous Models



## Detailed Models



## Rate-based Models





# Tray model

$$\frac{dM_{i,j}}{dt} = L_{j-1}x_{i,j-1} + V_{j+1}y_{i,j+1} - L_jx_{i,j} - V_jy_{i,j} \quad , \quad i = 1, \dots, NC$$

$$\frac{dU_j}{dt} = L_{j-1}h_{j-1}^L + V_{j+1}h_{j+1}^V - L_jh_j^L - V_jh_j^V$$

$$M_{i,j} = M_j^L x_{i,j} + M_j^V y_{i,j} \quad , \quad i = 1, \dots, NC$$

$$U_j = M_j^L h_j^L + M_j^V h_j^V - P_j v_j$$

$$\frac{M_j^L}{\rho_j^L} + \frac{M_j^V}{\rho_j^V} = v_j$$

$$\sum_{i=1}^{NC} x_{i,j} = \sum_{i=1}^{NC} y_{i,j} = 1$$

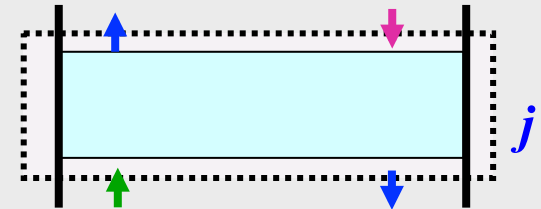
$$y_{i,j} = K_{i,j} x_{i,j} \quad , \quad i = 1, \dots, NC$$

$$h_j^L = h^L(T_j, P_j, \underline{x}_j) \quad ; \quad h_j^V = h^V(T_j, P_j, \underline{y}_j)$$

$$\rho_j^L = \rho^L(T_j, P_j, \underline{x}_j) \quad ; \quad \rho_j^V = \rho^V(T_j, P_j, \underline{y}_j)$$

$$K_{i,j} = K_i(T_j, P_j, \underline{x}_j, \underline{y}_j) \quad , \quad i = 1, \dots, NC$$

**Single tray only!!!!**



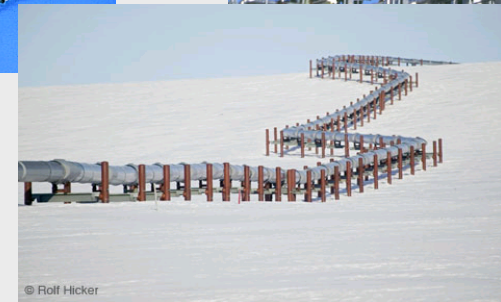
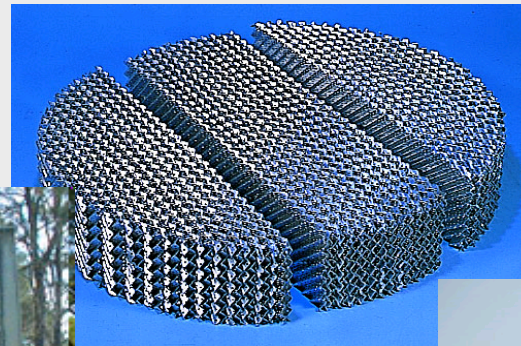


# Model details – spatial variations



Properties vary with respect to one or more spatial dimensions as well as with time:

- Tubular reactors
- Packed bed columns (adsorption/absorption/distillation chromatography)
- Pipelines
- *etc*

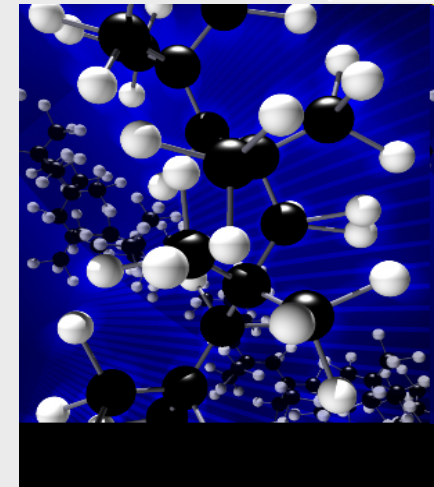
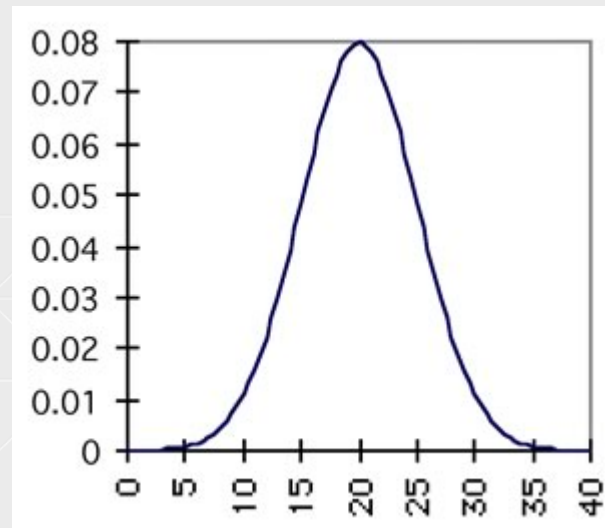
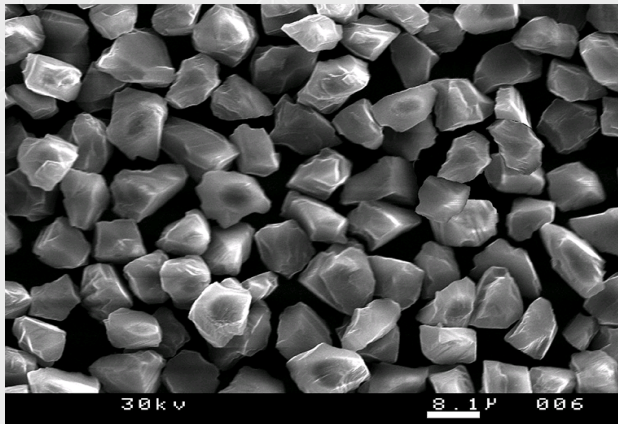


© Rolf Hicker



# Model details – Probability density

- Probability density functions instead of single scalar values:
  - Crystallisation units (size of crystals)
  - Polymerisation reactors (length of polymer chains)
- For such processes, the properties may *also* vary with both time and spatial position.



# Optimisation Problem Formulation



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**Minimise***Objective function 1*

&amp;

**Minimise***Objective function 2*

&amp;

....

**Subject to:***Model equations**Design variable bounds**Operational variable bounds**DAE/PDAE, nonlinear  
discrete and continuous  
continuous***To determine:***Design variables (constant)**Operation variables (time dependent)*

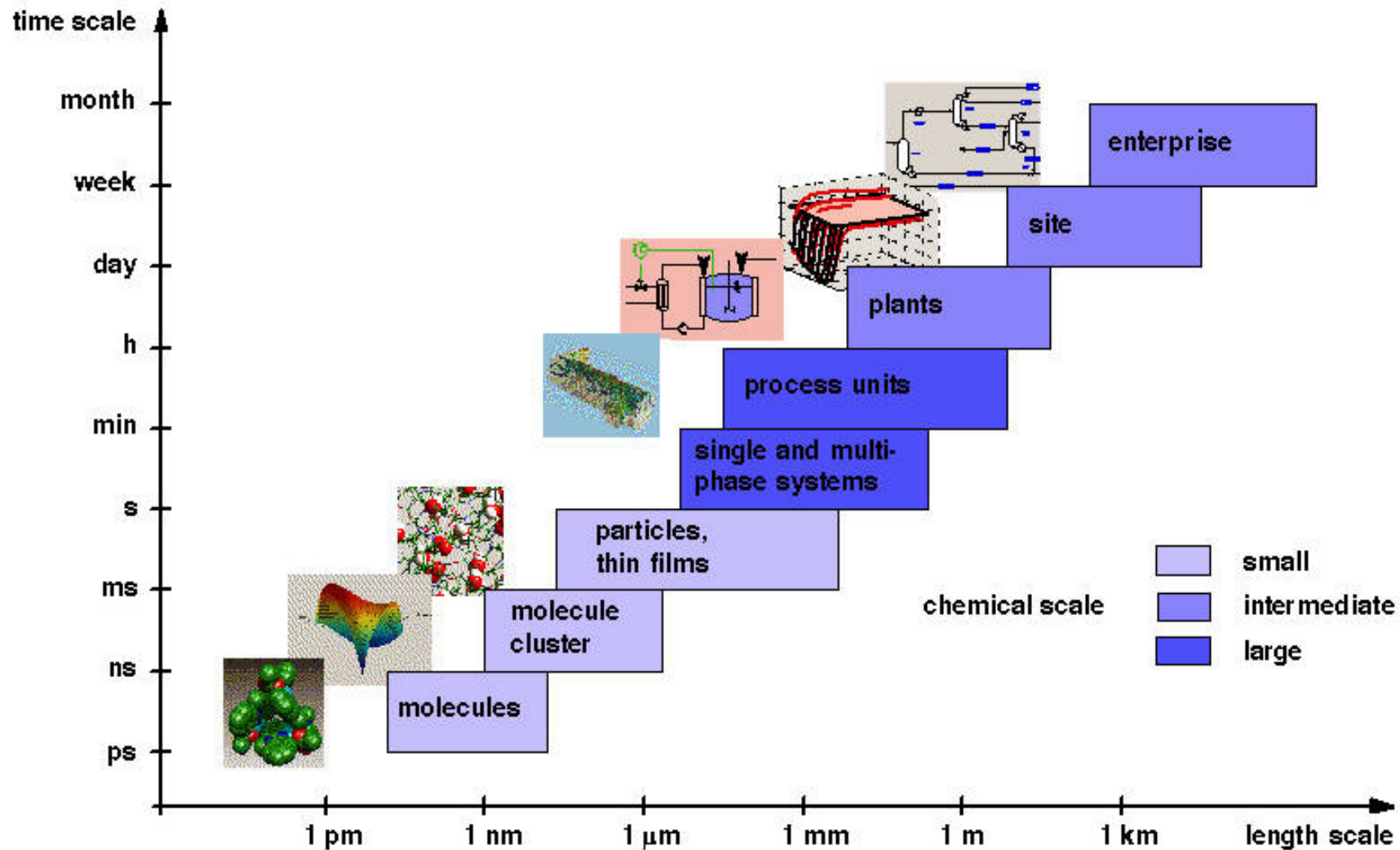


# EXAMPLES





# Multi-scale considerations

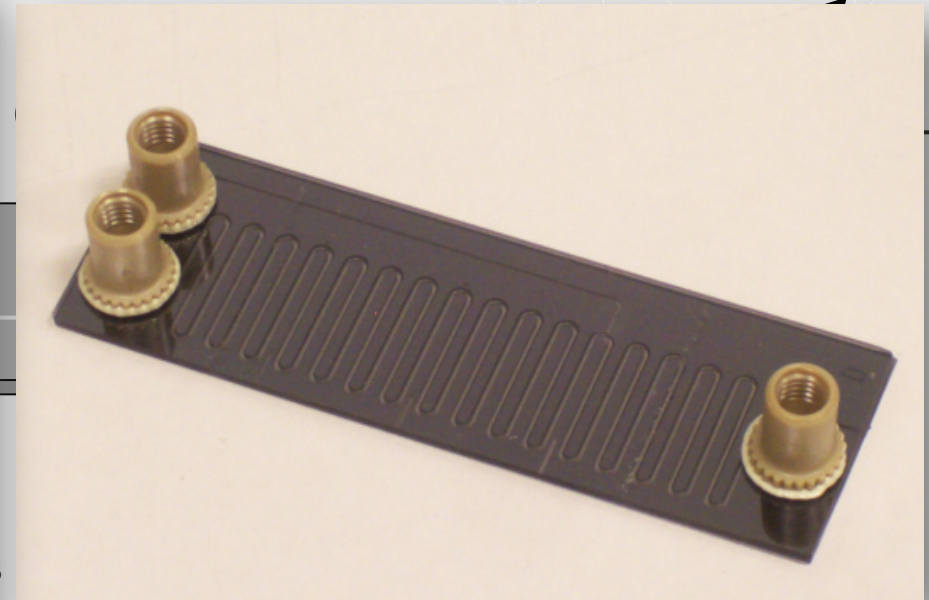
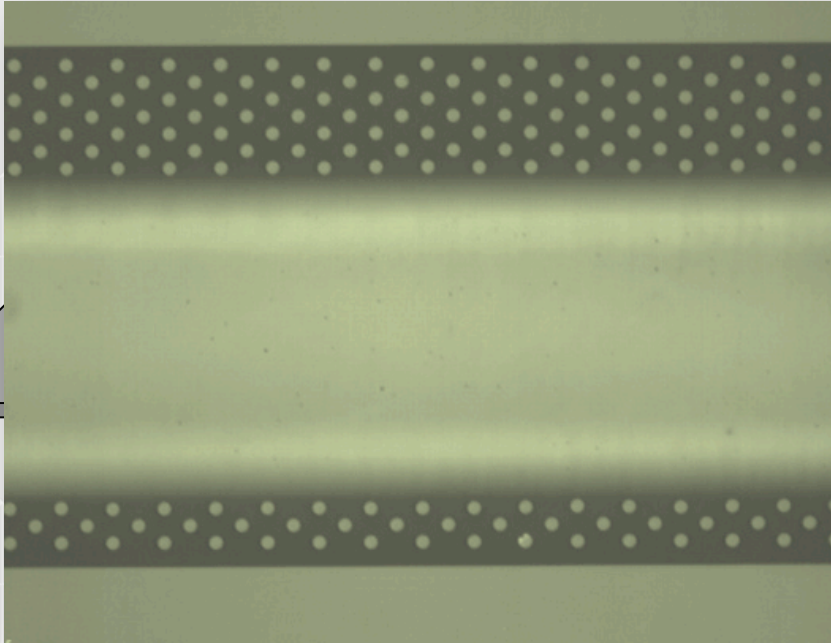




# Micro distillation

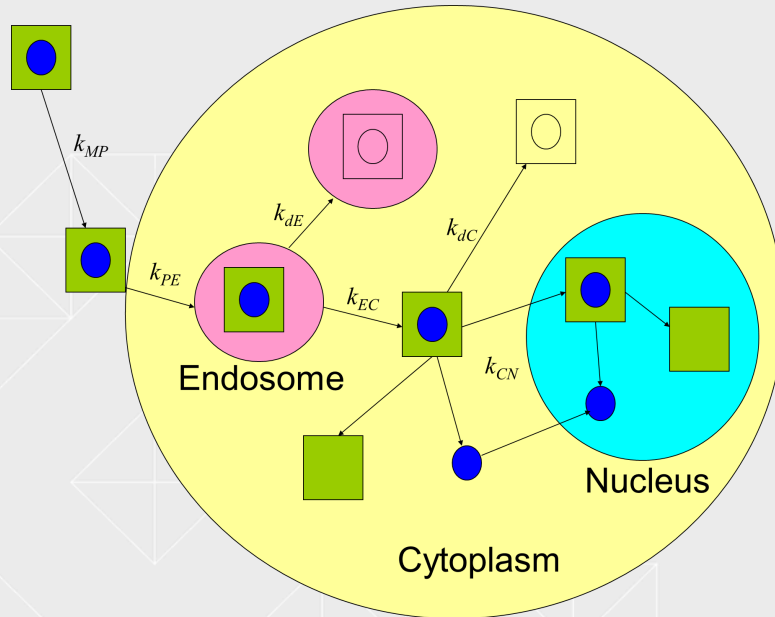


- To fabricate micro-distillation chips based on *zero-gravity distillation*



- To demonstrate fluid separation and assess separation performance

# Optimal control of Gene Delivery



$$\frac{dM}{dt} = -k_{MP}M + u$$

$$\frac{dP}{dt} = k_{MP}M - (k_{PE} + k_{PC} + k_{dP})P$$

$$\frac{dE}{dt} = k_{PE}P - (k_{EC} + k_{dE})E$$

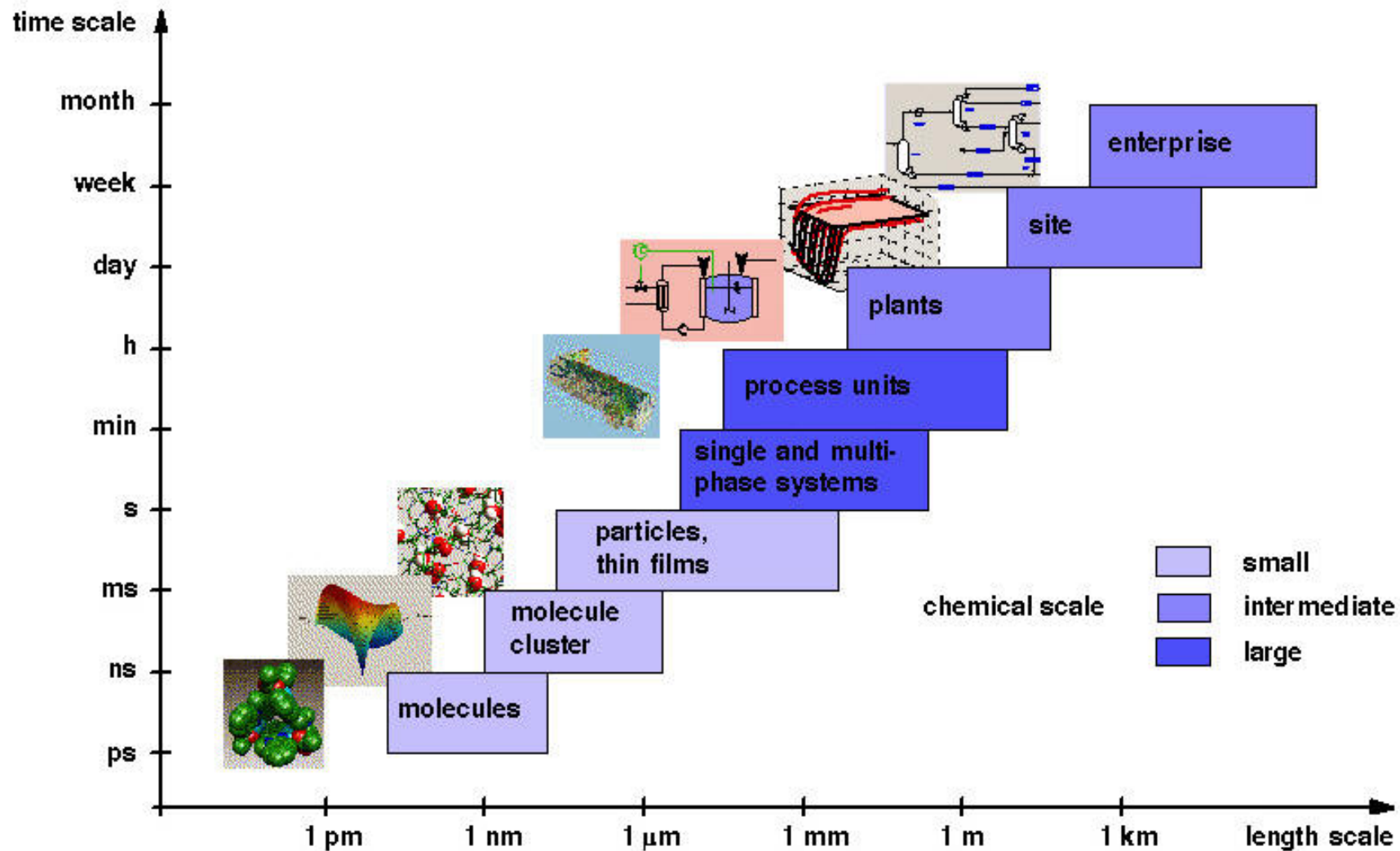
$$\frac{dC}{dt} = k_{EC}E - (k_{CN} + k_{dC})C$$

$$\frac{dN}{dt} = k_{CN}N$$

Compute optimal gene delivery profile to maximise efficacy and minimise toxicity subject to model equations and constraints.

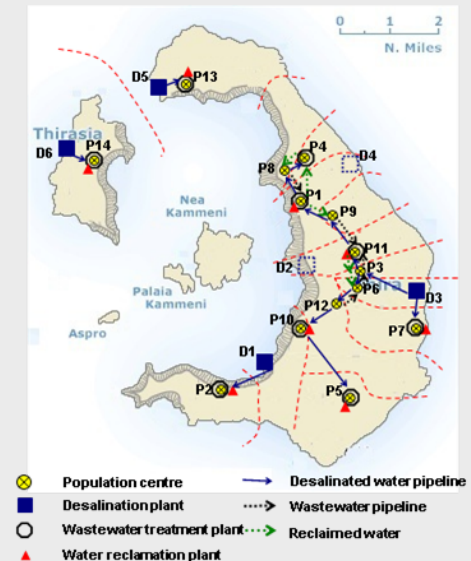
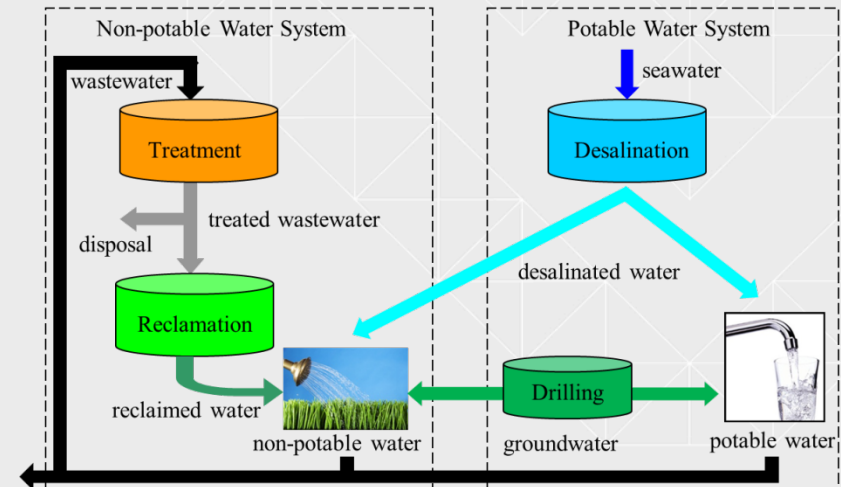


# Recall: Multi-scale considerations



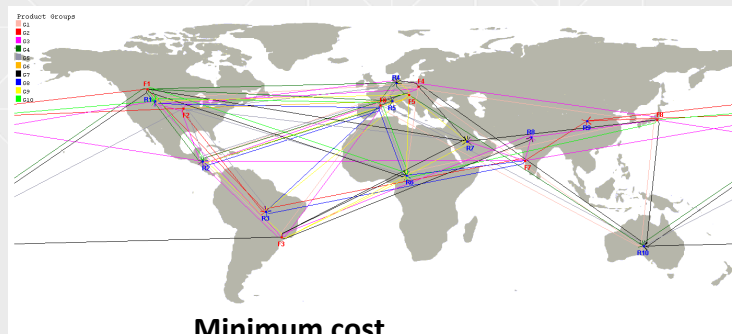
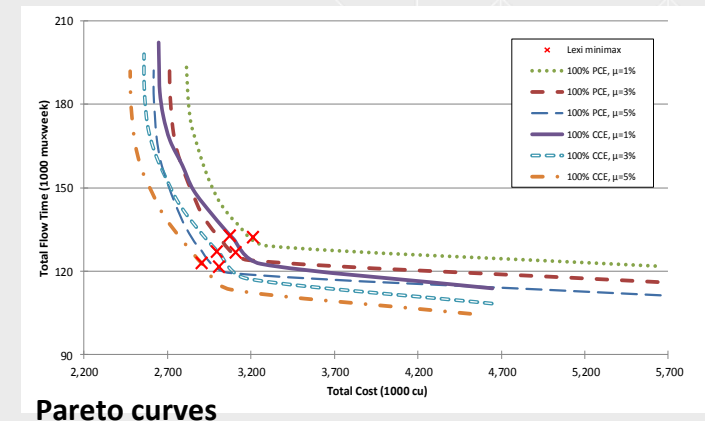
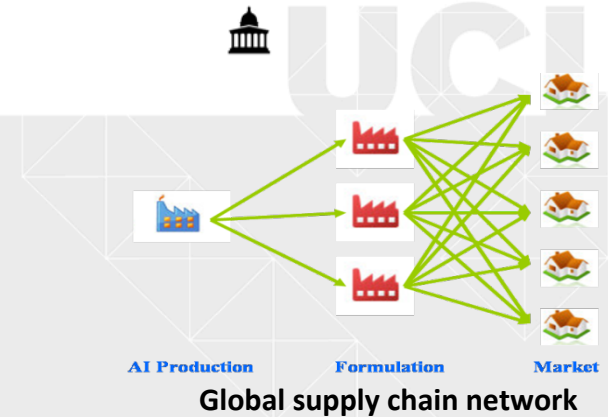
# Water Resource Management

- Non-conventional water resources are crucial to water deficient areas
  - Desalinated water
  - Treated wastewater
  - Reclaimed water
- MIP approaches proposed for integrated water resources management
- Decisions:
  - Plants locations, capacities and productions
  - Pipeline networks and flows
  - Pumping stations
  - Storage tanks
- Objective: annualised total cost
  - Capital costs (plant/pipeline/pumping station/storage tank)
  - Operating costs (production/pumping)

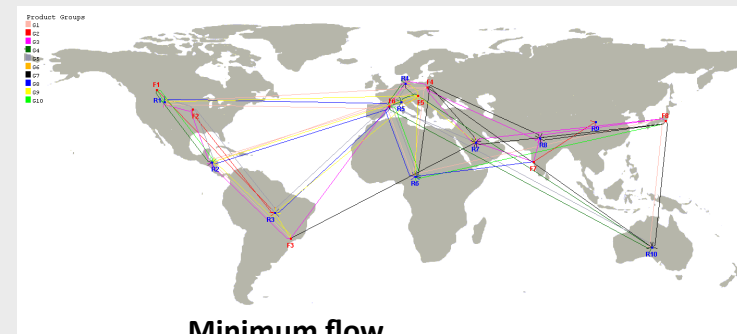


# Global Supply Chain Planning

- Three key supply chain performance metrics are considered:
  - Cost
  - Responsiveness (flow time)
  - Customer service level (lost sales)
- Two solution approaches for multiobjective MILP model:
  - $\epsilon$ -constraint method  $\rightarrow$  Pareto solutions
  - Lexicographic minimax method  $\rightarrow$  Fair solution



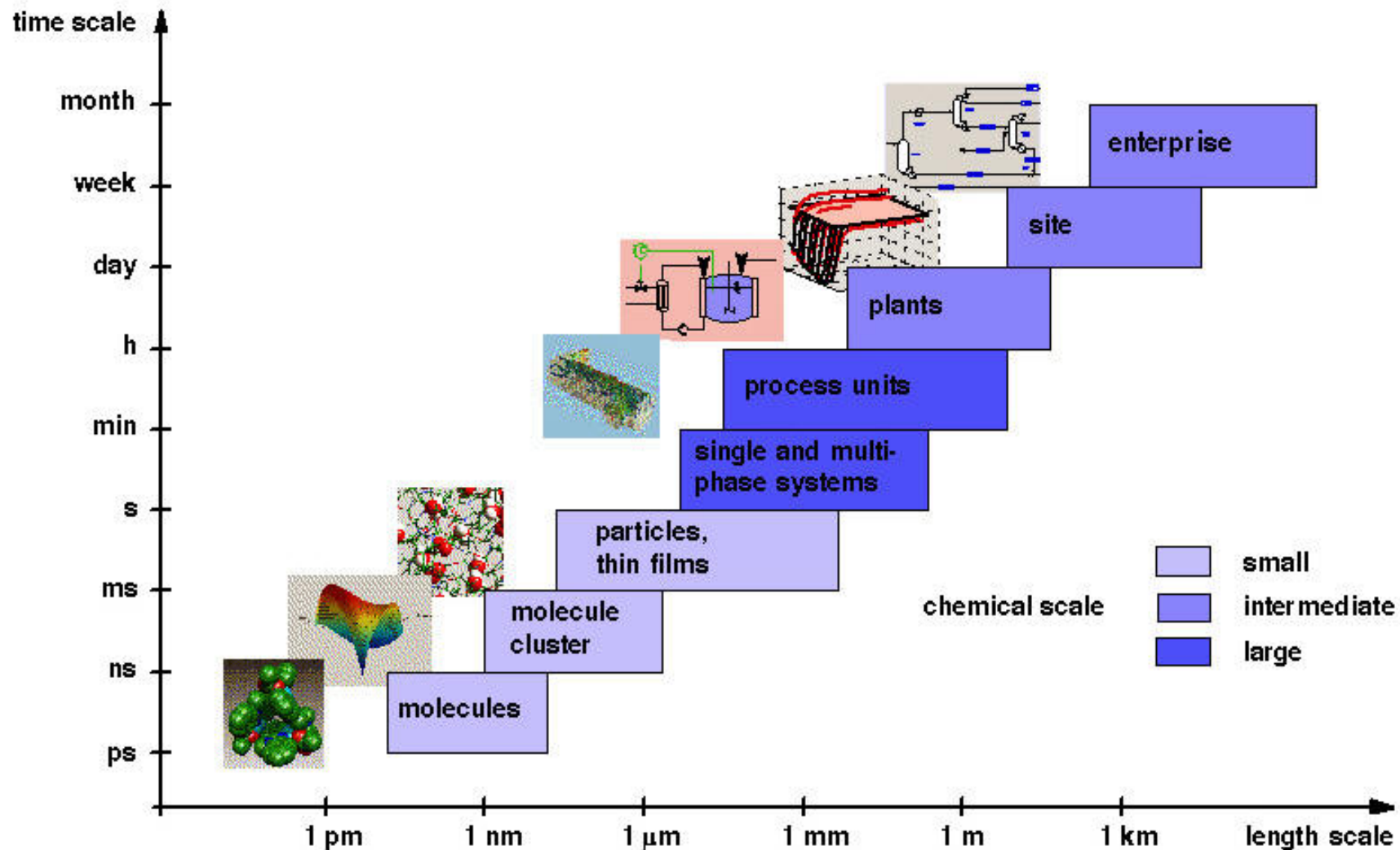
Minimum cost



Minimum flow



# Recall: Multi-scale considerations



# Main constraints



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**PHYSICS/CHEMISTRY**



**SAFETY**



**SUSTAINABILITY**



# Optimisation problem



## Integer:

# columns  
# plants  
etc

## Continuous:

Energy  
Flowrates  
etc



## Integer:

# columns  
# plants  
etc

## Continuous:

Energy =  $f(t)$   
Flowrates =  $f(t)$   
etc

# MINLP

# MIDO



# Product and Process Systems Engineering



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