Optimizing Geometric Forms

Trends and Challenges

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Geometric Modeling and Computer Graphics

Thursday, 14 February 13
Research Team

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Consumers of Digital 3D Models
Consumers of Digital 3D Models

games/movies

virtual worlds

[images from various online sources]
Consumers of Digital 3D Models

- games/movies
- architectural design
- virtual worlds
- manufacturing

[images from various online sources]
Consumers of Digital 3D Models

- games/movies
- architectural design
- virtual worlds
- manufacturing
- assisted surgery

[images from various online sources]
Consumers of Digital 3D Models

- games/movies
- architectural design
- digital archival
- virtual worlds
- manufacturing
- assisted surgery

[images from various online sources]
Creating Geometry: 3D Modelers

Bonzai3D screenshot
Creating Geometry: 3D Modelers

Bonzai3D screenshot

[images from various online sources]
Digitizing Physical Models

laser scanner
Digitizing Physical Models

laser scanner → 3D geometry

Optimizing Geometric Forms

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Digitizing Physical Models

laser scanner \rightarrow 3D geometry \rightarrow entertainment
Relations $\Rightarrow$ Redundancy
Relations $\Rightarrow$ Redundancy

symmetry
Relations $\Rightarrow$ Redundancy

symmetry
Relations $\Rightarrow$ Redundancy

symmetry
Relations $\Rightarrow$ Redundancy

symmetry
Relations $\Rightarrow$ Redundancy

symmetry

contact
Optimizing Geometric Forms

Relations $\Rightarrow$ Redundancy

symmetry

contact

$(a_x, a_y)$

$(b_x, b_y)$

$a_h$

$b_h$
Relations $\Rightarrow$ Redundancy

\[ |a_y - b_y| = \frac{a_h + b_h}{2} \]

Symmetry

Contact
Structure among Relations
Structure among Relations

\[ aRb \quad \text{and} \quad bRc \Rightarrow aRc \]
Structure among Relations

\[ aRb \quad \text{and} \quad bRc \Rightarrow aRc \]

\[ T_{ab} \cdot T_{bc} \cdot T_{ca} = I \]
Structure among Relations

\[ aRb \text{ and } bRc \Rightarrow aRc \]

\[ T_{ab} \cdot T_{bc} \cdot T_{ca} = I \]

\[
\begin{bmatrix}
a_{00} & a_{10} & a_{20} & a_{30} & a_{40} \\
a_{01} & a_{11} & a_{21} & a_{31} & a_{41} \\
a_{02} & a_{12} & a_{22} & a_{32} & a_{42} \\
a_{03} & a_{13} & a_{23} & a_{33} & a_{43} \\
a_{04} & a_{14} & a_{24} & a_{34} & a_{44}
\end{bmatrix}
\]
Structure among Relations

\[ aRb \quad \text{and} \quad bRc \Rightarrow aRc \]

\[ T_{ab} \cdot T_{bc} \cdot T_{ca} = I \]

\[
\begin{bmatrix}
  a_{00} & a_{10} & a_{20} & a_{30} & a_{40} \\
  . & a_{11} & a_{21} & a_{31} & a_{41} \\
  . & . & a_{22} & a_{32} & a_{42} \\
  . & . & . & a_{33} & a_{43} \\
  . & . & . & . & a_{44}
\end{bmatrix}
\]
Structure among Relations

\[ aRb \quad \text{and} \quad bRc \Rightarrow aRc \]

\[ T_{ab} \cdot T_{bc} \cdot T_{ca} = I \]

\[
\begin{bmatrix}
  a_{00} & a_{10} & \cdot & a_{30} & \cdot \\
  \cdot & a_{11} & a_{21} & \cdot & a_{41} \\
  \cdot & \cdot & a_{22} & a_{32} & a_{42} \\
  \cdot & \cdot & \cdot & a_{33} & a_{43} \\
  \cdot & \cdot & \cdot & \cdot & a_{44}
\end{bmatrix}
\]
Structure among Relations

\[ aRb \text{ and } bRc \Rightarrow aRc \]

\[ T_{ab} \cdot T_{bc} \cdot T_{ca} = I \]

exploit ‘structure’ for
compactness (redundancy)
robustness (constraints)
Discrete vs. Continuous

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Typical Optimization Problems
Typical Optimization Problems

• **Discrete**: Selection among ambiguous relations
Typical Optimization Problems

- **Discrete**: Selection among ambiguous relations
- **Continuous**:  
  - Global optimization  
  - Refine/couple parameters
Typical Optimization Problems

- **Discrete**: Selection among ambiguous relations

- **Continuous**:
  - Global optimization
  - Refine/couple parameters

- **Mixed integer formulations**
Typical Optimization Problems

- **Discrete**: Selection among ambiguous relations
- **Continuous**:
  - Global optimization
  - Refine/couple parameters
- **Mixed integer formulations**
- **Common Challenges**
  - Large systems (order of 50k-100k variables)
  - Robust initialization and relative weightings
  - Large search space based on parameterization
1) Heterogeneous data
1) Heterogeneous data

incomplete, sparse, noisy pointsets
2) Massive data volumes
3) Simplified interactions
3) Simplified interactions
3) Simplified interactions
Future of Geometry Processing?
Future of Geometry Processing?
geometry ➔ high level structures, respect global constraints
Future of Geometry Processing?

intelligently reuse existing content

geometry → high level structures, respect global constraints
Research Theme

- visualization
- acquisition
- shape analysis
- interaction
- synthesis
- form-finding
Research Theme

shape analysis

visualization

acquisition

interaction

synthesis

form-finding
low-level geometry $\rightarrow$ (structure+element) + variations
Given object $S$, extract regions $s_1$ and $s_2$, such that:

$$s_1 \sim T(s_2) : s_1, s_2 \in S$$
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Detecting Symmetries

[Siggraph 2006]

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Detecting Symmetries

Abstract

In this paper we present a novel method for detecting meaningful symmetries of the object, followed by a verification step. Based on a statistical sampling analysis, we provide guarantees on the success rate of our algorithm. The algorithm that processes geometric models efficiently discovers symmetries of varying degree. This allows the user to select the subset of symmetries at all scales, including approximate or imperfect symmetries. Examples include scan registration and alignment, shape matching, and art. For example, in geometry, the Erlanger program of Felix Klein has fueled for over a century mathematicians' interest in invariance under certain group actions as a key principle for understanding geometric spaces. Numerous biological, physical, or artistic examples can be seen in this example, symmetry implies certain economies and efficiencies of structure that make it universally appealing.

Figure 1: Symmetry detection on a sculpted model. From left to right: Original model, detected partial and approximate symmetries.

Keywords: geometric modeling, shape analysis, symmetry detection, indexing for retrieval, etc.

CR Categories:

I.3.5 [Computer Graphics]: Computational geometry, shape compression, segmentation, consistent editing, symmetry detection, indexing for retrieval, etc.

1 Introduction

Many natural and man-made objects exhibit symmetries as a fundamental design principle. For example, in nature, the symmetry of a face is an important factor in our perception of beauty. As Thompson (1961) states, "Symmetry is a complexity-reducing concept [...]; seek it everywhere."
Detecting Symmetries

low-level geometry ➔ (structure+element) + variations

[Siggraph 2006]

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Optimizing Geometric Forms
low-level geometry $\rightarrow$ (structure+element) + variations

[Siggraph 2008]
Detecting Regularities

low-level geometry \rightarrow (structure + element) + variations

[Siggraph 2008]
Detecting Regularities

low-level geometry $\rightarrow$ (structure+element) + variations

[Siggraph 2008]
Detecting Regularities

low-level geometry $\rightarrow$ (structure+element) + variations

[Siggraph 2008]
‘Find and Replace’
‘Find and Replace’

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‘Find and Replace’
Enhancing Repetitions

reuse of fabrication molds

identical torus molds

identical cylinder molds

panel types

plane
cylinder
paraboloid
torus
cubic

[Siggraph 2010]
Enhancing Repetitions

identical torus molds
identical cylinder molds
reuse of fabrication molds
panel types
plane
cylinder
paraboloid
torus
cubic

typical cost saving ~ 70-80%

[Siggraph 2010]
Enhancing Repetitions

low-level geometry $\rightarrow$ (structure+element) + variations

identical torus molds
identical cylinder molds
reuse of fabrication molds
panel types

typical cost saving $\sim$ 70-80%

[Siggraph 2010]
Fabrication-aware Design

Scalable Acquisition

Figure 7: Photographs can have significant parts occluded due to trees and other obstacles. The occlusions, however, are usually different across views thus resulting in improved geometry and texture consolidation when we use more images.

Figure 8: Consolidation result on a data of very tall skyscraper.
Scalable Acquisition

Figure 7: Enhanced geometry and texture consolidation when we use more input data.

Figure 8: Consolidation result on a data of very tall skyscrapers.

[ICCV 2011]

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Reconstructing From Images

Input: Collection of images of a building facade (with repetitions)

optimal geometric forms

Reconstruction From Images

Input: Collection of images of a building facade (with repetitions)

Initial repetition detection

Line and transformation initialization

Repetition completion

Initial alignment

Detected edges

Close edges

[Eurographics 2012]
repetition detection & optimization

line and transformation initialization
Smart Interactions
Relations across Features

Detail-preserving deformation

Speed x2

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Algorithm

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Sample Edit Session

Edit mode: [grab-and-drag]

Speed x3

Toy jeep model
574 wires
80 groups
208 components
Geometry to Motion

Input Model

How Things Work Visualization

[Siggraph 2010]
Geometry to Motion

Input Model

[ SIGGRAPH 2010 ]

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Geometry to Motion

Input Model

Motion and Interaction Analysis

[Siggraph 2010]
Geometry to Motion

Input Model

Motion and Interaction Analysis

[Siggraph 2010]
Geometry to Motion

Input Model

Motion and Interaction Analysis

How Things Work Visualization

[Siggraph 2010]
Geometry to Motion

live capture
speed 1x

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Optimizing Geometric Forms
why does this work?
Constrained Meshes

• Given:
  single \textit{constrained} mesh (mesh + constraints)
Constrained Meshes

• Given:
  single *constrained* mesh (mesh + constraints)

• Goal:
Constrained Meshes

• Given:
  single *constrained* mesh (mesh + constraints)

• Goal:
  – characterize/navigate *neighboring* constrained meshes
Constrained Meshes

• Given:
  single *constrained* mesh (mesh + constraints)

• Goal:
  – characterize/navigate *neighboring* constrained meshes
  – navigate only the *good* ones
Constrained Shape Space

\[ \mathbf{x} = (v_1, \ldots, v_n) \in \mathbb{R}^D \]
Constrained Shape Space

\[ \mathbf{x} = (v_1, \ldots, v_n) \in \mathbb{R}^D \]

- mesh → point

[SiggraphA 2011]
Constrained Shape Space

\[ \mathbf{x} = (v_1, \ldots, v_n) \in \mathbb{R}^D \]

- mesh $\rightarrow$ point
- combinatorics remain fixed

[SiggraphA 2011]
Constrained Shape Space

\[ \mathbf{x} = (v_1, \ldots, v_n) \in \mathbb{R}^D \]

- mesh \mapsto point
- combinatorics remain fixed
- starting mesh \( x_0 \) satisfies (nonlinear) constraints

[SiggraphA 2011]
Each face constraint

\[ \Gamma_i := \{ \mathbf{x} \in \mathbb{R}^D : E_i(\mathbf{x}) = 0 \} \quad \forall \quad i = 1, \ldots, m \]
Intersection Surface

Each face constraint

\[ \Gamma_i := \{ \mathbf{x} \in \mathbb{R}^D : E_i(\mathbf{x}) = 0 \} \quad \forall \quad i = 1, \ldots, m \]

\[ \mathbf{d} \Rightarrow \mathbf{x}_0 + \mathbf{d} \]
Osculant Surface

\[ S(u) = x_0 + \sum_{i=1}^{D-m} u_i e_i + \frac{1}{2} \sum_{j=1}^{m} (u^T \cdot A_j \cdot u)n_j \]
Osculant Surface

\[ S(u) = x_0 + \sum_{i=1}^{D-m} u_i e_i + \frac{1}{2} \sum_{j=1}^{m} (u^T \cdot A_j \cdot u) n_j \]

\[ E_i(x) = E_i(x_0) + \nabla E_i^T \cdot (x - x_0) + \frac{1}{2} (x - x_0)^T \cdot H_i \cdot (x - x_0) + o(\|x - x_0\|^2) \]
Osculant Surface

\[ S(u) = x_0 + \sum_{i=1}^{D-m} u_i e_i + \frac{1}{2} \sum_{j=1}^{m} (u^T \cdot A_j \cdot u)n_j \]

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Osculant Surface

\[
S(u) = x_0 + \sum_{i=1}^{D-m} u_i e_i + \frac{1}{2} \sum_{j=1}^{m} (u^T A_j u) n_j
\]

\[
E_i(x) = E_i(x_0) + \nabla E_i^T(x - x_0) + \frac{1}{2} (x - x_0)^T H_i (x - x_0) + o(\|x - x_0\|^2)
\]

\[
E_i(u) = E_i(x_0)
\]
Osculant Surface

\[ S(u) = x_0 + \sum_{i=1}^{D-m} u_i e_i + \frac{1}{2} \sum_{j=1}^{m} (u^T \cdot A_j \cdot u)n_j \]

\[ E_i(x) = E_i(x_0) + \nabla E_i^T \cdot (x - x_0) + \frac{1}{2} (x - x_0)^T \cdot H_i \cdot (x - x_0) \]
\[ + o(\|x - x_0\|^2) \]

\[ E_i(u) = E_i(x_0) + \frac{1}{2} \sum_{j=1}^{m} (\nabla E_i^T \cdot n_j)(u^T \cdot A_j \cdot u) \]
Osculant Surface

\[ S(u) = x_0 + \sum_{i=1}^{D-m} u_i e_i + \frac{1}{2} \sum_{j=1}^{m} (u^T \cdot A_j \cdot u) n_j \]

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\[ E_i(u) = E_i(x_0) + \frac{1}{2} \sum_{j=1}^{m} (\nabla E_i^T \cdot n_j)(u^T \cdot A_j \cdot u) \]

\[ + \frac{1}{2} \sum_{p=1}^{D-m} \sum_{q=1}^{D-m} (e_p^T \cdot H_i \cdot e_q) u_p u_q + o(\|u\|^2) \]
Walking on the Tangent Space

PQ mesh manifold

$\mathcal{M}$
Walking on the Tangent Space

PQ mesh manifold

deformation field

average displacement: vertex 1 mm^2
Walking on the Tangent Space
Walking on the Tangent Space
Walking on the Tangent Space

- **PQ mesh manifold**
- **deformation field**
- **max$_i$ |$E_i$|**
- **average displacement/vertex (mm)**
- **0 to 10mm**
Flat Circular Mesh Exploration
Beyond Model-Pairs
Exploring Data Collections

low-level geometry $\rightarrow$ (structure + element) + variations

(a) input collection
Exploring Data Collections

low-level geometry $\rightarrow (\text{structure} + \text{element}) + \text{variations}$

(a) input collection
(b) template deformation model

[Siggraph 2011]
Exploring Data Collections

low-level geometry $\rightarrow$ (structure+element) + variations

(a) input collection  (b) template deformation model  (c) constrained exploration

[Siggraph 2011]

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Exploring Data Collections

low-level geometry \rightarrow (structure \, + \, element) + \text{variations}

(a) input collection \quad \quad \quad (b) template deformation model \quad (c) constrained exploration

\textbf{without correspondences}

[Siggraph 2011]
Exploring Data Collections

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Optimizing Geometric Forms
With Fuzzy Correspondence

3D Database Exploration: selecting(3), mode(0)
When Force Drives Form

Figure 2: (Left) Modeling interface consisting of modeling and weight modes. (Right) The modeling interface with typical elements. These approaches propose final structural feasibility by ensuring non-negative force between brick joints. Our system continuously runs validity check in the background.

Modeling user interface. Each suggestion when clicked appears in the modeling panel. If the shape is invalid, the system shows a big red (b). When the model falls down, the system shows a big red (b) or becomes unstable, i.e., topples (c).

Warning flagged for invalid configurations: Joints get disconnected (a), are not durable (b), or unstable (c). Explicitly showing the valid range reduces the need of trial and errors to stay within or return to valid state during direct manipulation editing.

Figure 3: Range indicators. Range is shown in black when the current configuration is valid, red (see Figure 4). Explicitly showing the valid range reduces the need of trial and errors to stay within or return to valid state during direct manipulation (mouse drag). When the current configuration is invalid, the system shows the valid range in red (b). When a joint becomes disconnected, the system shows how to make it connected again (Figure 5a).

In addition to checking whether the current configuration is valid or invalid, the system learns visual cues, e.g., the user cannot fix the undurability or instability moving each DOF individually. For example in Figure 6, if the user slides the top board of the table toward left, the angle of the left leg becomes unstable, i.e., the user cannot fix the undurability or instability moving each DOF individually, these feedbacks.

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[Siggraph 2012]
When Force Drives Form

Figure 2:

- **(Left)** Modeling interface consisting of modeling and make a new rectangular plank in the drawing mode defined by the Figure 2-right shows the basic modulation of a plank and placement of a weight.
- **(Right)** The modeling interface with typical suggestion panels. (Right) The modeling interface shows connectivity, durability, and stability. As a design ground and shows whether the current configuration satisfies the result of the analysis appears as an annotation in the main panel.

**3 System Overview**

In the context of design rationalization, researchers have worked on an existing plank or aligned to canonical xyz-axis. A joint is a set of such constraints for a furniture. The set of such constraints for a furniture can be redundant (e.g., Parker and Ambrose 1997). Hence, in our framework, we first describe how to continuously updated during the user’s dragging operation, so the user need of trial and errors to stay within or return to valid state during.

Section 5 describes how we measure and analyze physical validity changes with respect to further geometric modifications, while Section 6 describes how we compute valid range of the current shape during user’s editing. Specifically, the system checks for inequality constraints on joint and contact forces. We first define joint forces and then explain amount to checking for inequality constraints on joint and contact.

When analyzing moment and forces, we first try to satisfy geometric constraints, i.e., joint constraints, the representative configuration appears in the modeling panel together with arrow marks indicating the coordinated editing gesture.

Example of coordinated editing using suggestions. The top board of the table toward left, the angle of the left leg becomes i.e., the user cannot fix the undurability or instability moving each control multiple DOFs of a model simultaneously while satisfying.

**Nails at a Nail-Joint, for Each Joint by Directly Minimizing the Total Potential Energy of the System**

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When Force Drives Form

\[ \Gamma := \{ X_1, X_2, \ldots \} \]  

[Siggraph 2012]

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Optimizing Geometric Forms
Guided Exploration

\[ \Gamma_{\text{stable}} \]
\[ \Gamma_{\text{valid}} \]
\[ \Gamma_{\text{durable}} \]

\( f_{\text{cont}}^2 \)
\( \tilde{\Gamma}_{\text{stable}} \)

Joint 1
Joint 2
Contact 1
Contact 2

Optimizing Geometric Forms
“Forms as Force Diagrams”
Optimizing Layouts

- parcel constraint
- boundary edges
- holes/courtyard
- built area
- courtyard area
- shadow
- thickness

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Optimizing Layouts

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Same for Indoors
Summary

• low-level geometry $\Rightarrow$ high-level abstraction
Summary

• low-level geometry $\Rightarrow$ high-level abstraction
• symmetry, relations, contacts, etc. are good candidates
Summary

• low-level geometry $\Rightarrow$ high-level abstraction
• symmetry, relations, contacts, etc. are good candidates
• capture the necessary dimensions
• low-level geometry $\Rightarrow$ high-level abstraction
• symmetry, relations, contacts, etc. are good candidates
• capture the necessary dimensions
Summary

- low-level geometry $\implies$ high-level abstraction
- symmetry, relations, contacts, etc. are good candidates
- capture the necessary dimensions
- discrete/continuous (global) optimization
Summary

• low-level geometry $\Rightarrow$ high-level abstraction
• symmetry, relations, contacts, etc. are good candidates
• capture the necessary dimensions

• discrete/continuous (global) optimization
• pattern finding in high dimensions
thank YOU

http://www.cs.ucl.ac.uk/staff/N.Mitra/