# ***Optimizing Geometric Forms***

## *Trends and Challenges*

*Niloy J. Mitra*

Reader (Associate Professor)
Geometric Modeling and Computer Graphics

# Research Team

- Youyi Zheng (postdoc)
- Sawsan Alhalawani
- Melinos Averkiou
- Bongjin Koo
- Han Liu

and ***collaborators*** ...

***Funding:*** Marie Curie Grant, Adobe Research Grant, UCL Impact grant

# Consumers of Digital 3D Models

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games/movies


virtual worlds

[images from various online sources]

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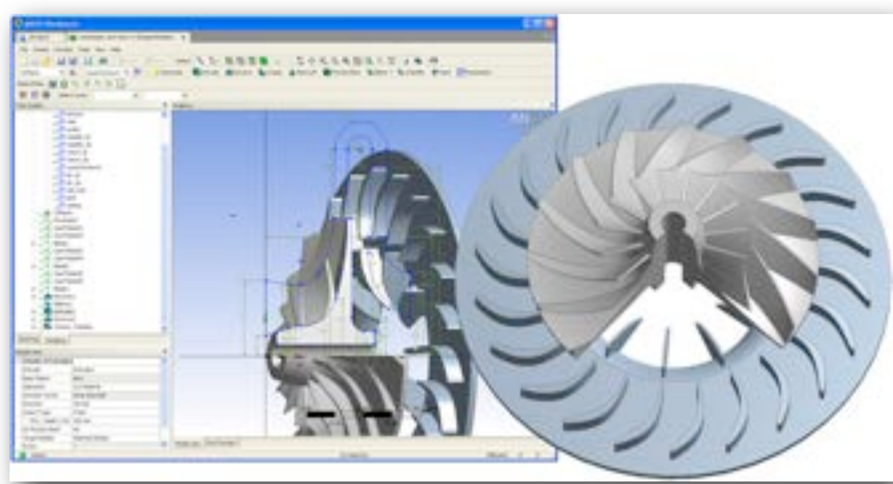

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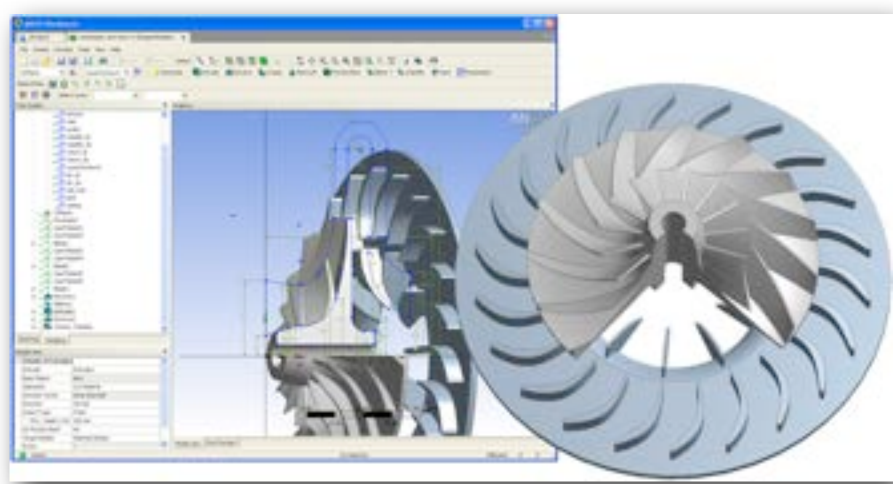

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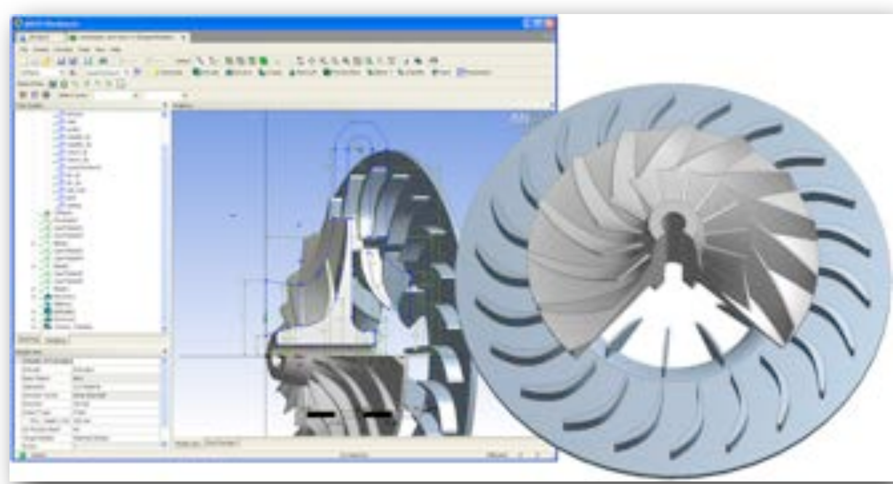

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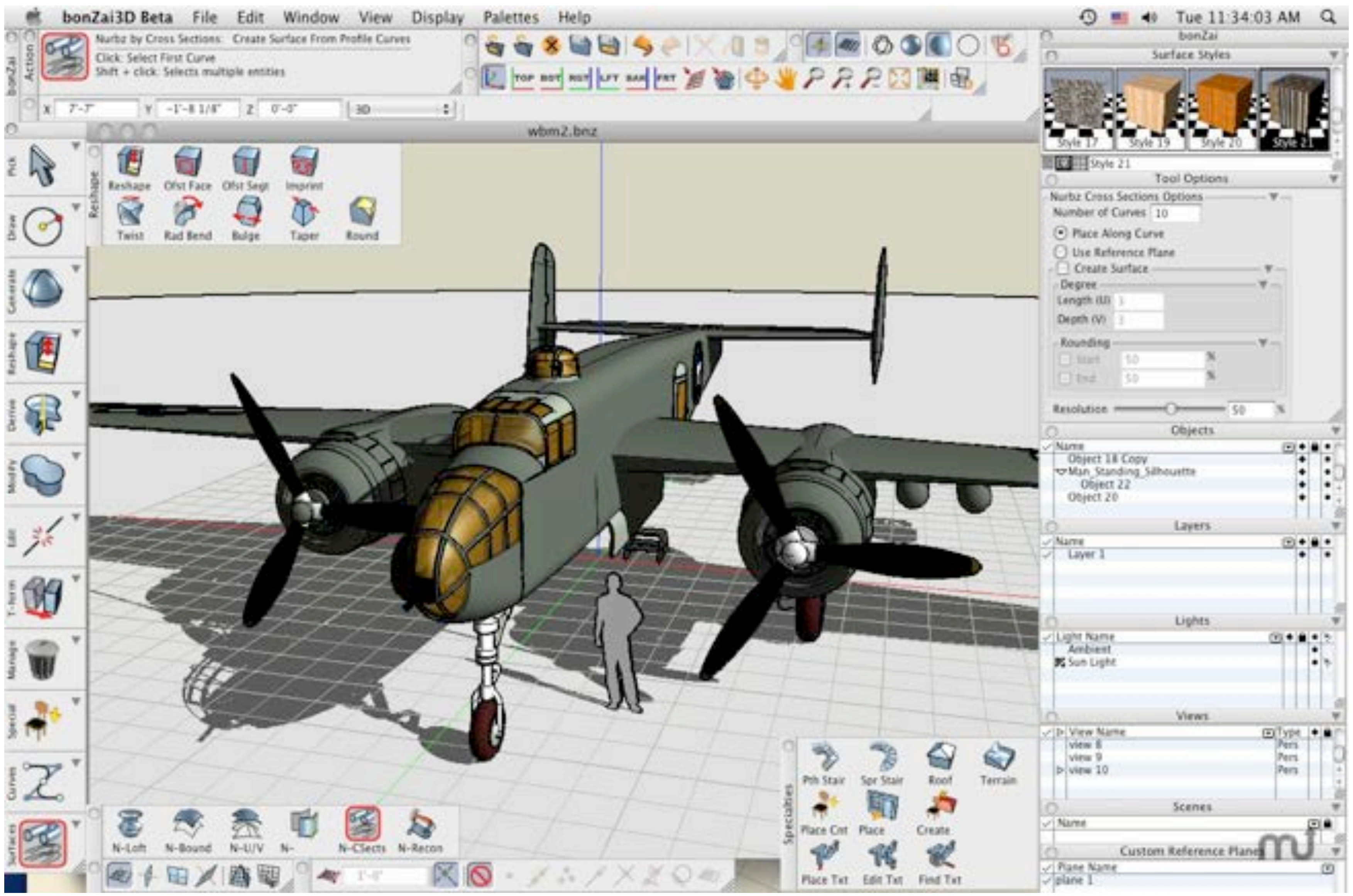
# Creating Geometry: 3D Modelers

Bonzai3D screenshot



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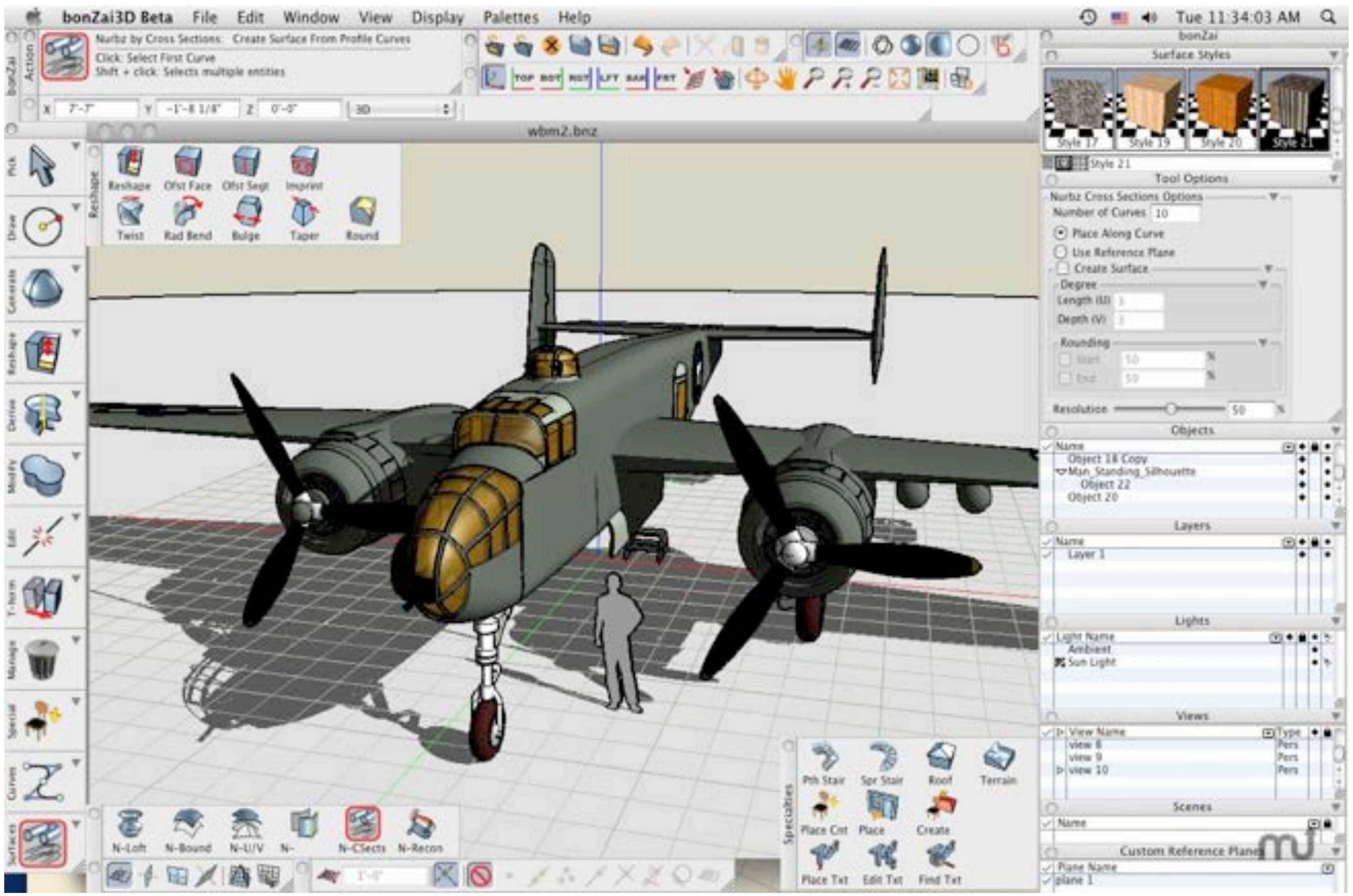


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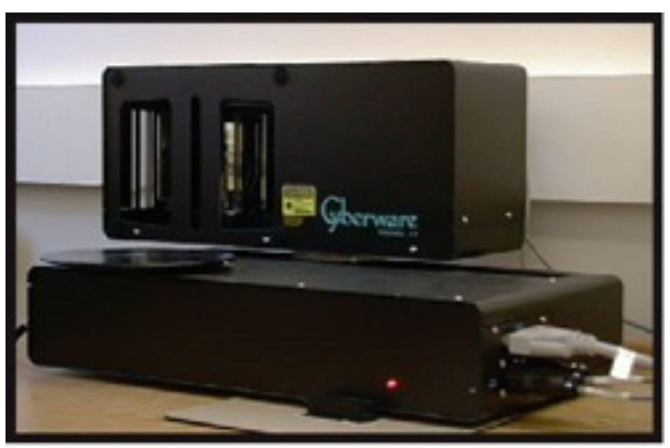


laser scanner

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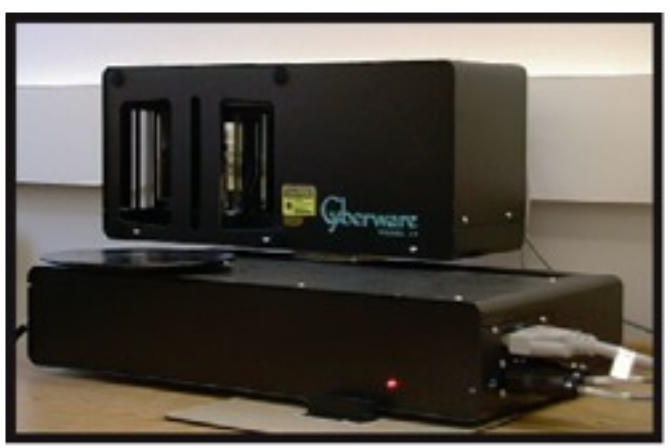


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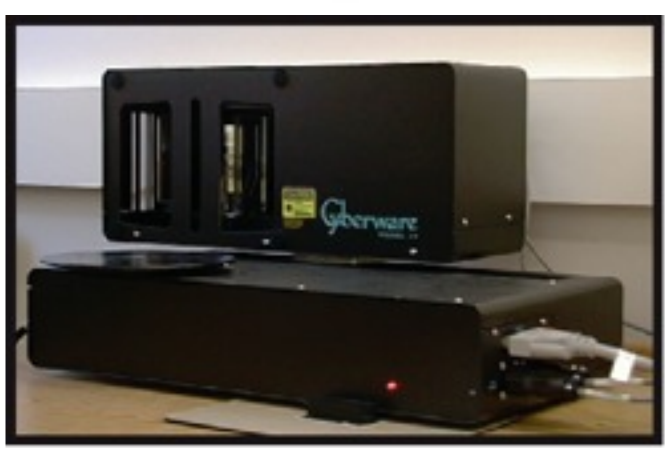


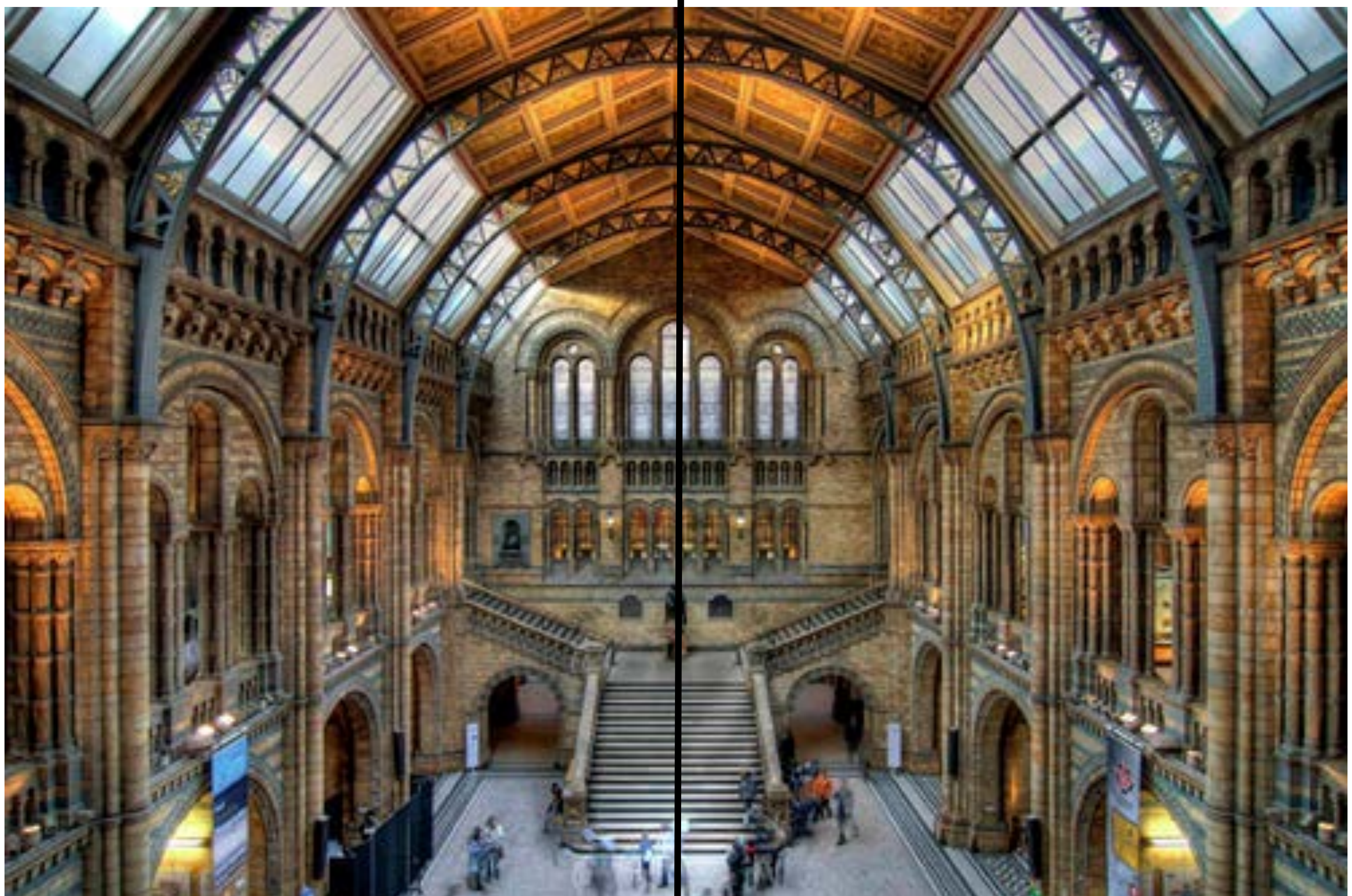

laser scanner

3D geometry

entertainment

# Relations $\Rightarrow$ Redundancy

# Relations ⇒ Redundancy



symmetry

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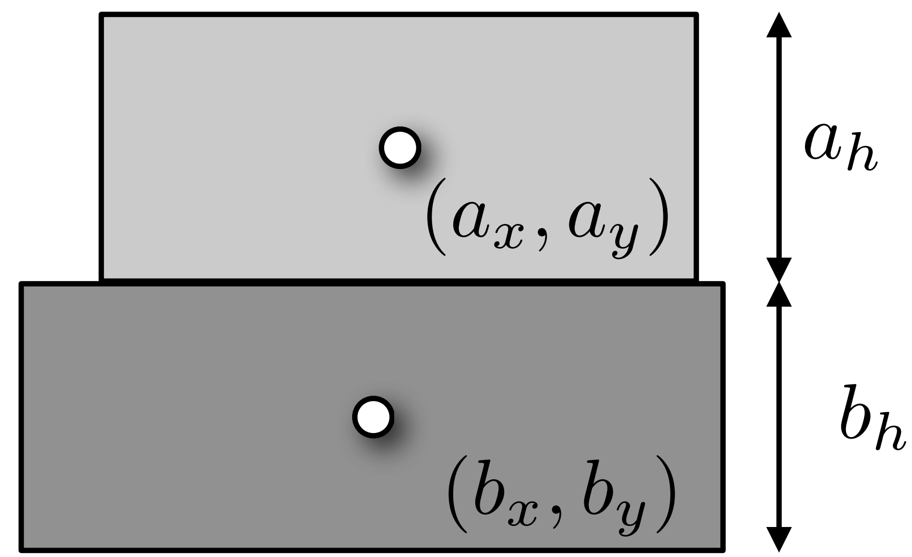


contact

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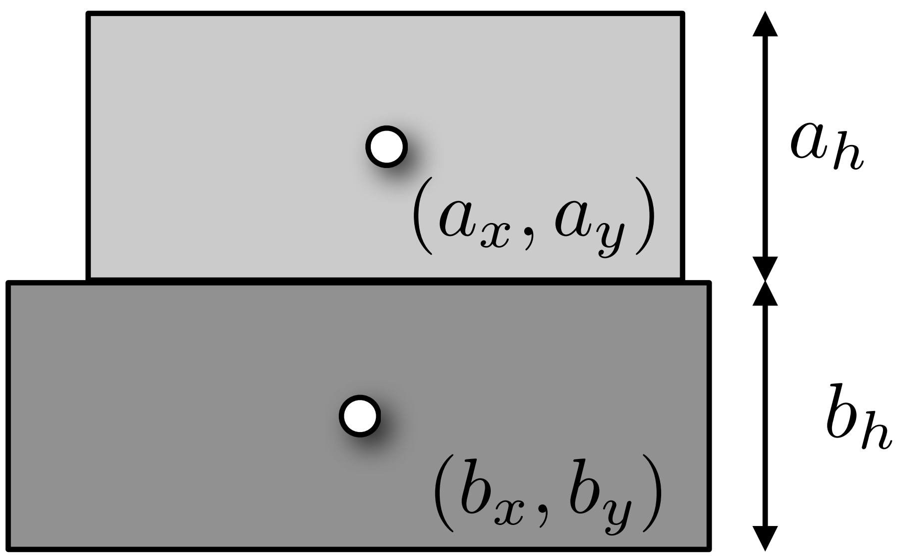
symmetry

$$|a_y - b_y| = \frac{a_h + b_h}{2}$$

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exploit ‘structure’
for

compactness (redundancy)
robustness (constraints)

# Discrete vs. Continuous

# Typical Optimization Problems

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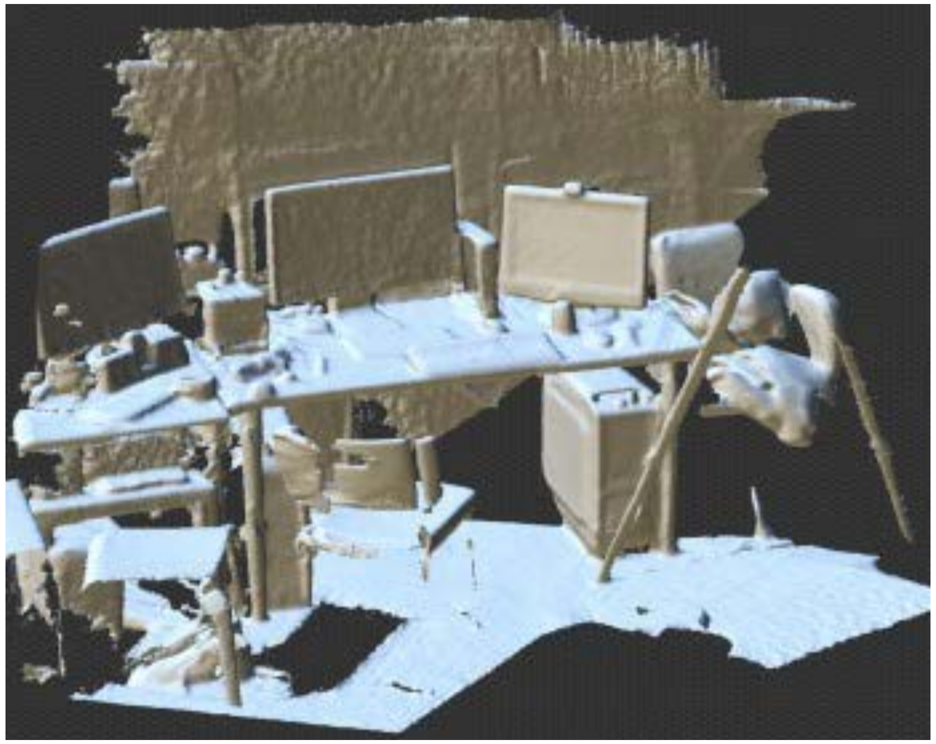
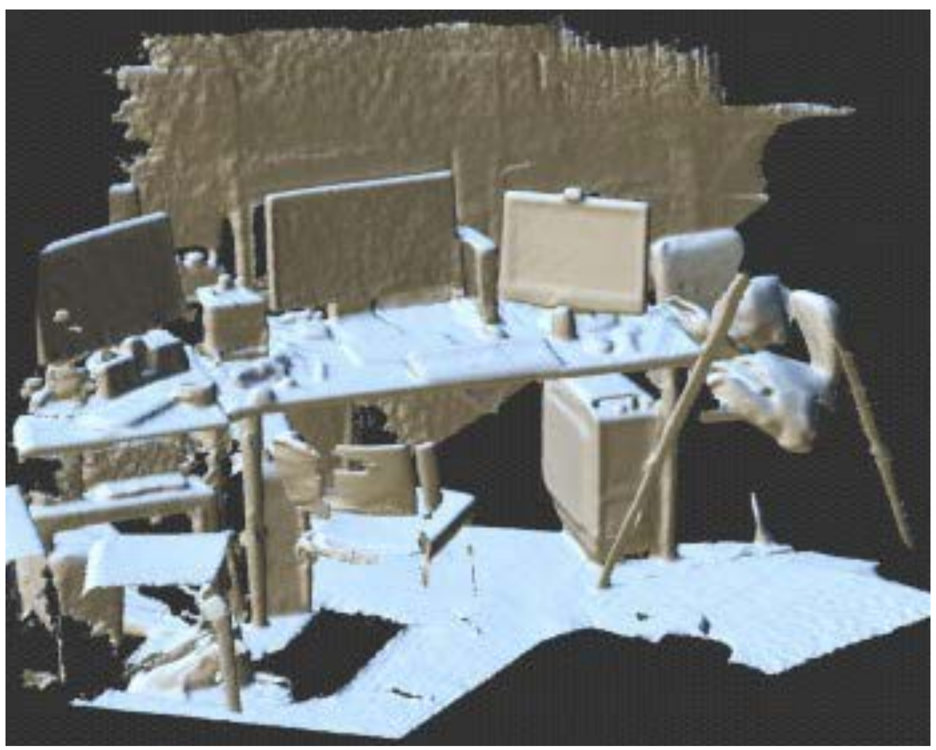
- **Mixed integer formulations**
- **Common Challenges**
  - Large systems (order of 50k-100k variables)
  - Robust initialization and relative weightings
  - Large search space based on parameterization

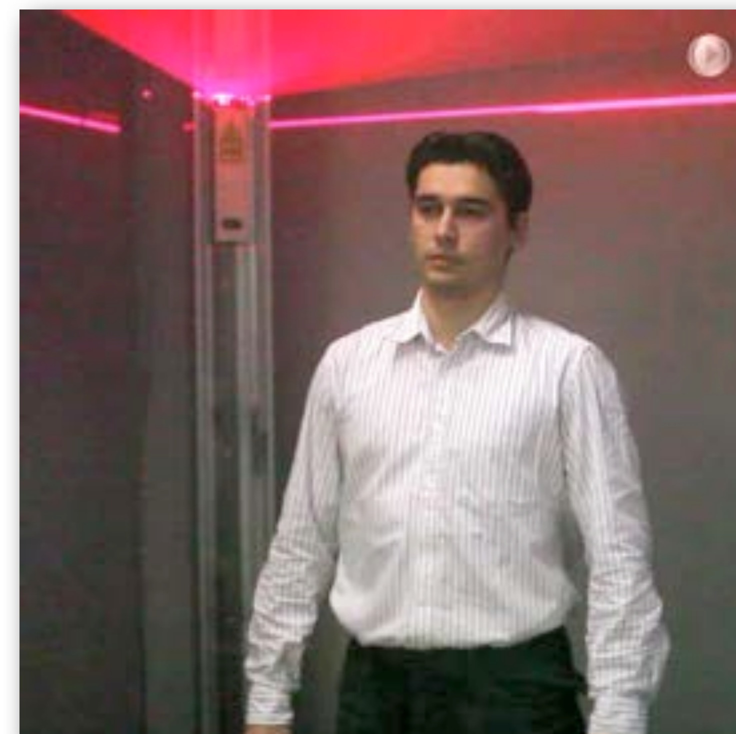
# 1) Heterogeneous data



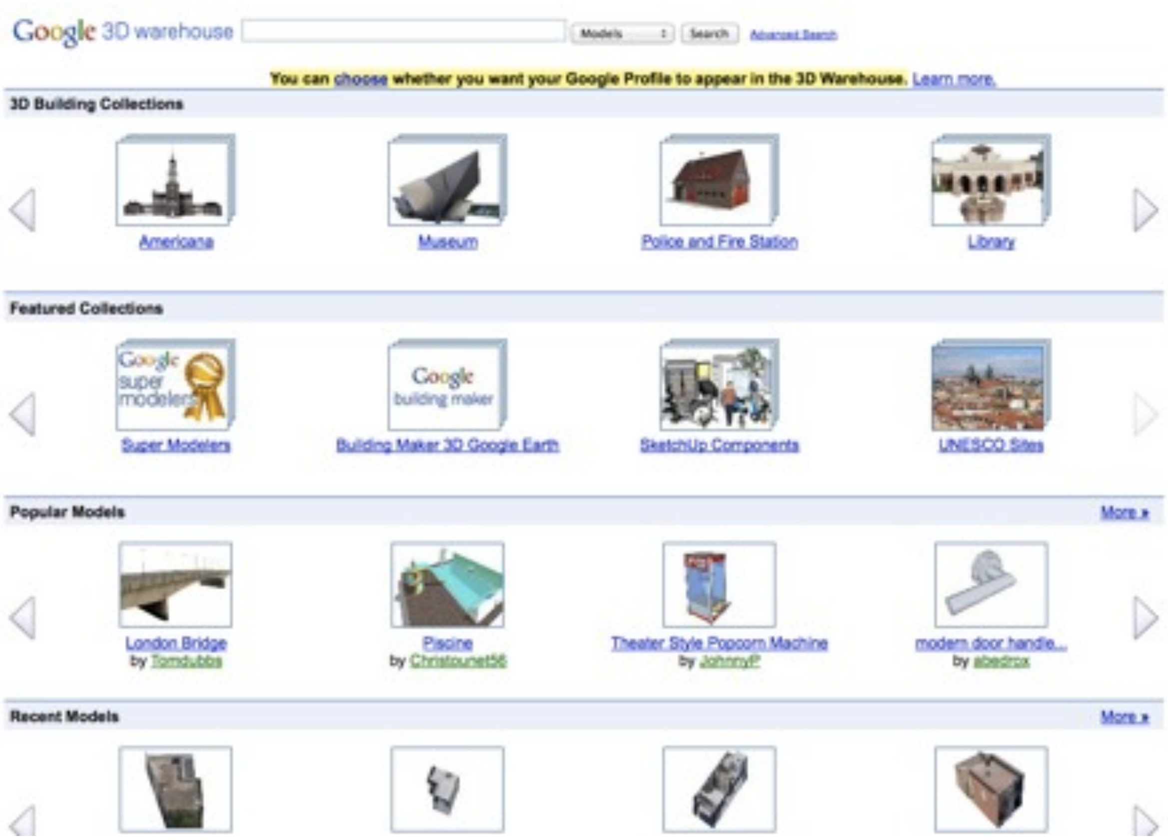
# 1) Heterogeneous data

*incomplete, sparse, noisy pointsets*





# 2) Massive data volumes




# 3) Simplified interactions

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# Future of Geometry Processing?

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geometry → high level structures,
respect global constraints

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*intelligently* reuse existing content

geometry → high level structures,
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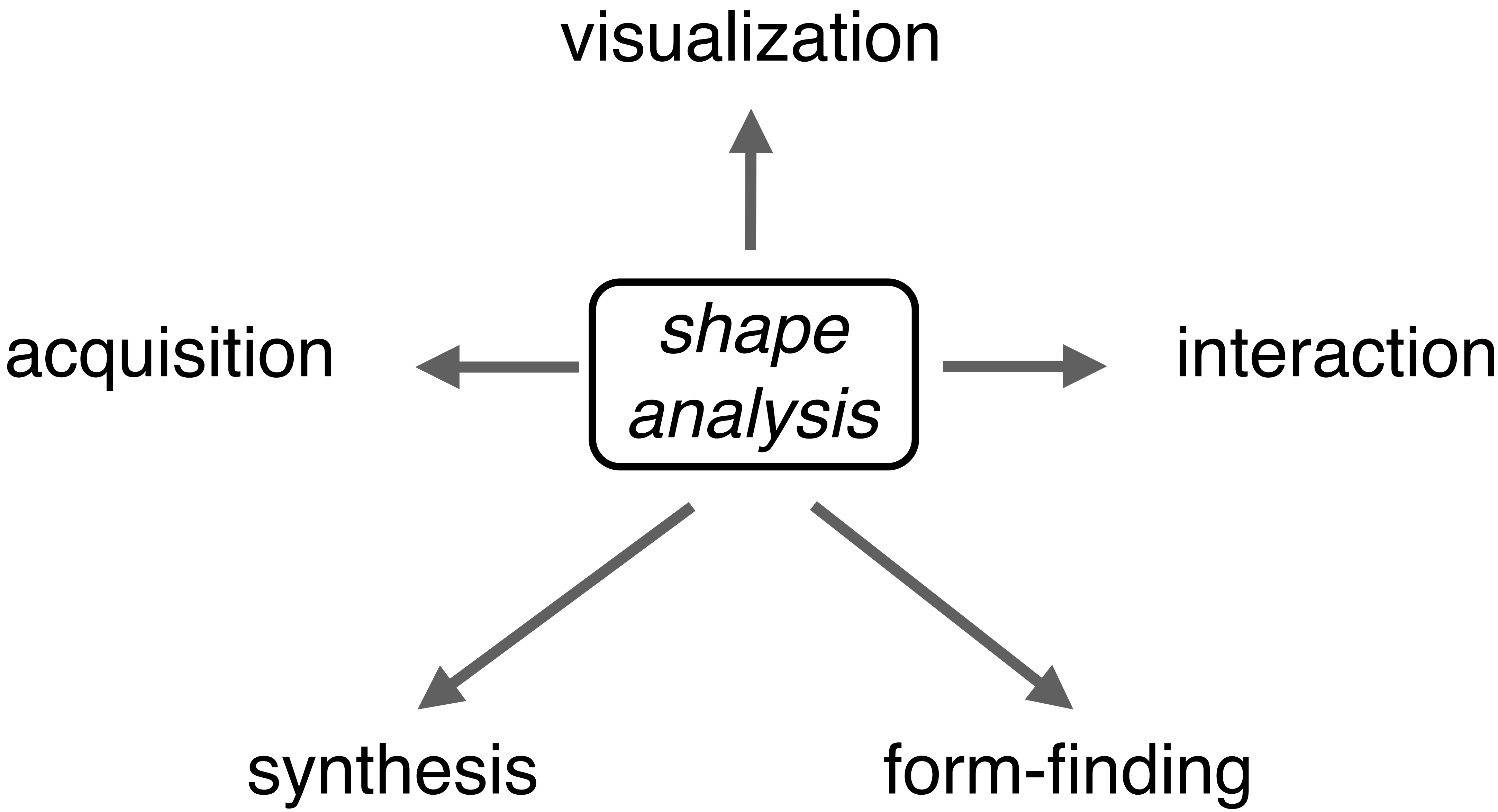
# Research Theme



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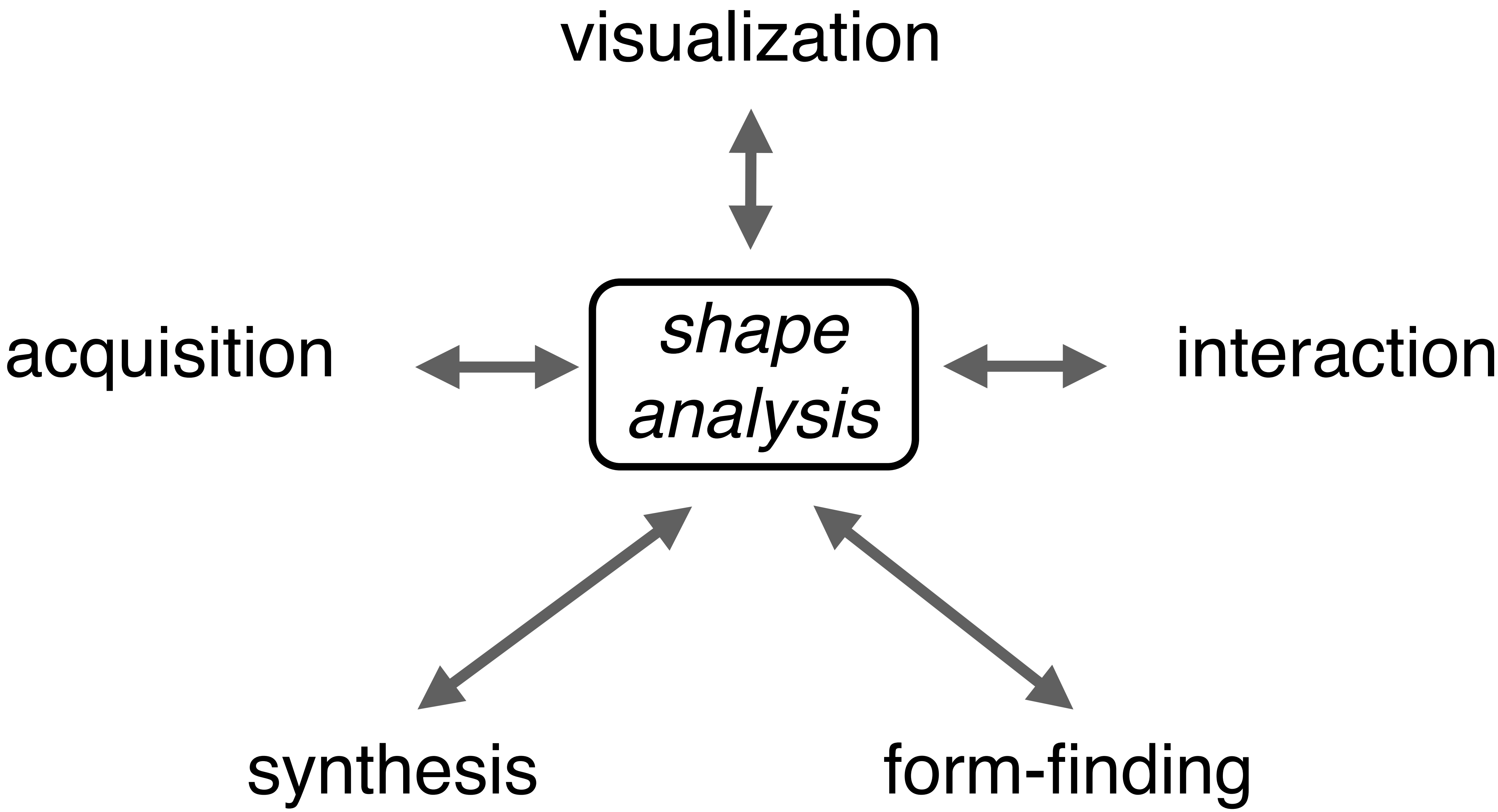
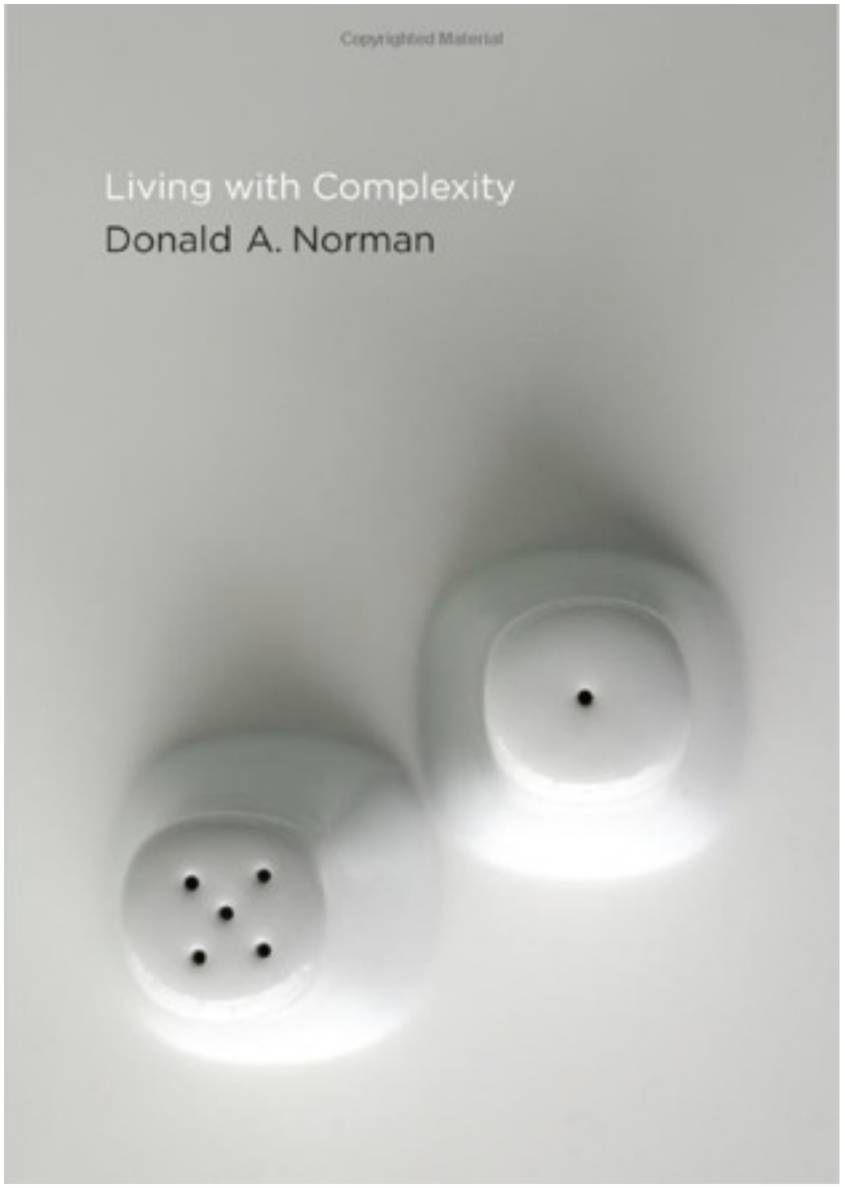


# ‘Models of Data’



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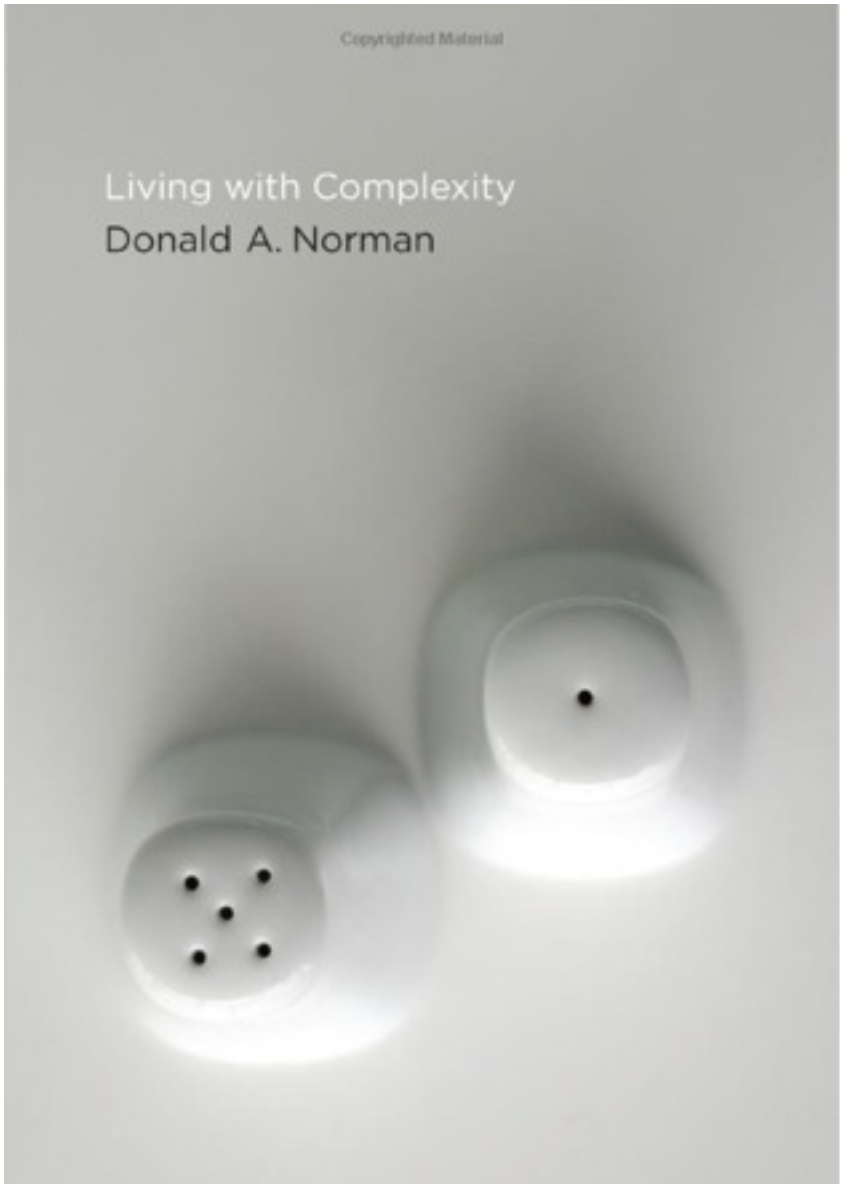


low-level geometry → (structure+element) + variations

# Repetitions or Redundancy

Given object S,
extract regions $s_1$ and $s_2$, such that:

$$s_1 \sim T(s_2) : s_1, s_2 \in S$$

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# Detecting Symmetries

[Siggraph 2006]

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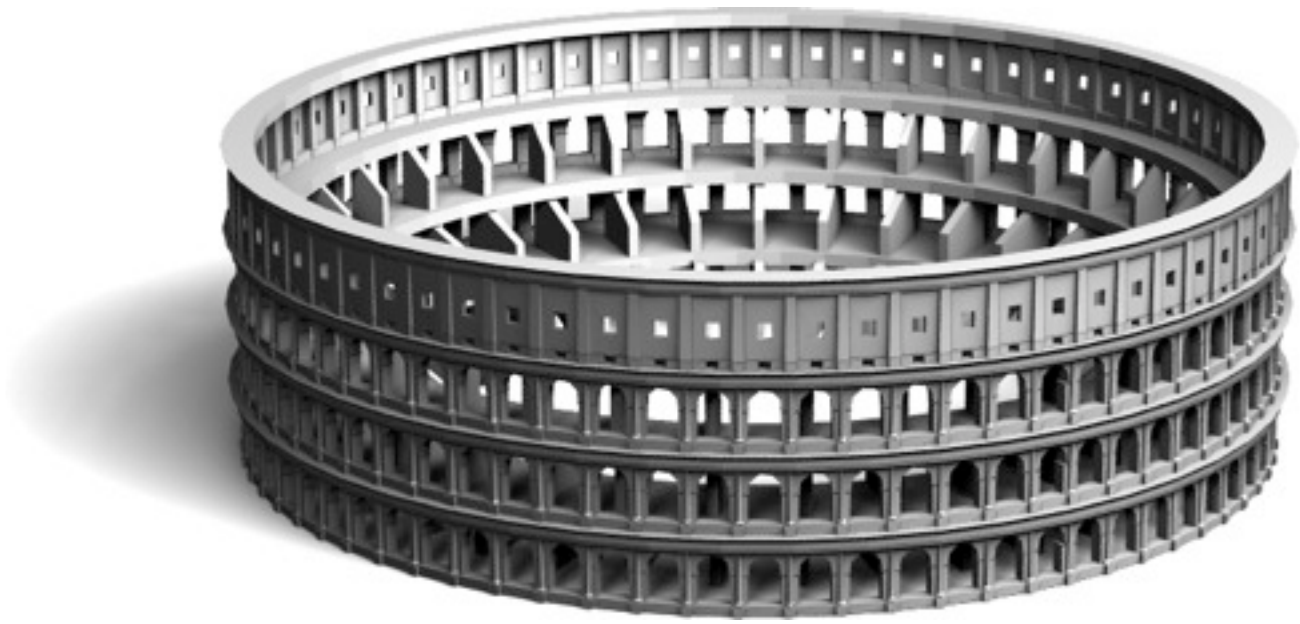

[Siggraph 2008]

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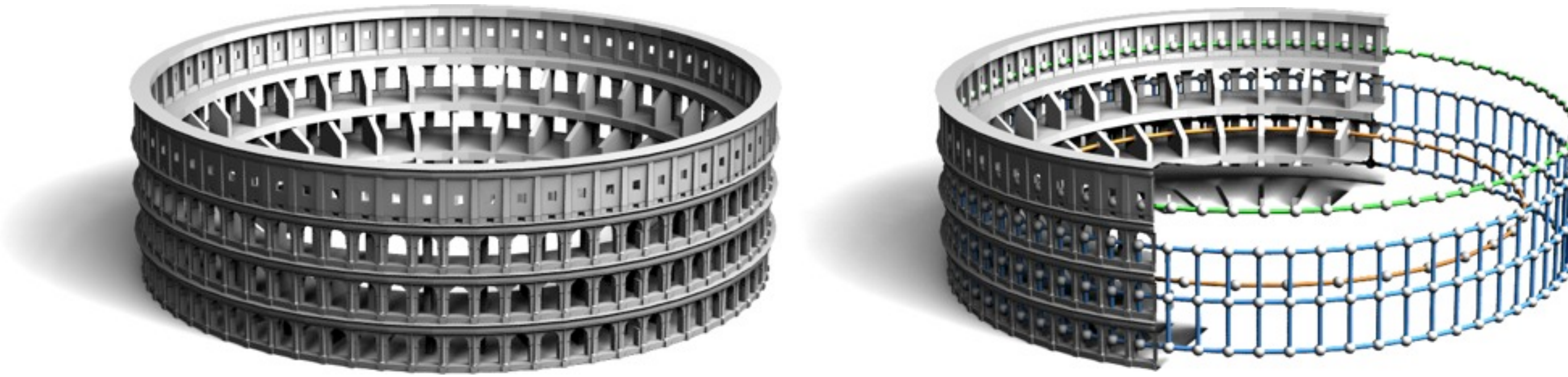

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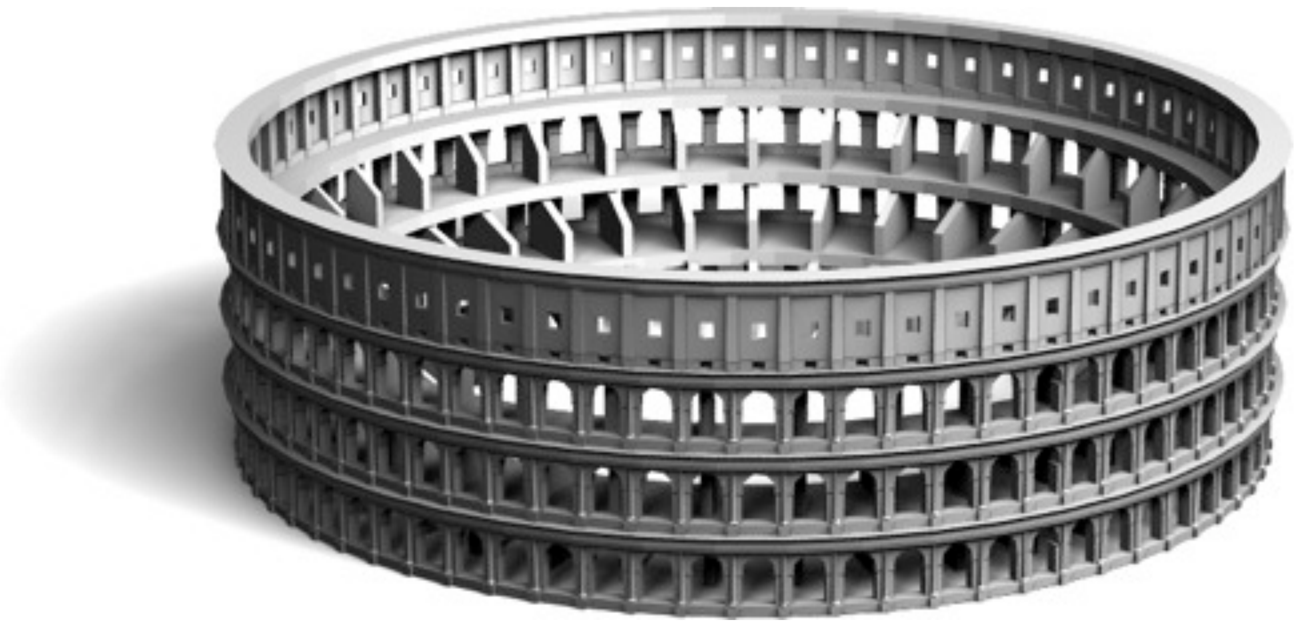

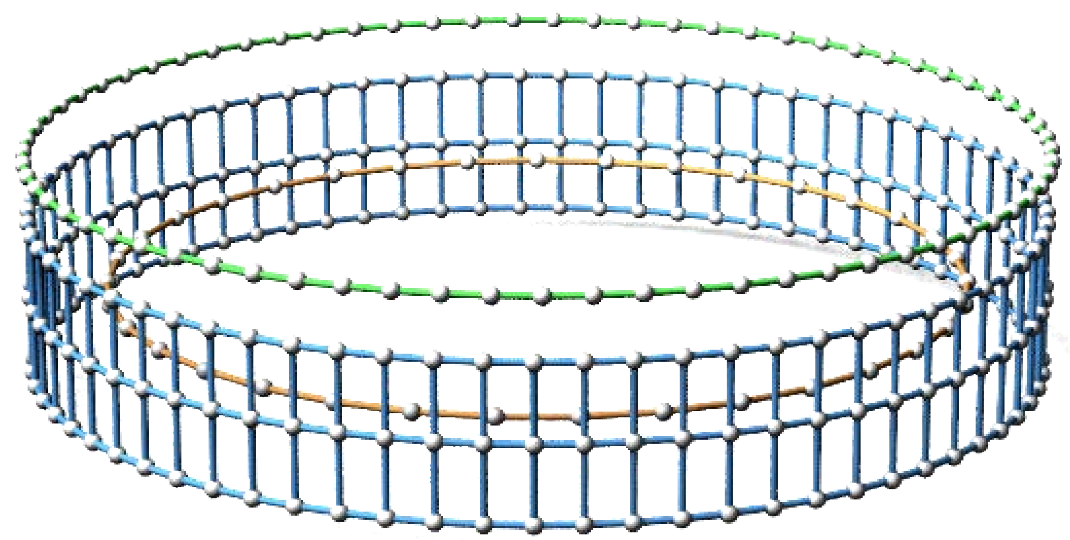

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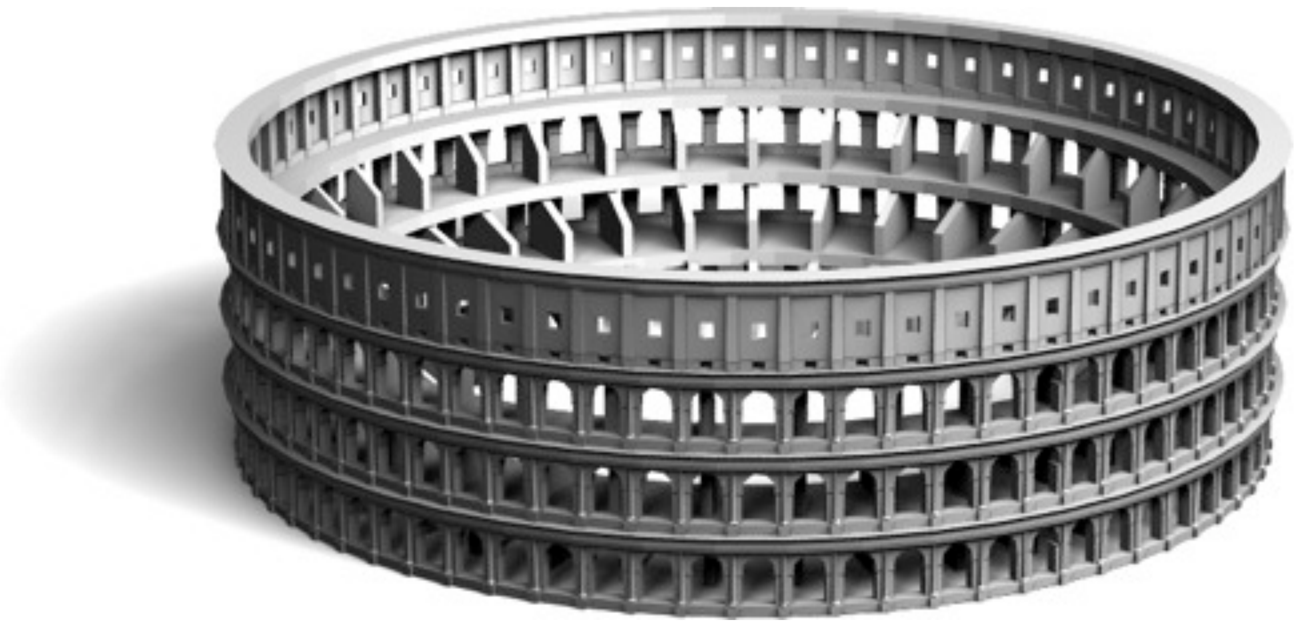
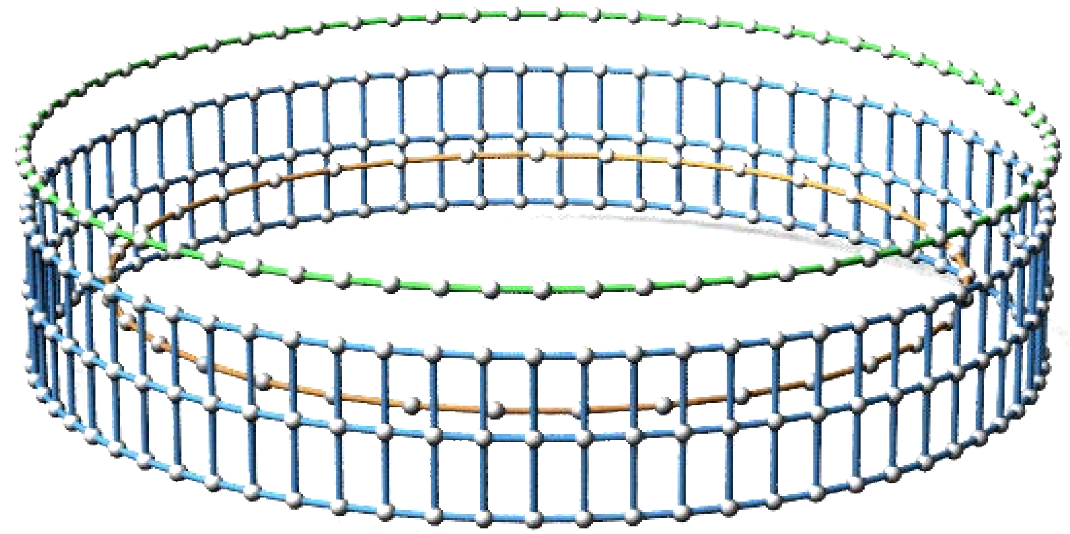




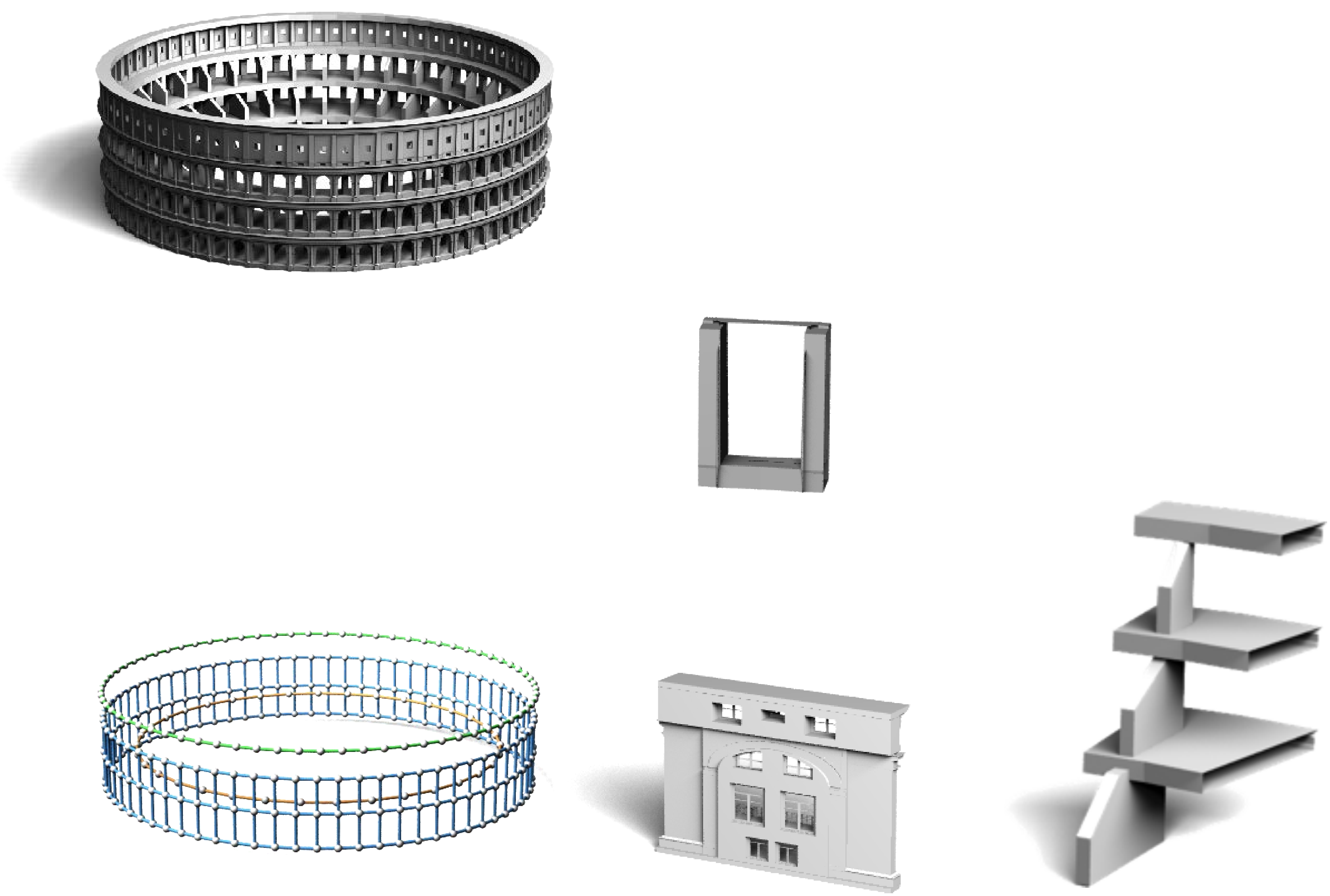
Rot: 72

Rot x Trans: 72 x 3

Rot: 35

[Siggraph 2008]

# ‘Find and Replace’



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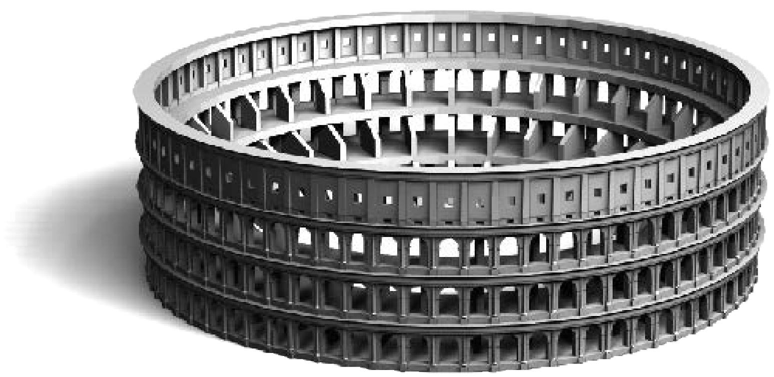

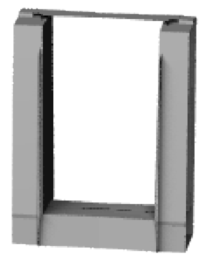

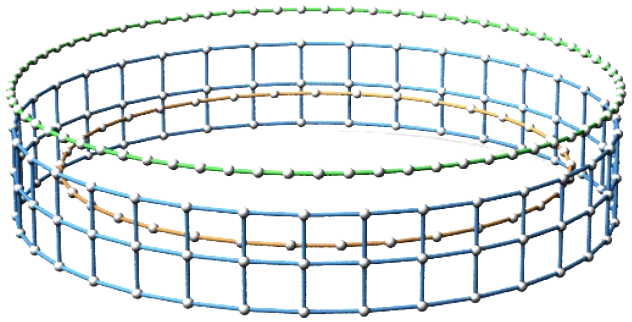

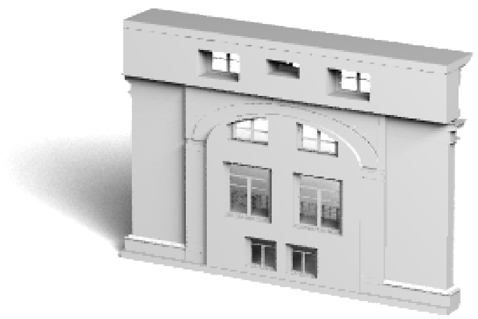

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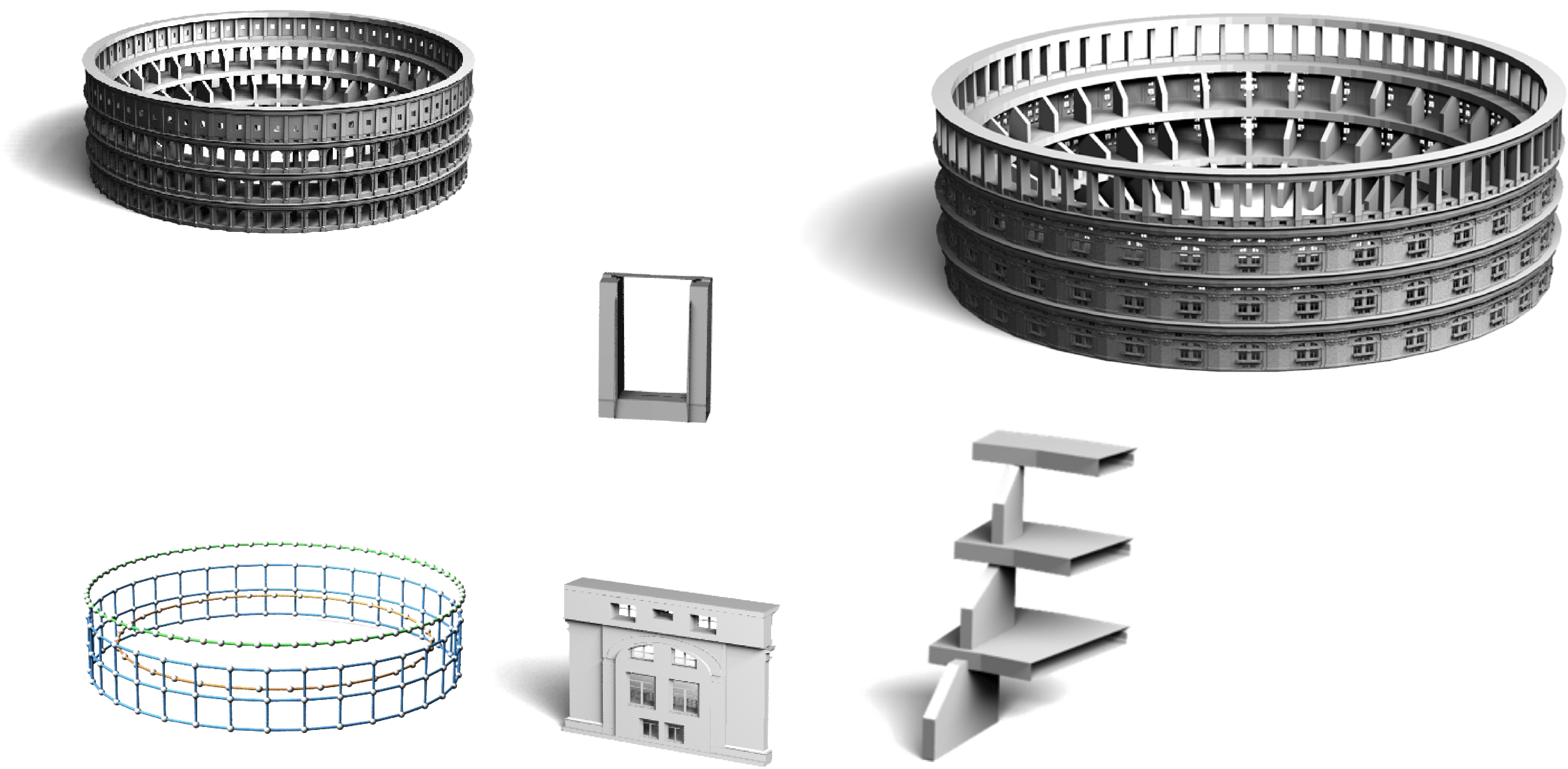
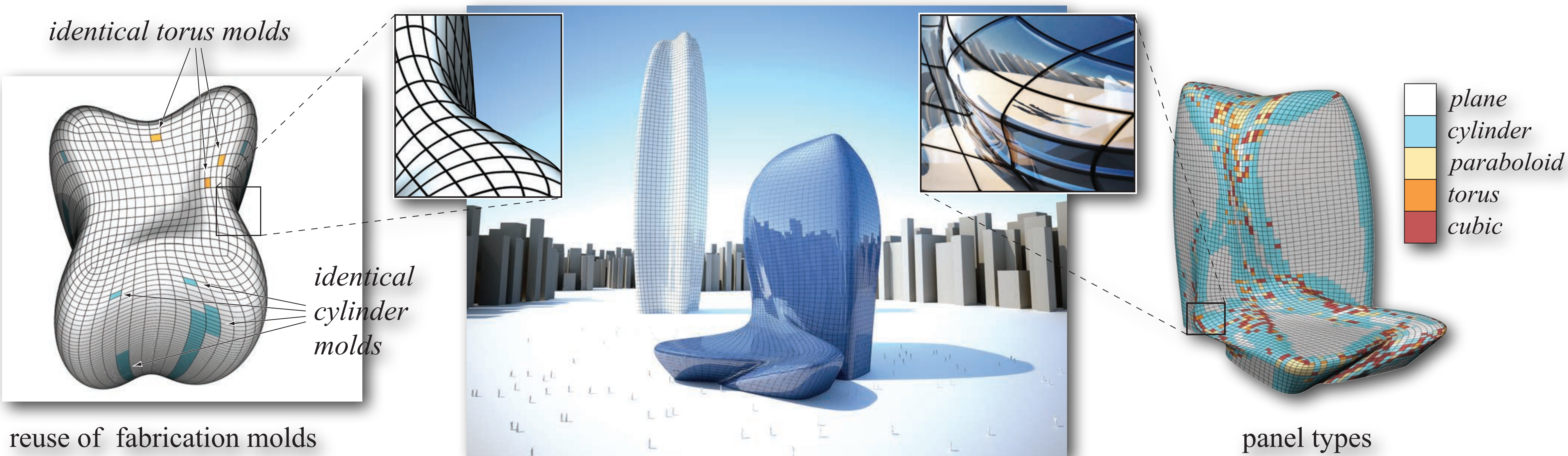


# Enhancing Repetitions




[Siggraph 2010]

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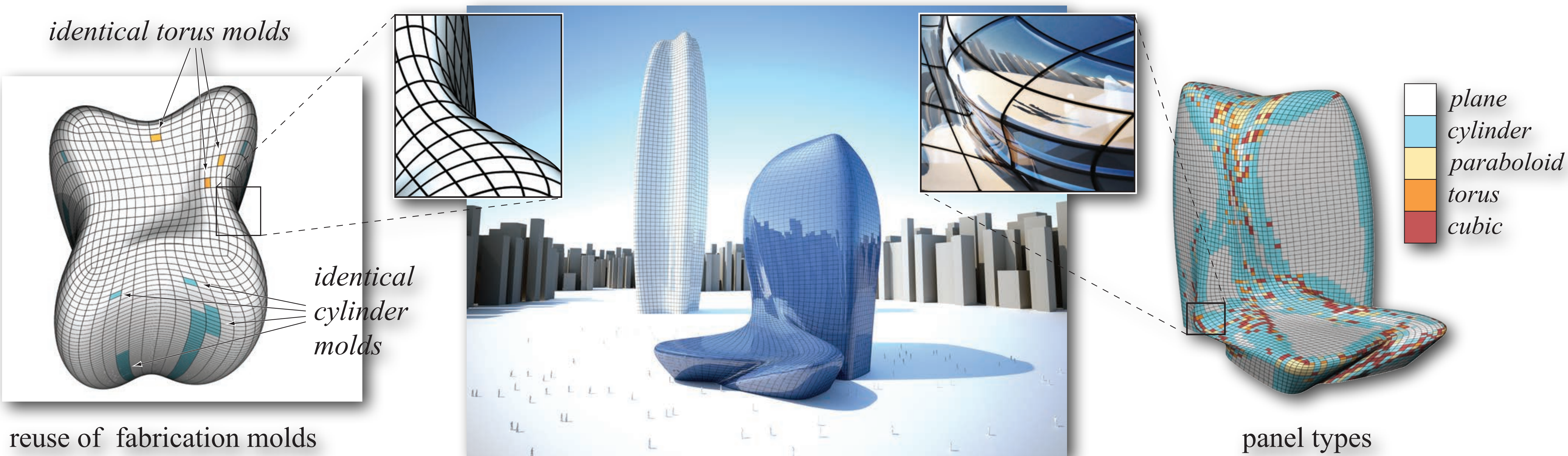


typical cost saving ~ 70-80%

[Siggraph 2010]

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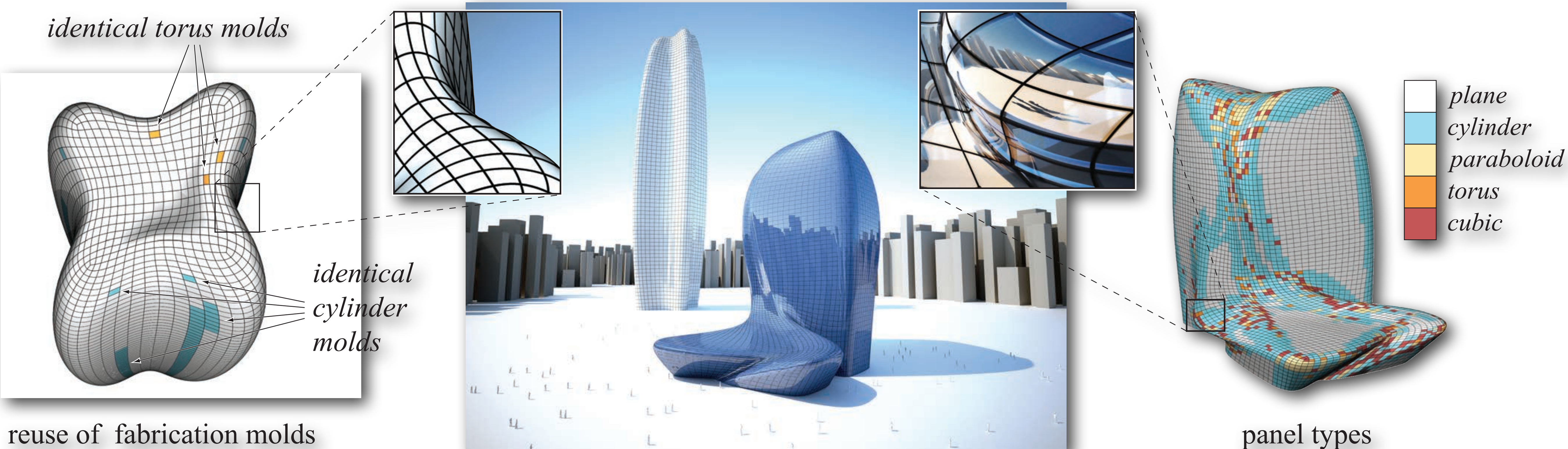


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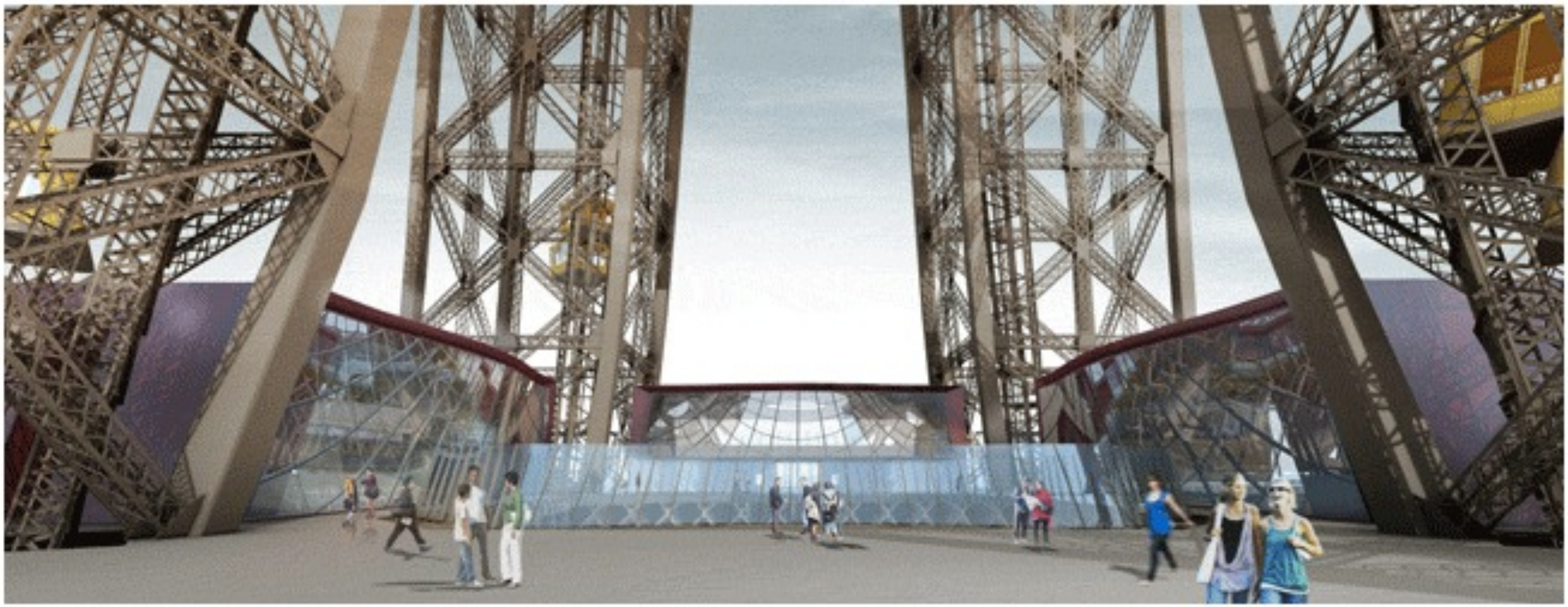
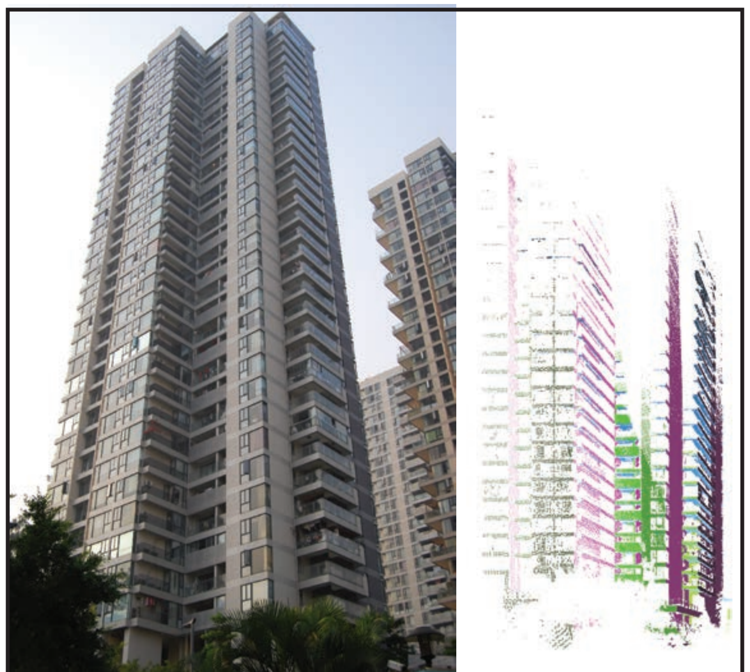
# Fabrication-aware Design



[http://www.eiffel-tower.com/news/top-story/199.html]

# Scalable Acquisition



input image    LiDAR scan

input data

[ICCV 2011]

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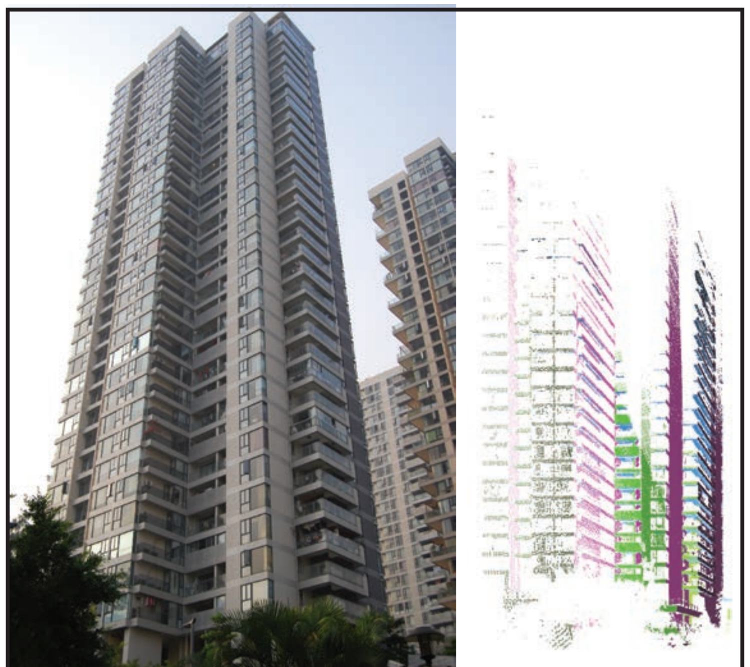

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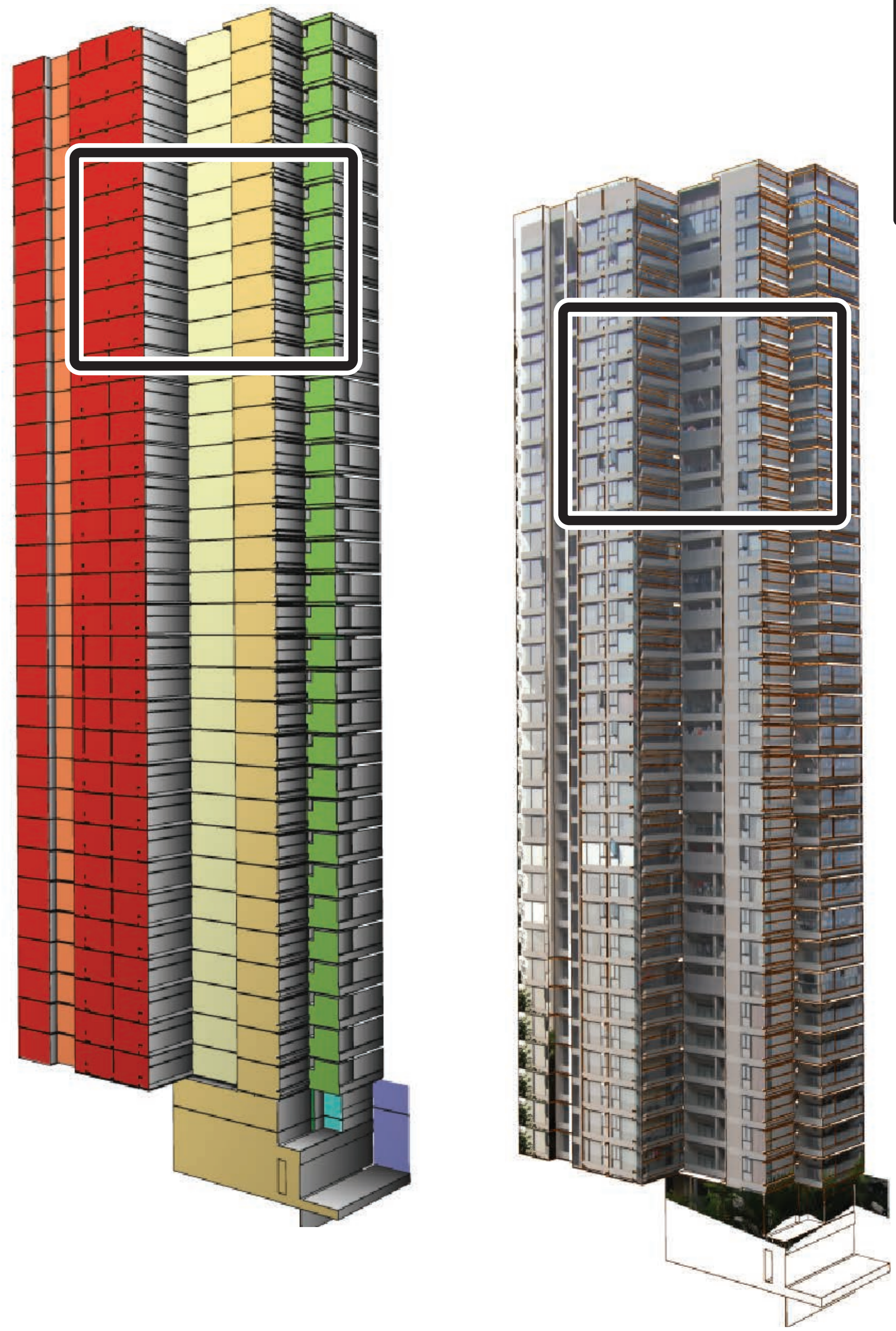
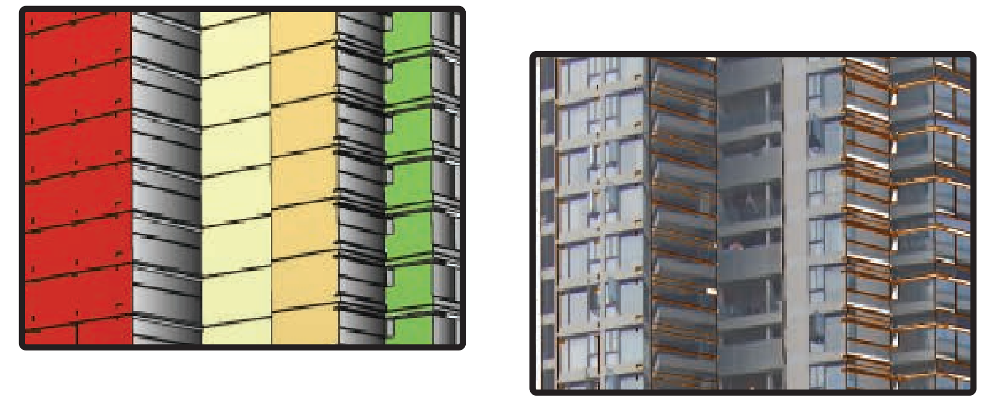


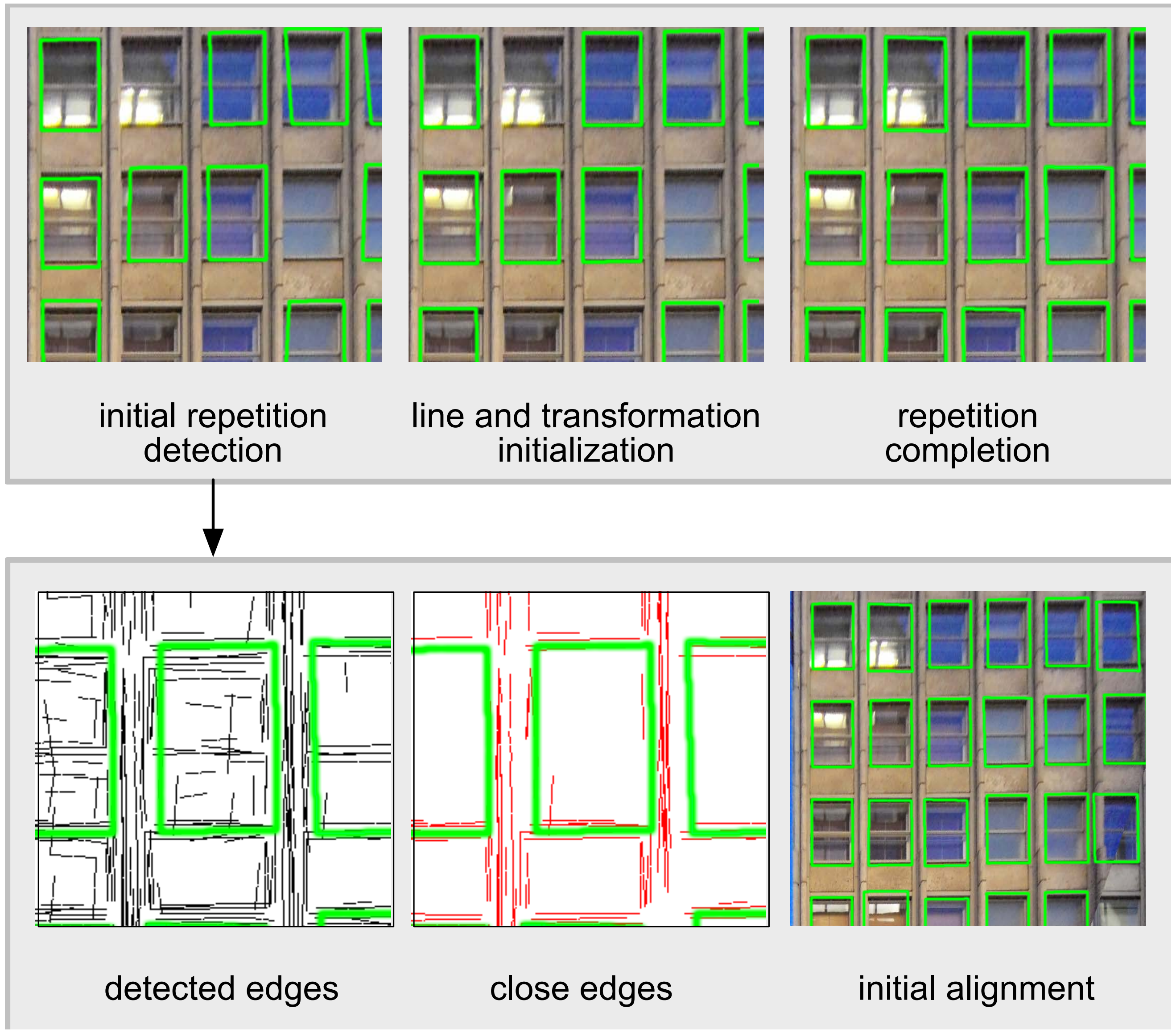


enhanced geometry
and texture

[ICCV 2011]

# Reconstructing From Images

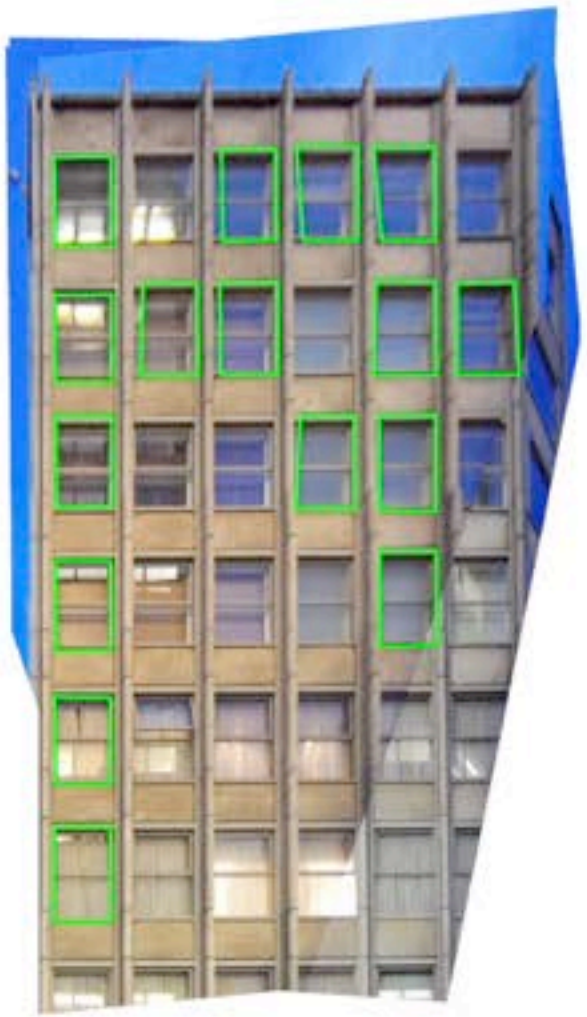
Input: Collection of images of a building facade (with repetitions)




[Eurographics 2012]

# Factored Facades

repetition detection & optimization

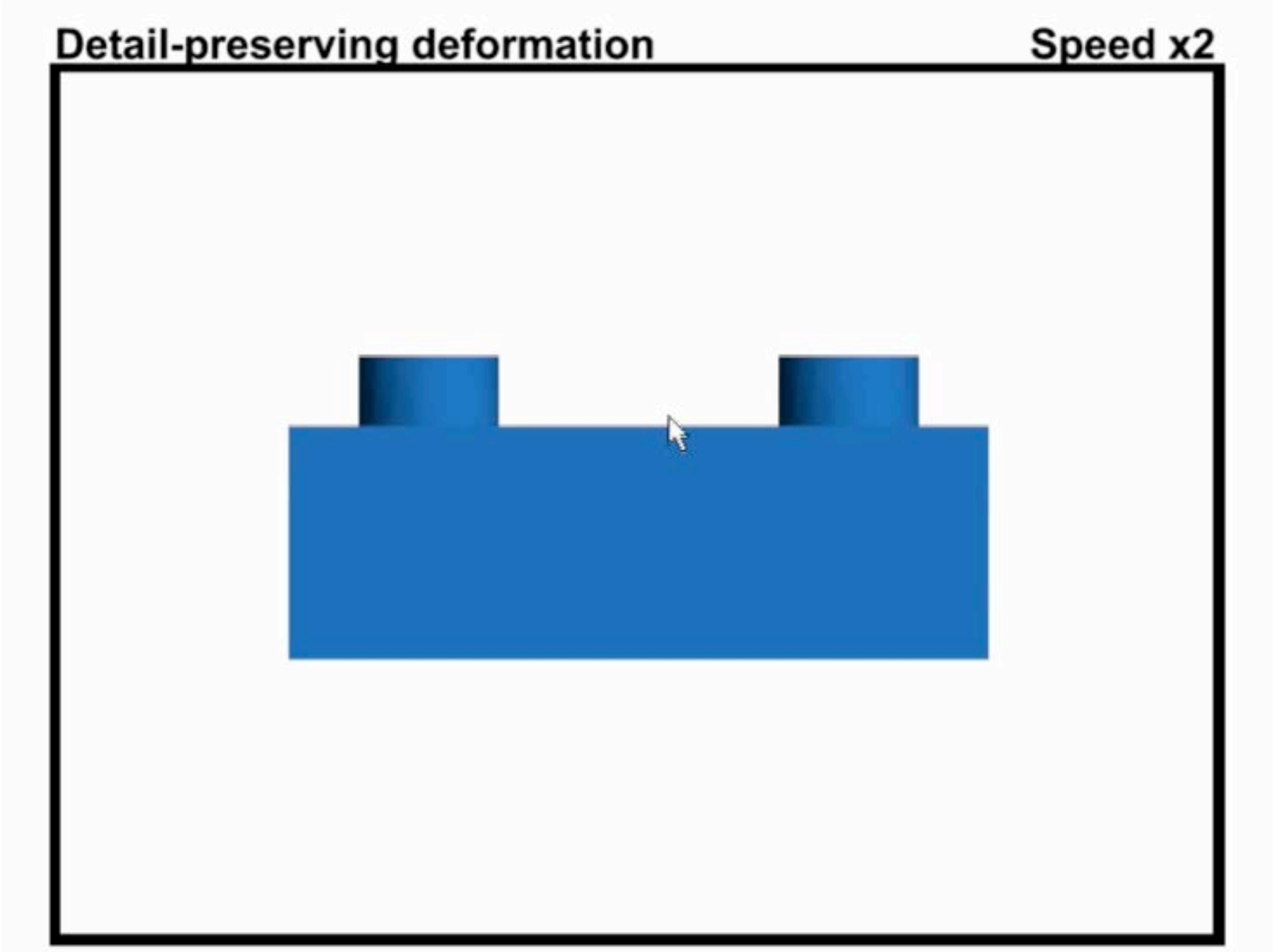


line and transformation initialization

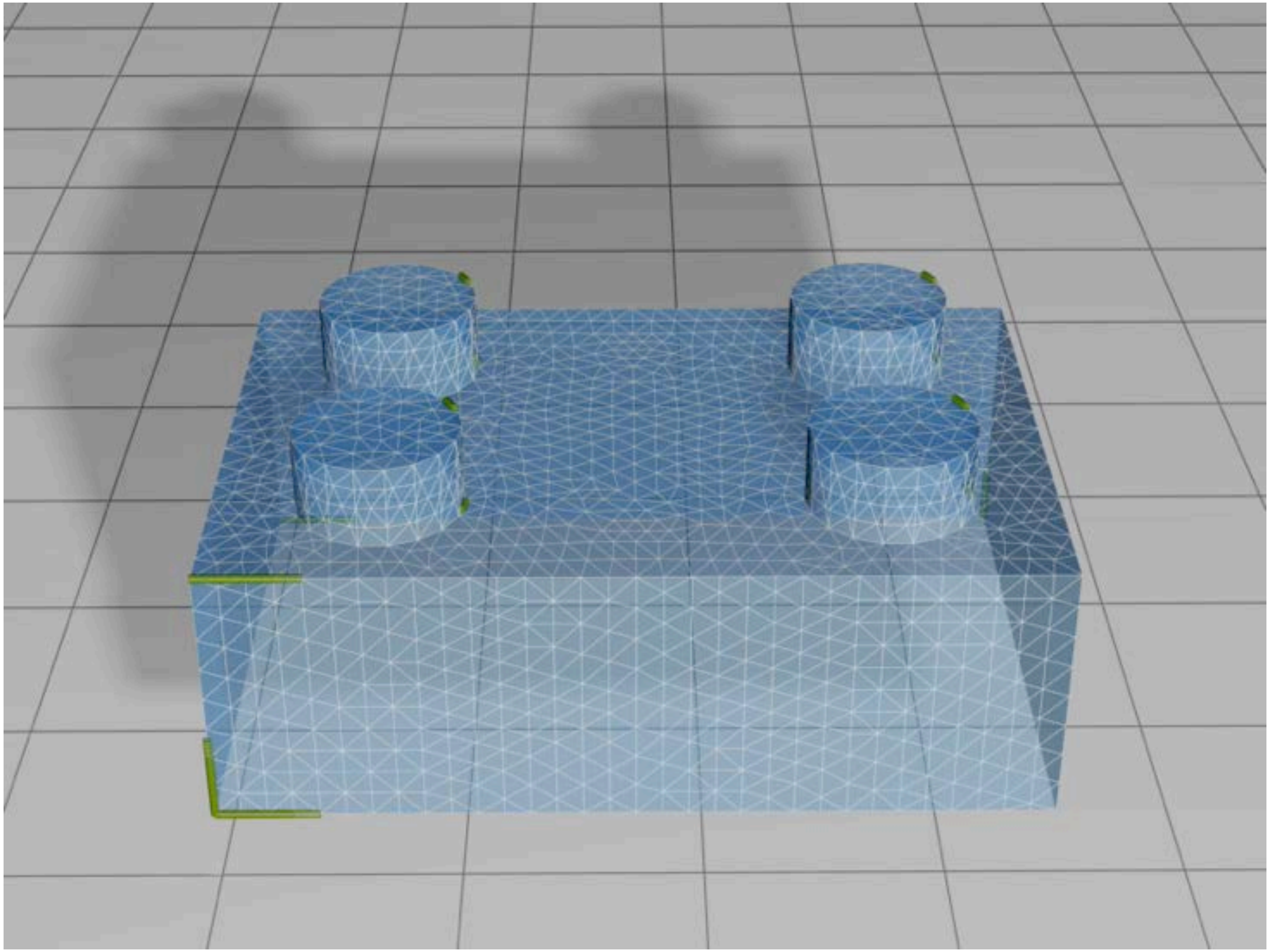
# Smart Interactions

# Relations across Features

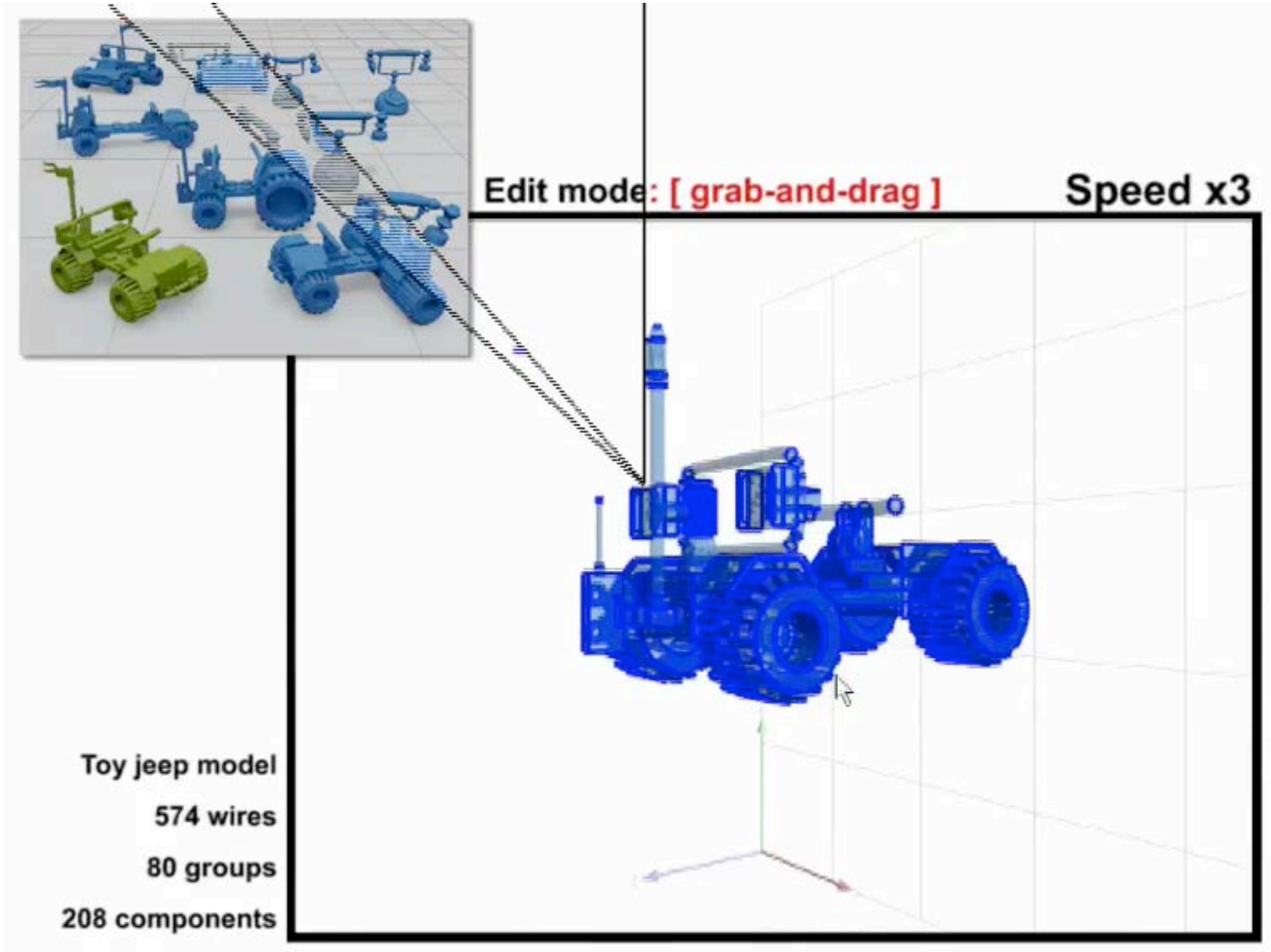
[Siggraph 2009]



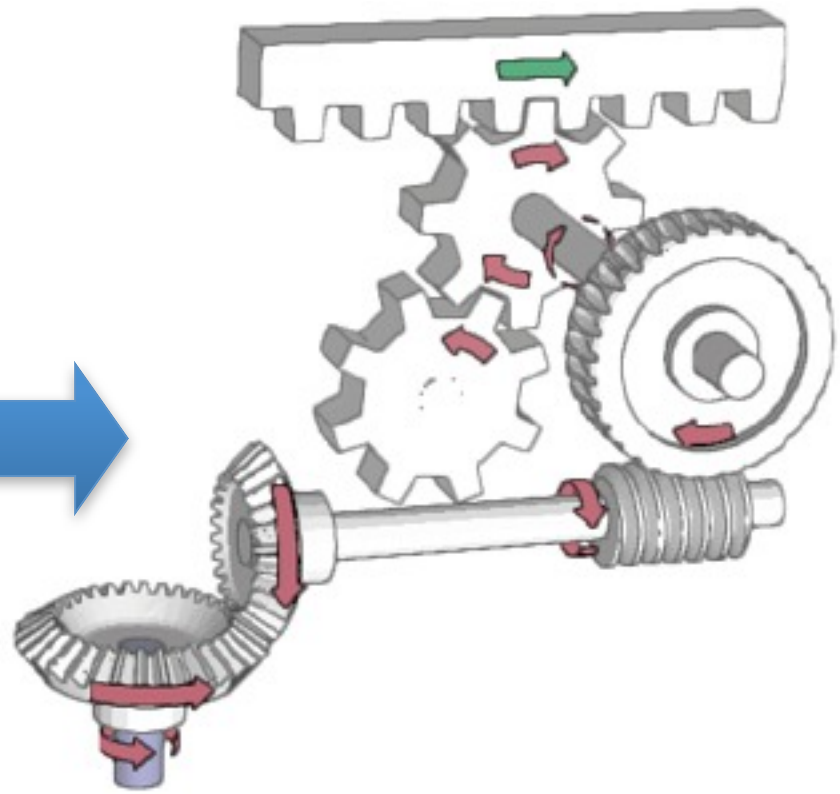
# Algorithm



# Sample Edit Session



# Geometry to Motion



**Input Model**

**How Things Work Visualization**

[Siggraph 2010]

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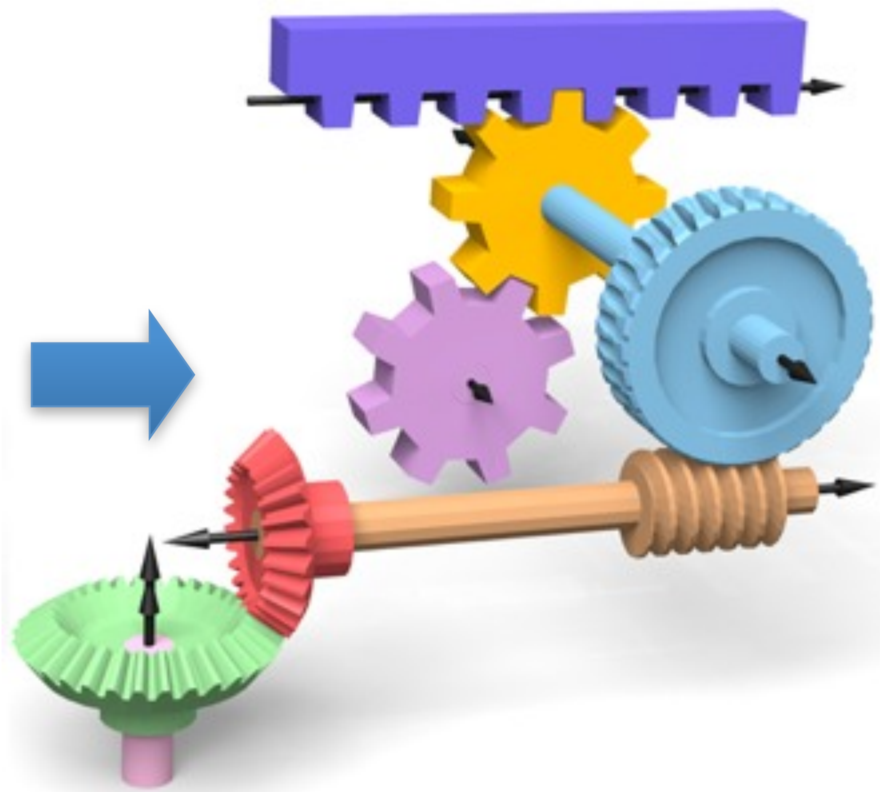

**Input Model**

**Motion and Interaction Analysis**

[Siggraph 2010]

# Geometry to Motion

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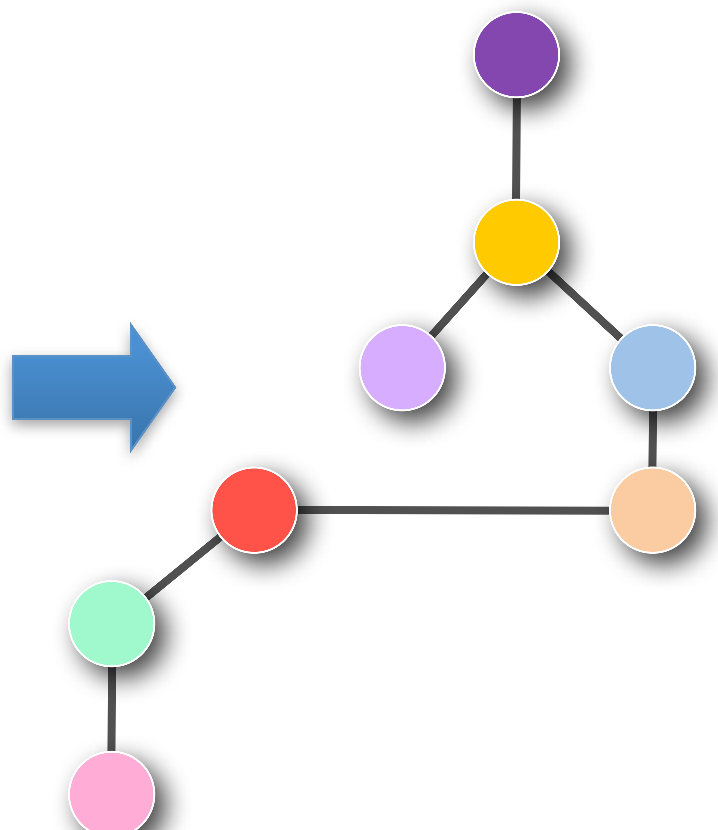

**Motion and Interaction Analysis**

[Siggraph 2010]

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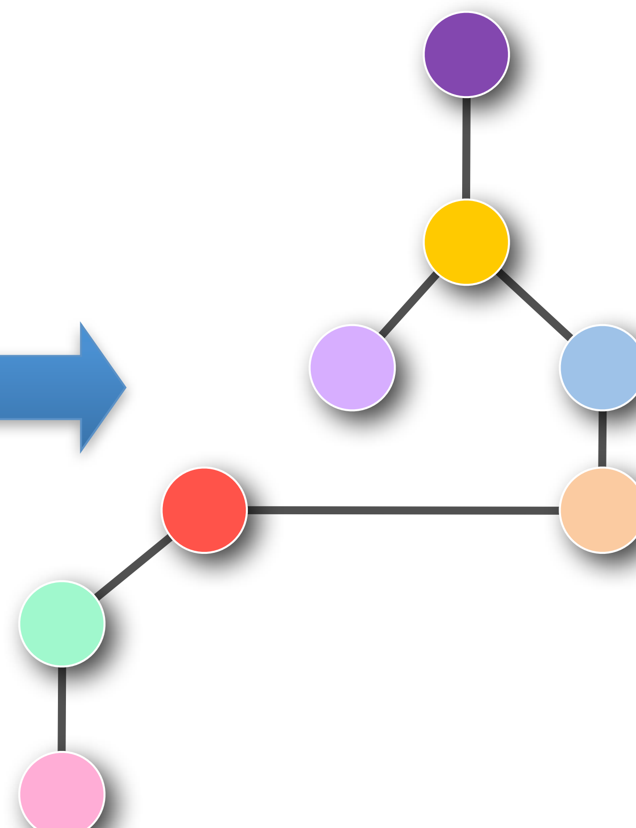
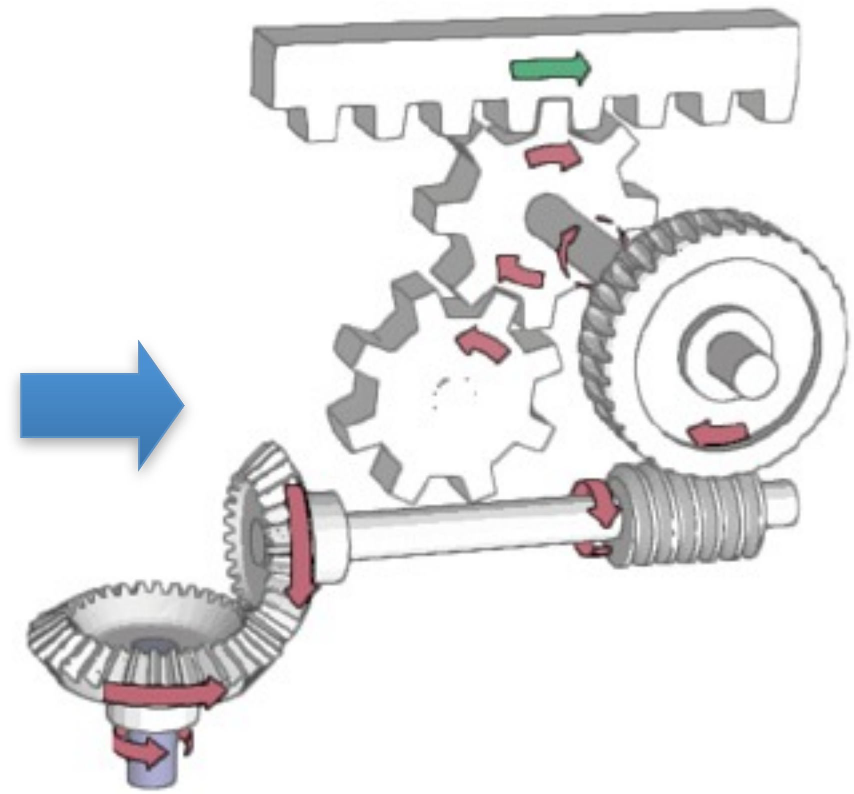


Motion and Interaction Analysis



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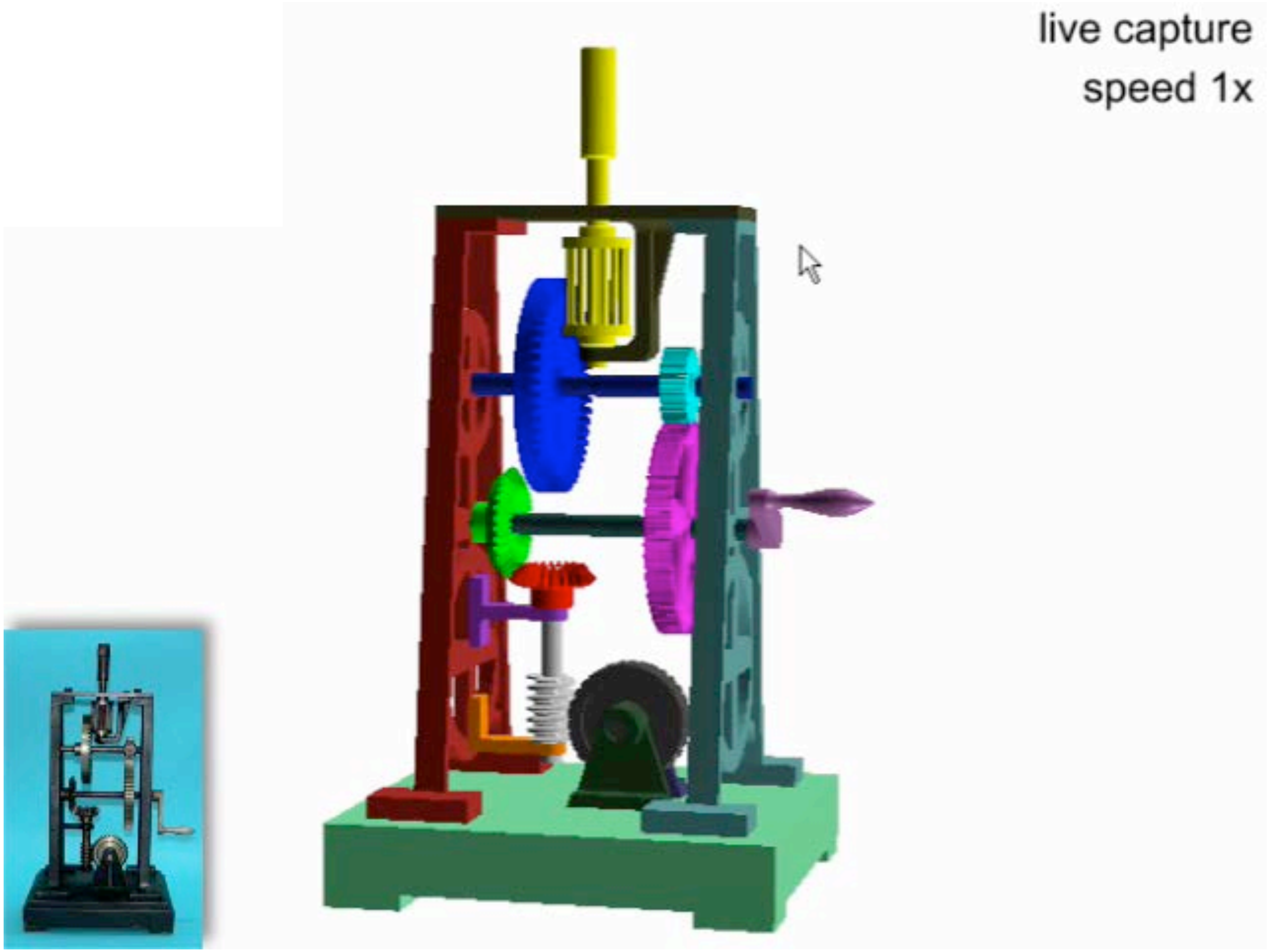

why does this work?

# Constrained Meshes

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- Goal:
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  - navigate only the *good* ones

# Constrained Shape Space

$$\mathbf{x} = (v_1, \ldots, v_n) \in \mathbb{R}^D$$

[SiggraphA 2011]

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- mesh $\longrightarrow$ point
- combinatorics remain fixed
- starting mesh $\mathbf{x}_0$ satisfies (nonlinear) constraints

[SiggraphA 2011]

# Intersection Surface

Each face constraint

$$\Gamma_i := \{\mathbf{x} \in \mathbb{R}^D : E_i(\mathbf{x}) = 0\} \quad \forall \quad i = 1, \ldots, m$$

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$$\mathbf{d} \Rightarrow \mathbf{x}_0 + \mathbf{d}$$

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$$\mathbf{S}(\mathbf{u}) = \mathbf{x}_0 + \sum_{i=1}^{D-m} u_i \mathbf{e}_i + \frac{1}{2} \sum_{j=1}^{m} (\mathbf{u}^T \cdot A_j \cdot \mathbf{u}) \mathbf{n}_j$$

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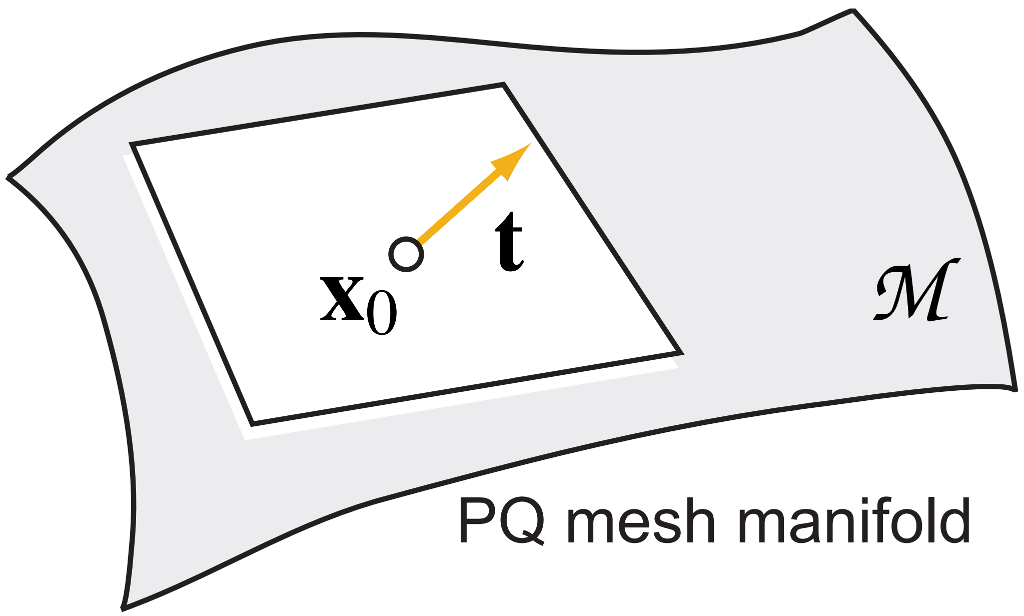
# Walking on the Tangent Space



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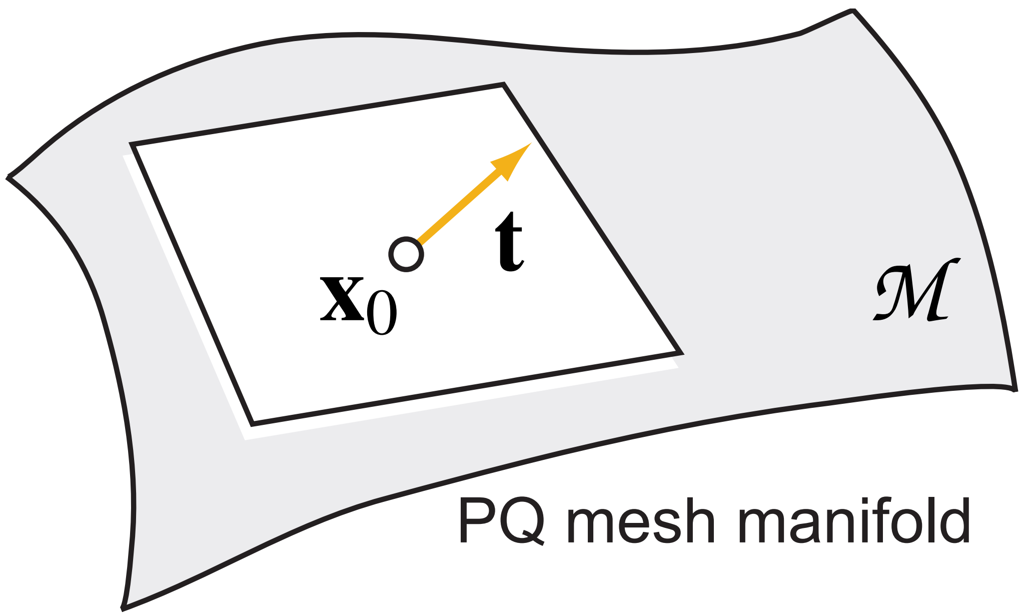


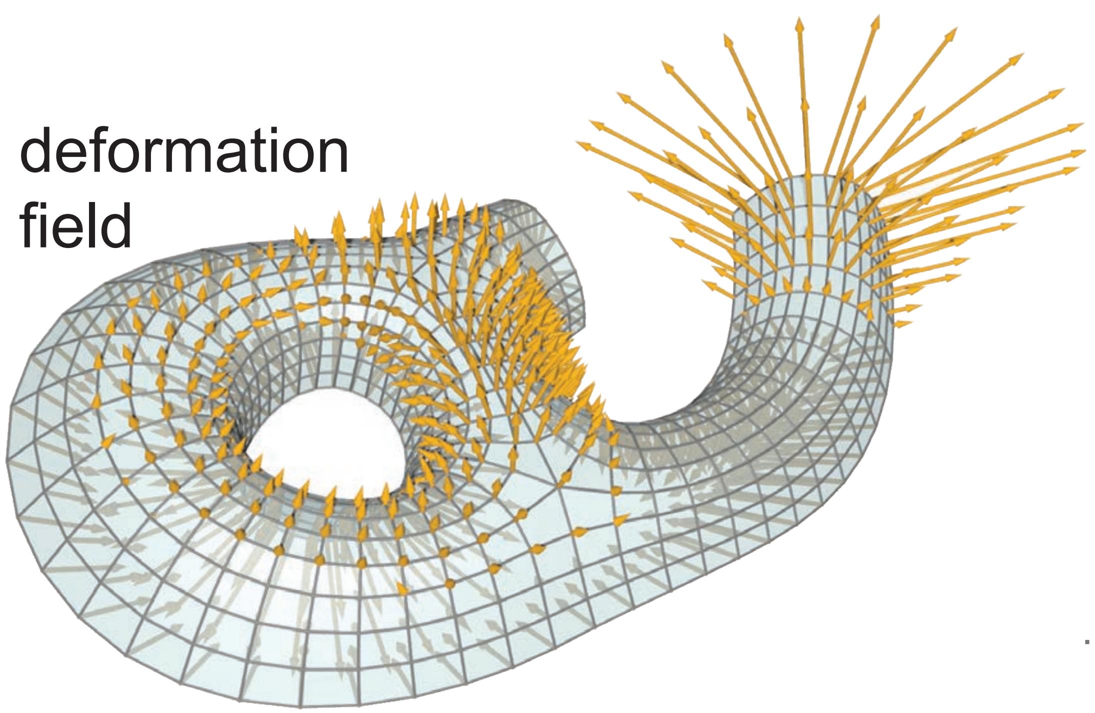

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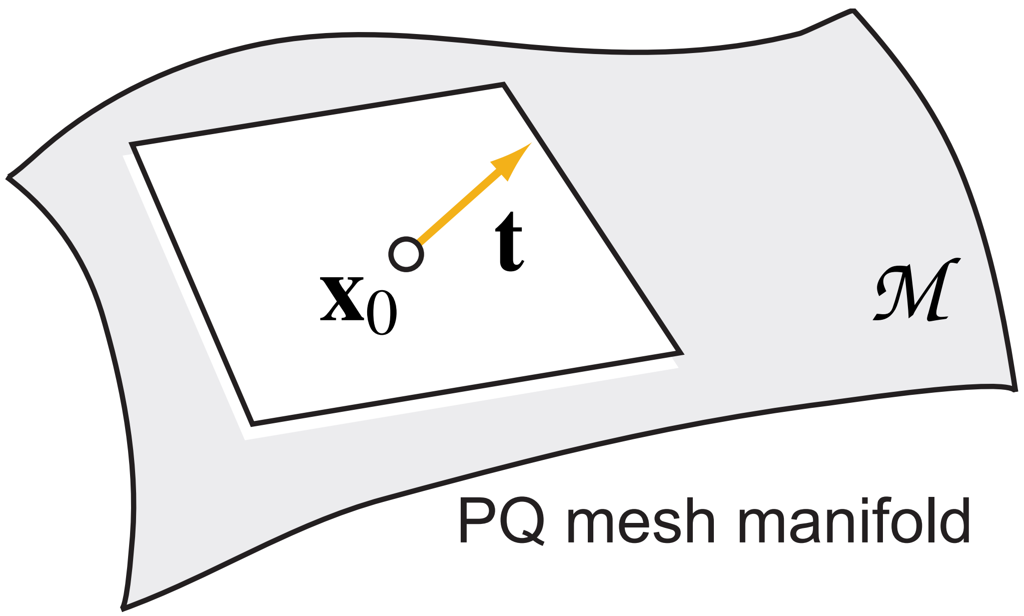


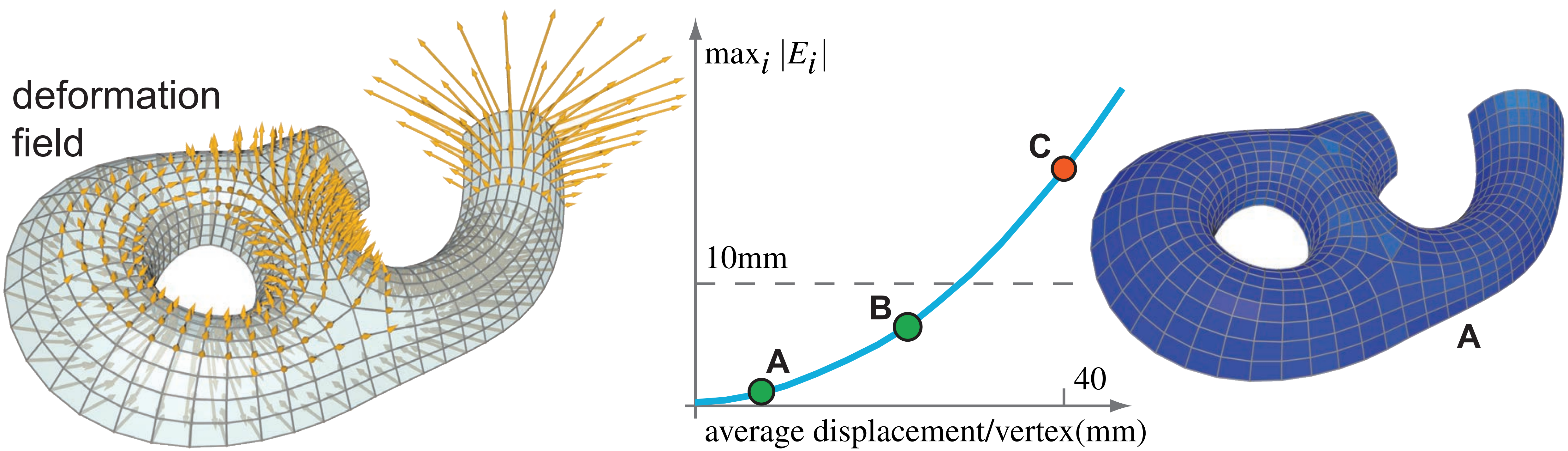

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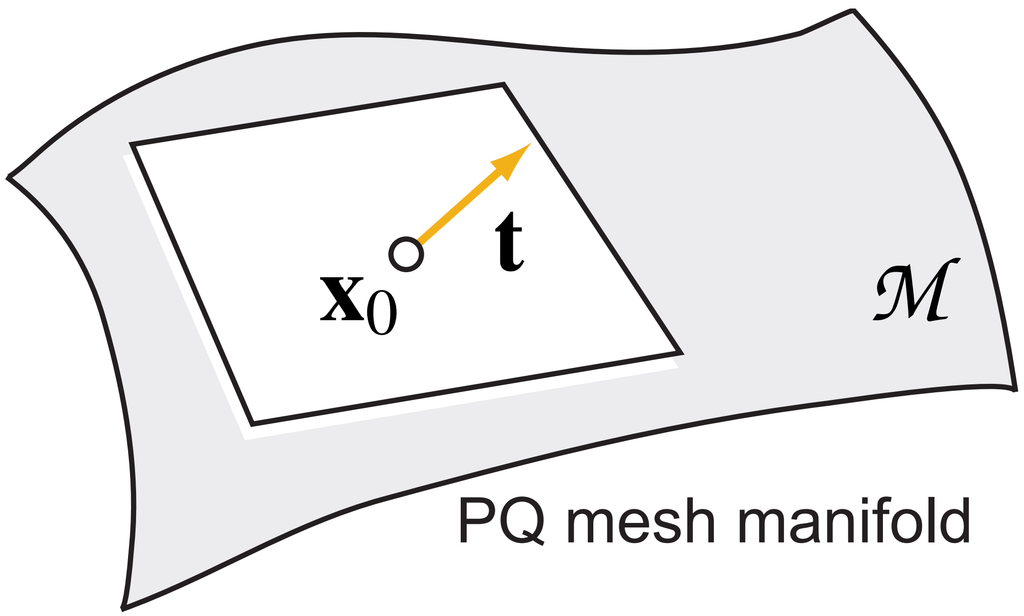


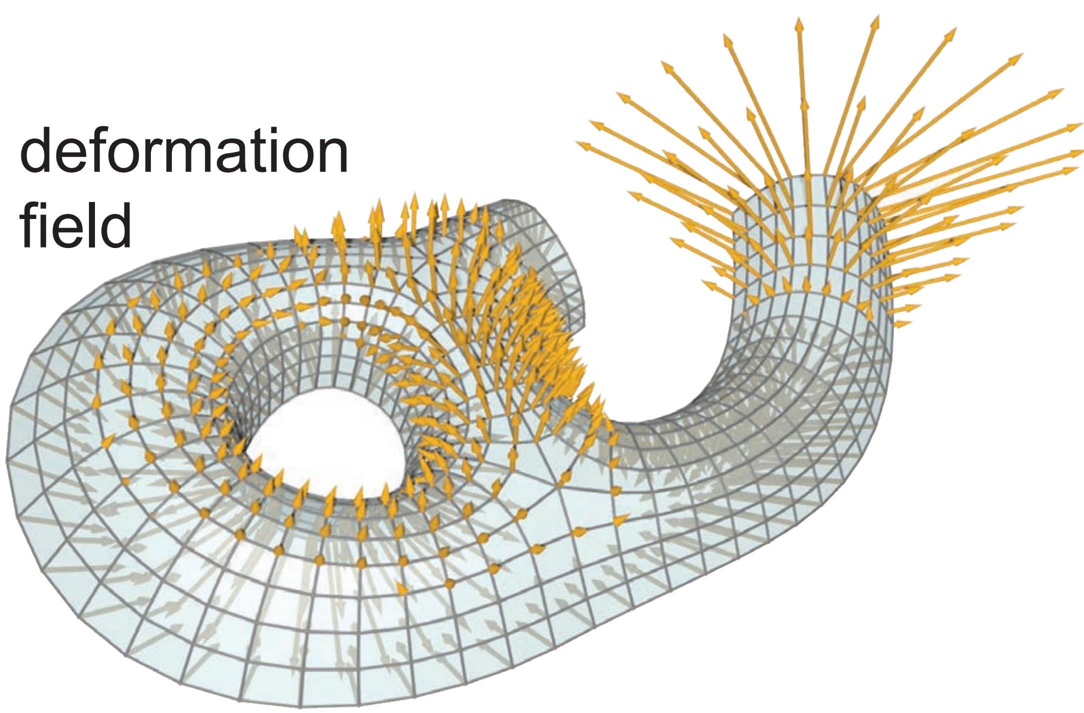


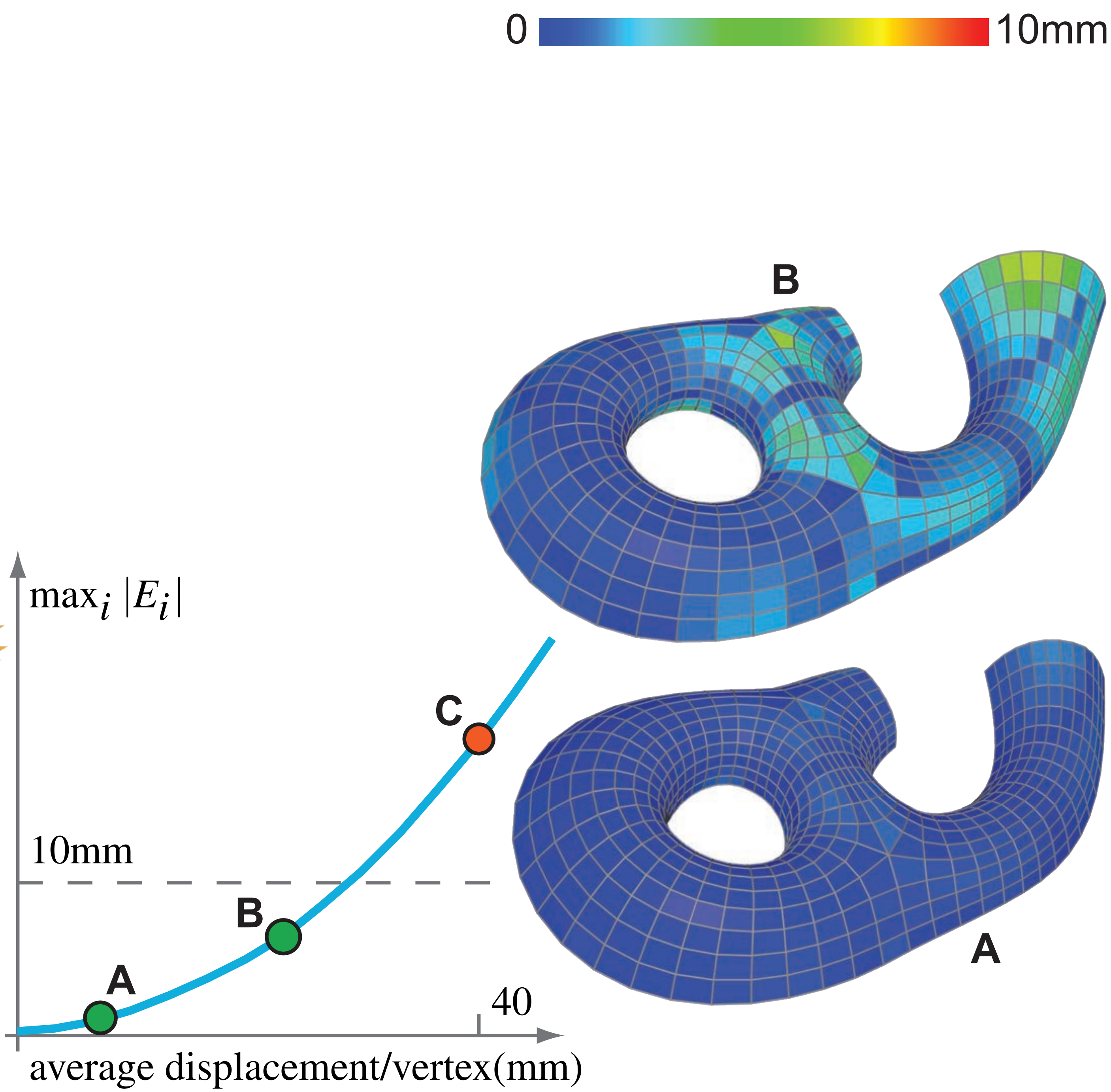

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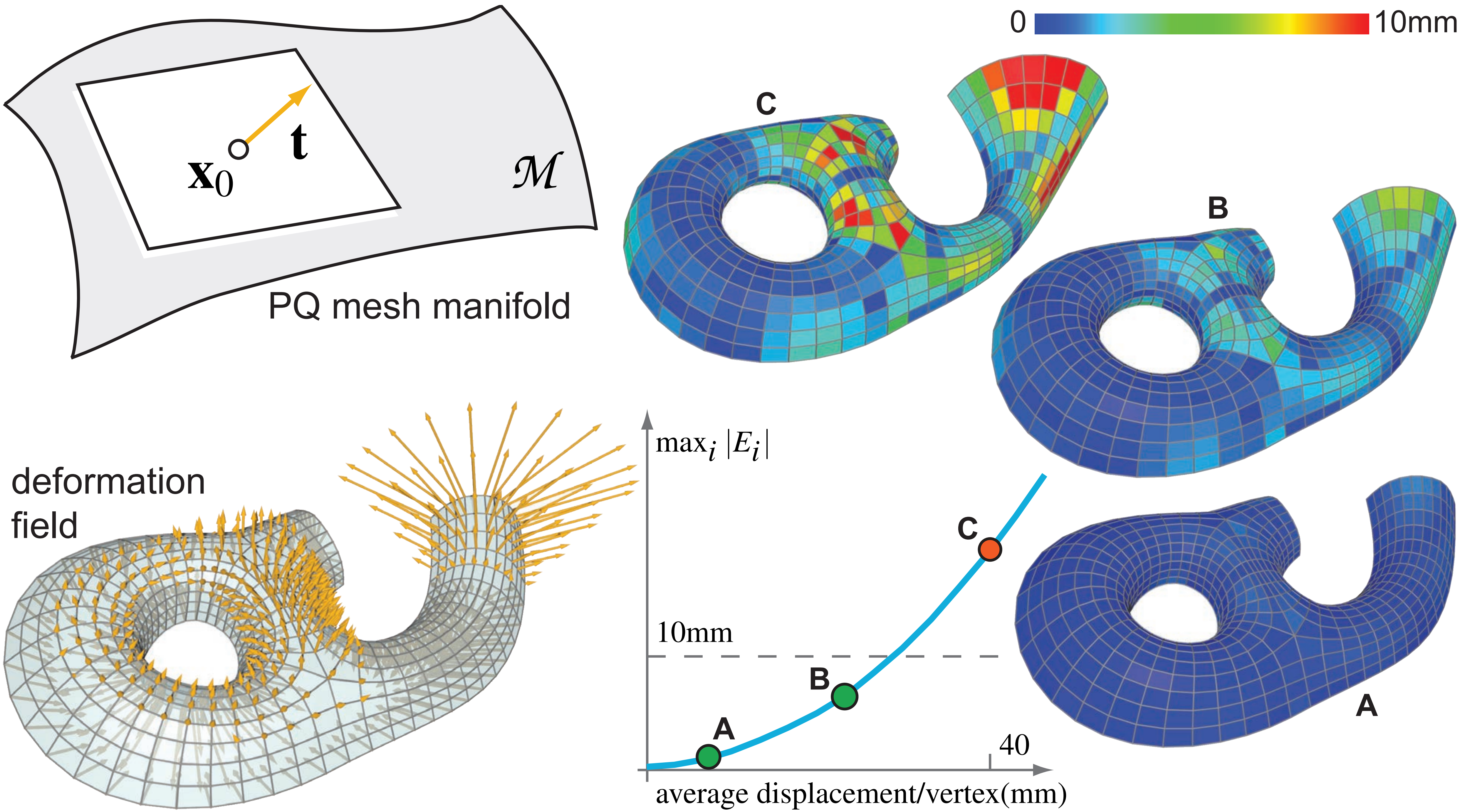


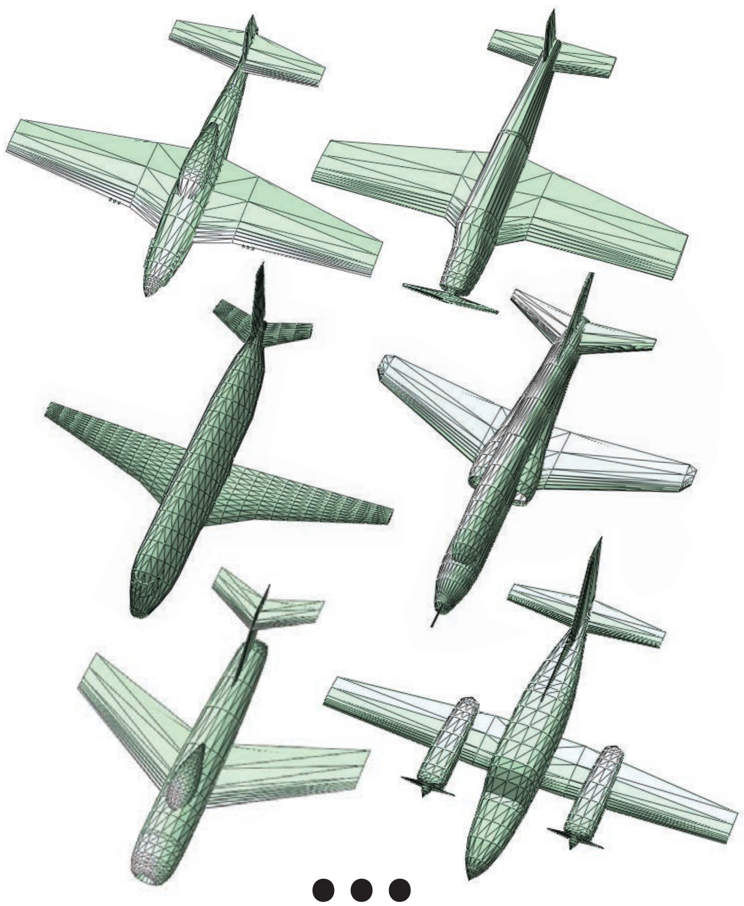
# Design Exploration

Flat Circular Mesh Exploration

# Beyond Model-Pairs

# Exploring Data Collections

low-level geometry → (structure+element) + variations



(a) input collection

[Siggraph 2011]

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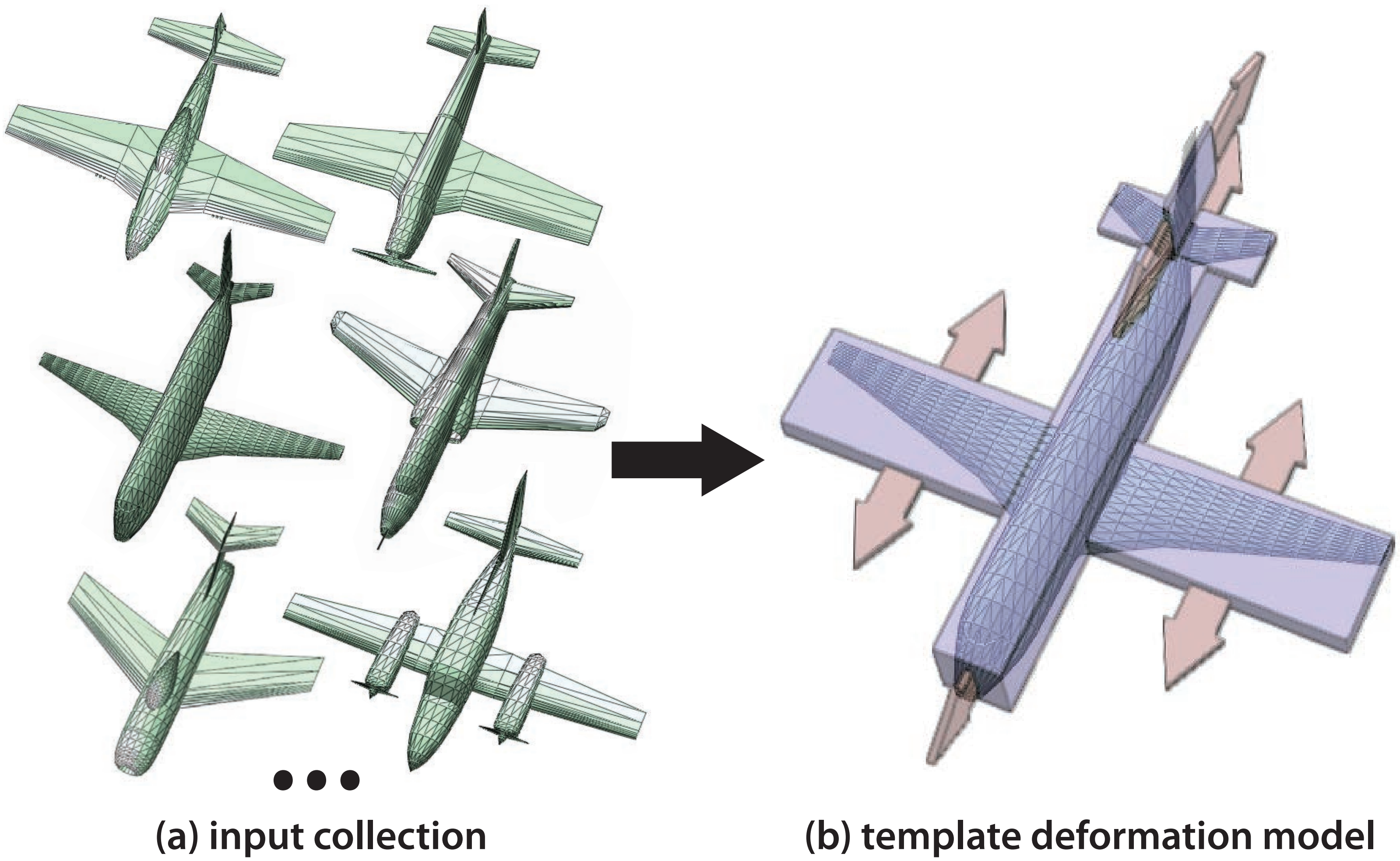
low-level geometry → (structure+element) + variations




[Siggraph 2011]

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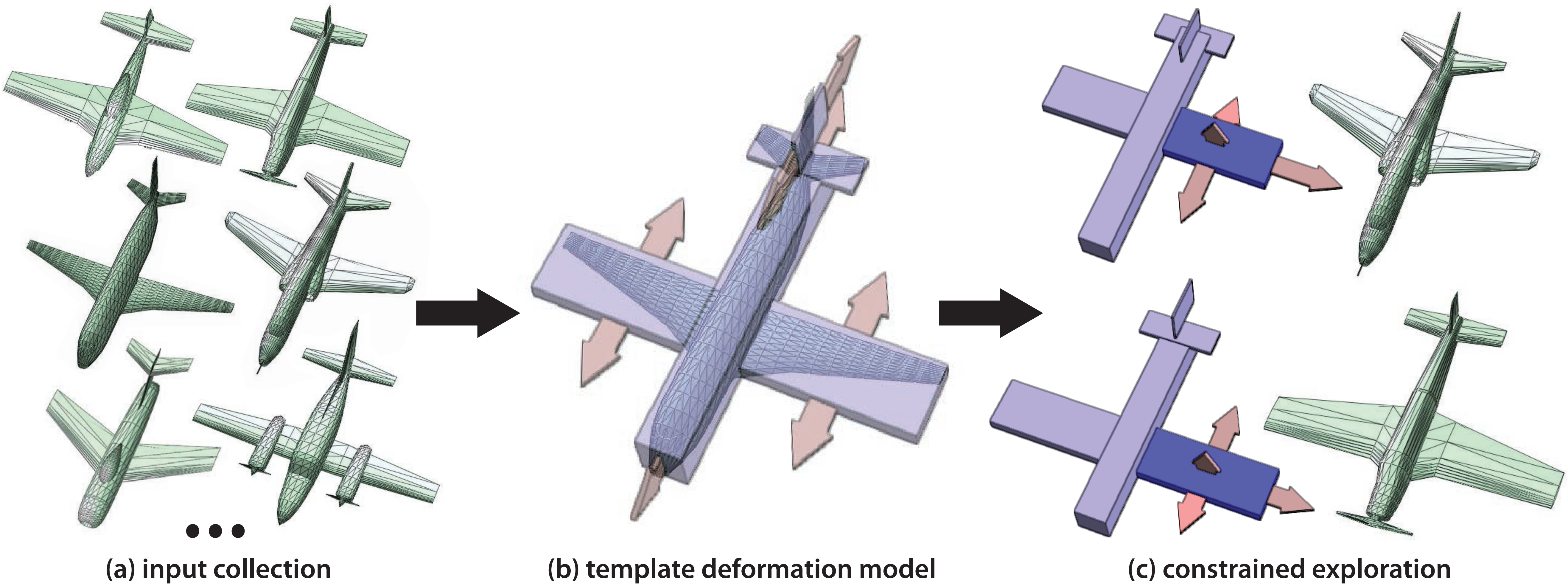


[Siggraph 2011]

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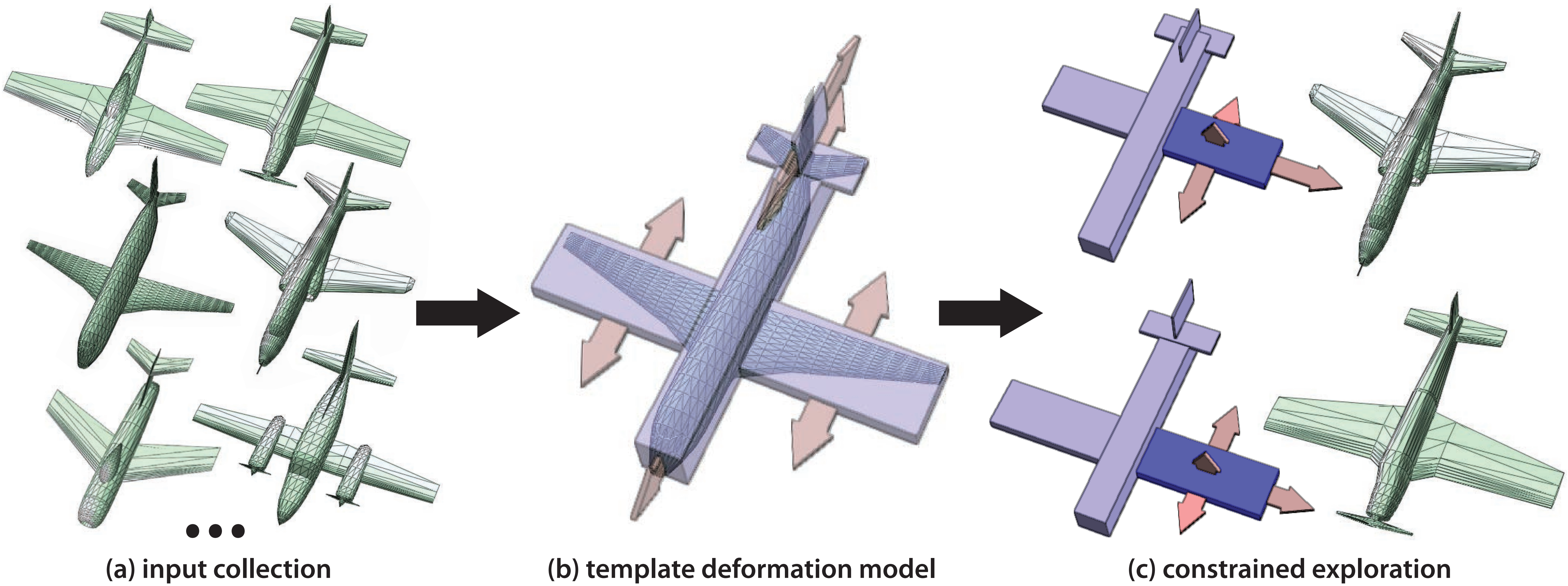
low-level geometry → (structure+element) + variations




***without correspondences***

[Siggraph 2011]

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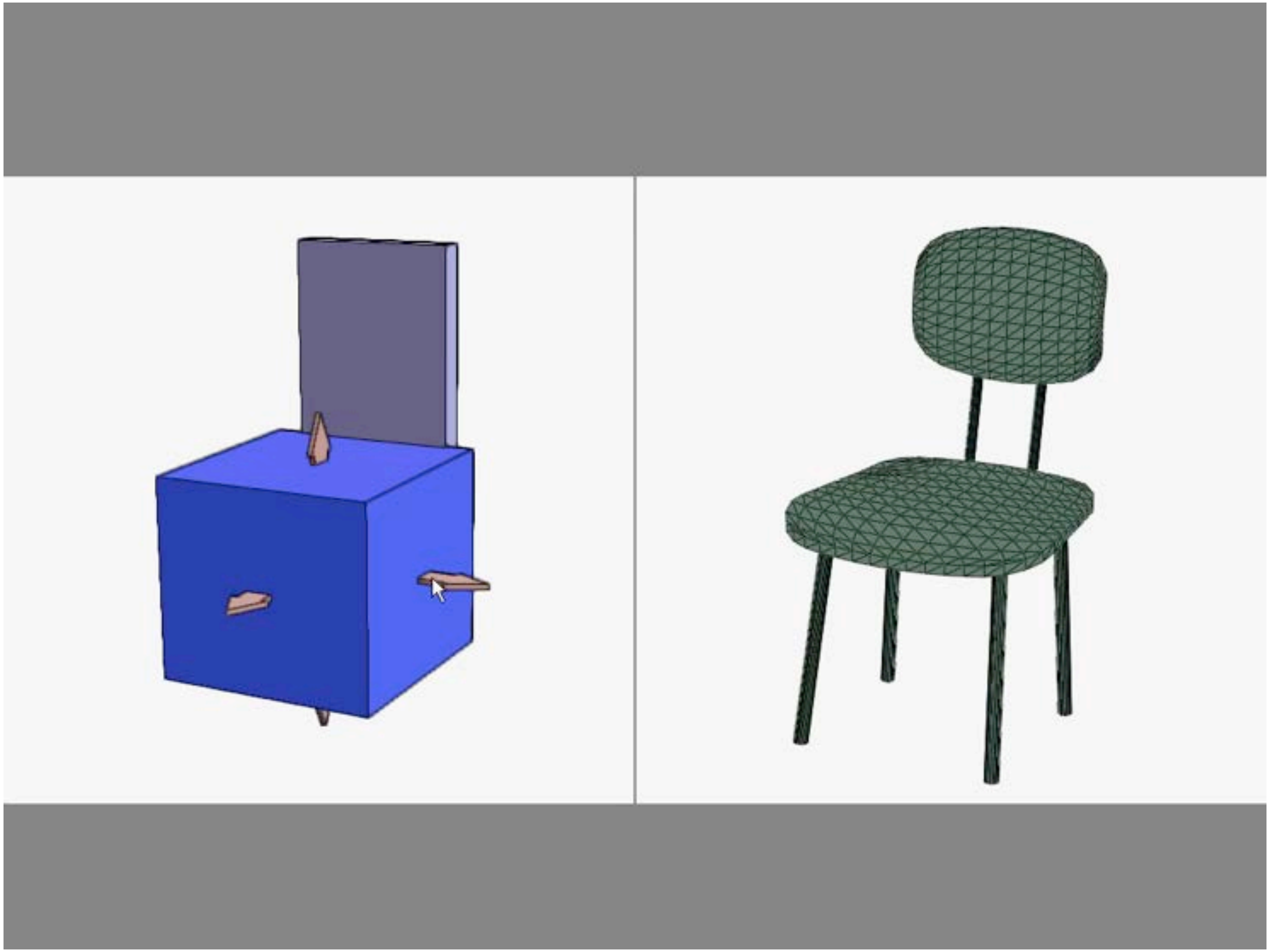
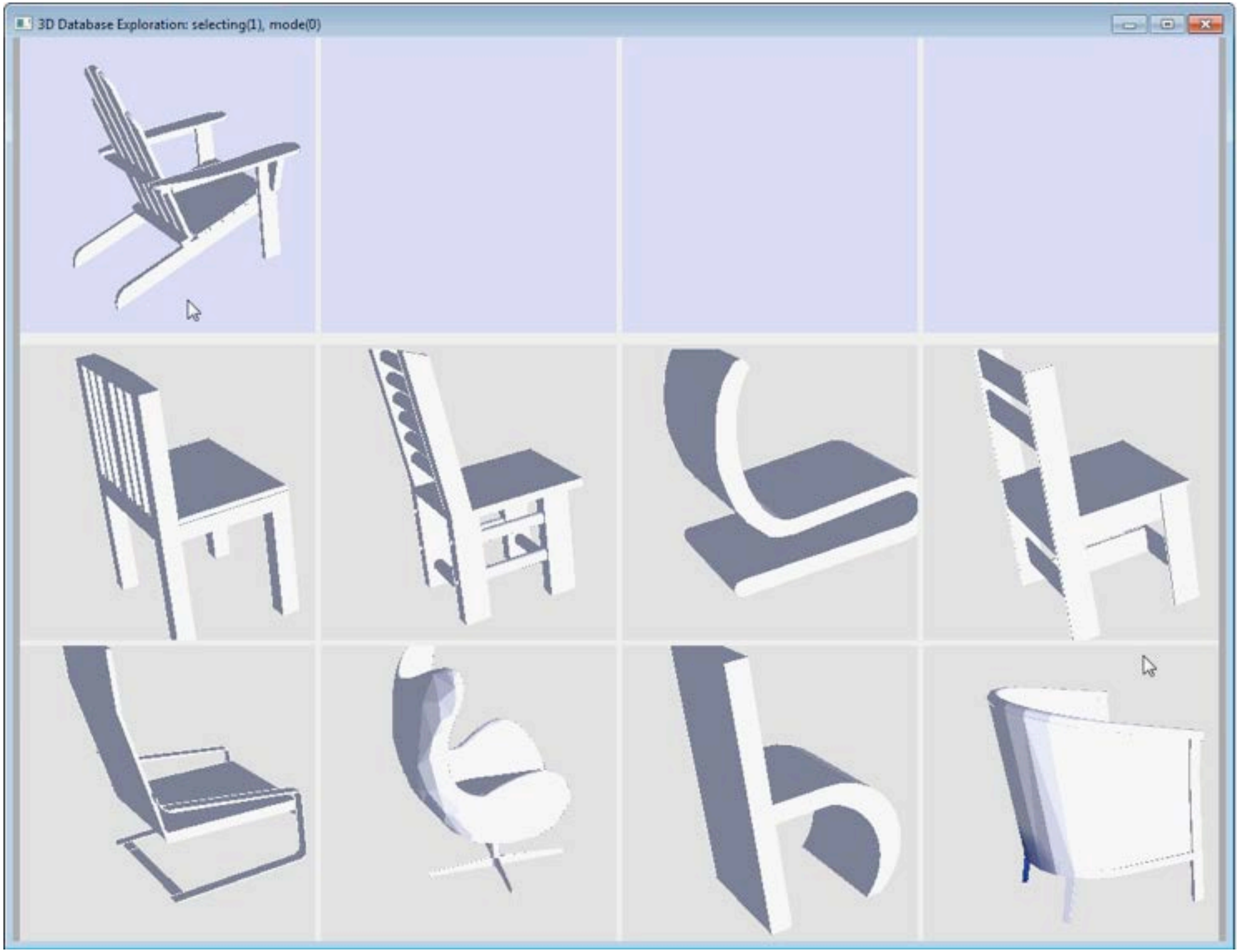
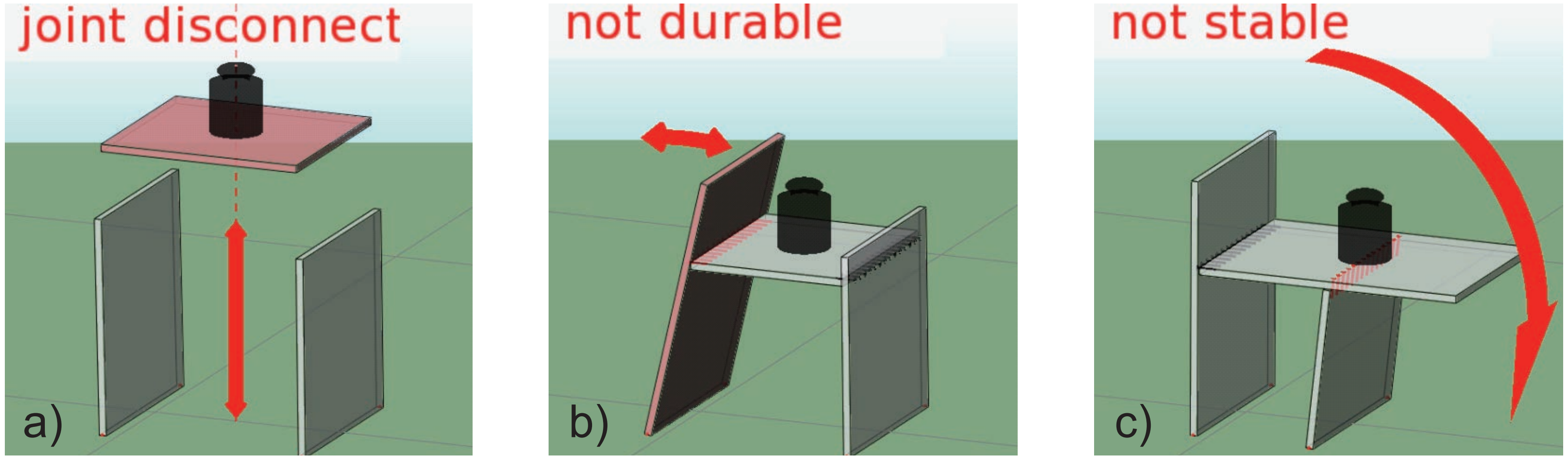


# With Fuzzy Correspondence



# When Force Drives Form




[Siggraph 2012]

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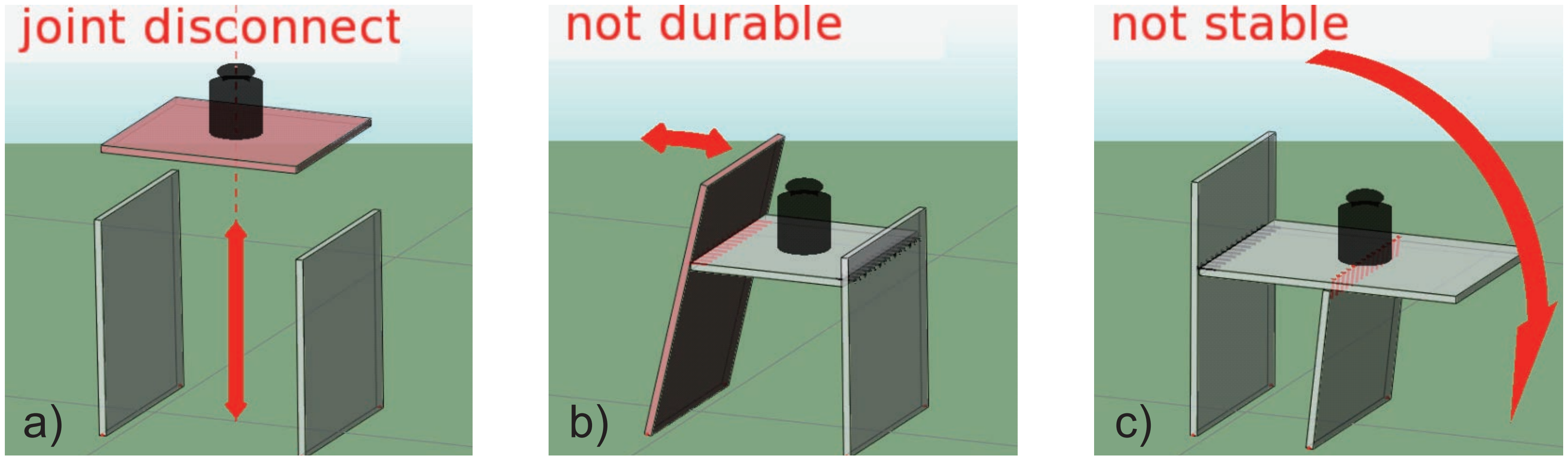


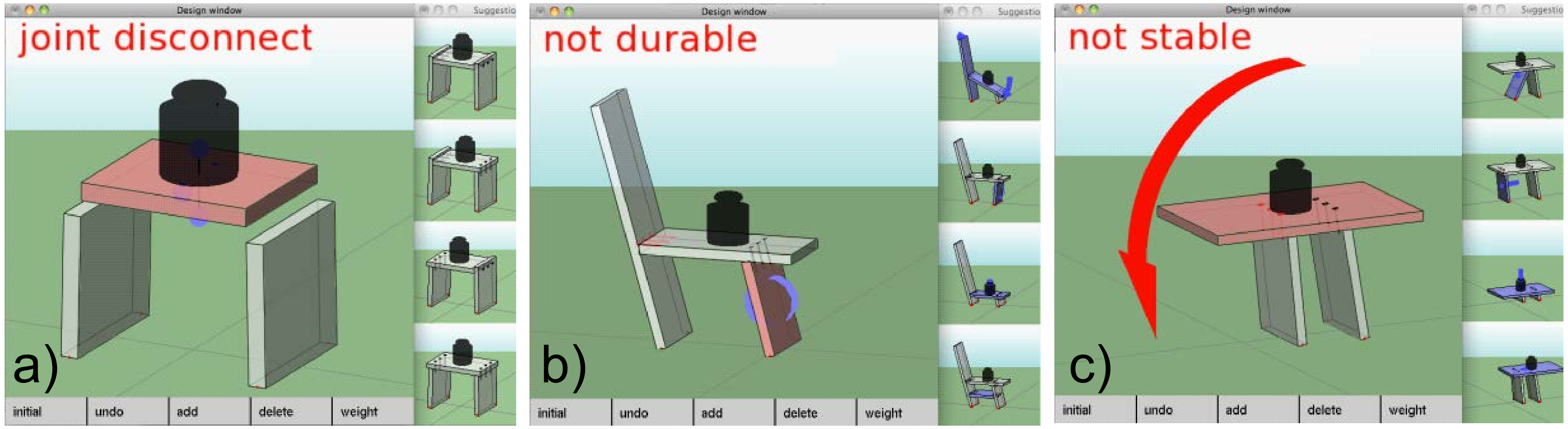


[Siggraph 2012]

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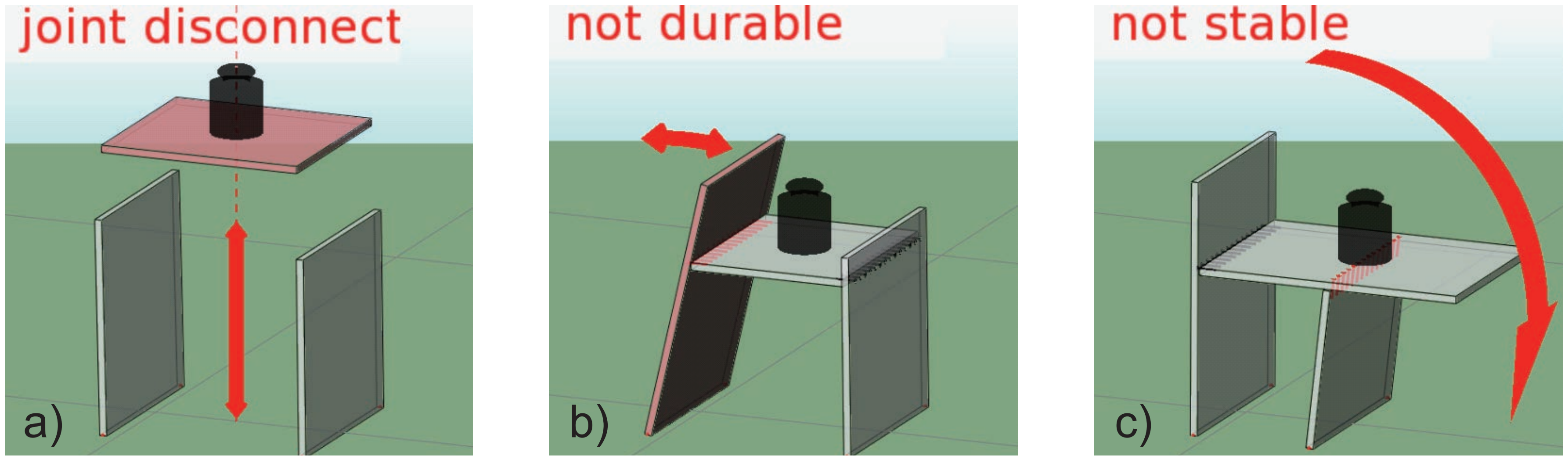
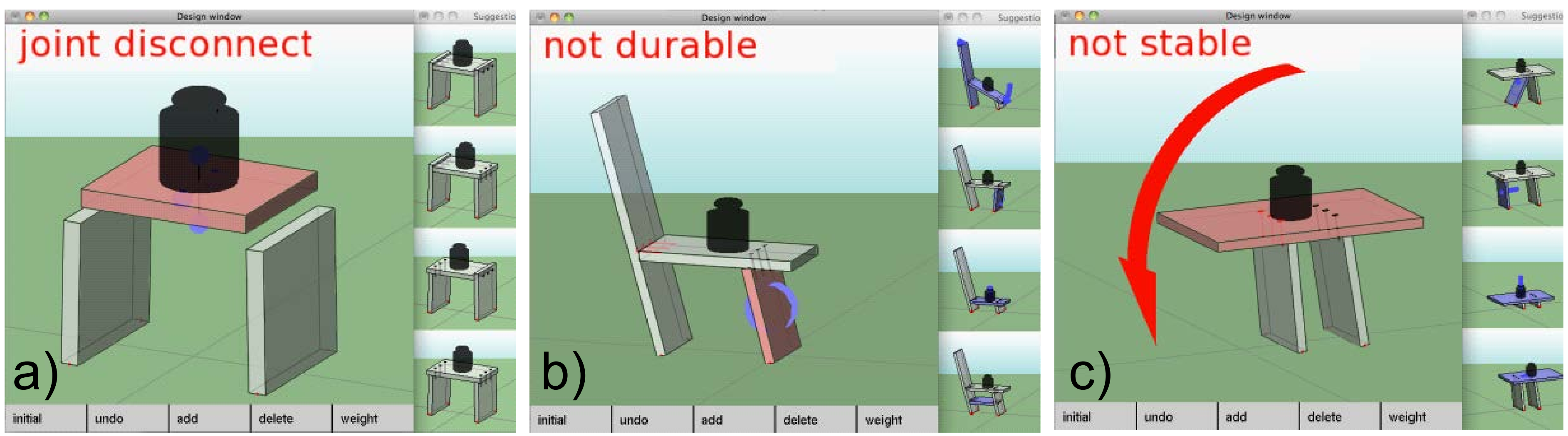
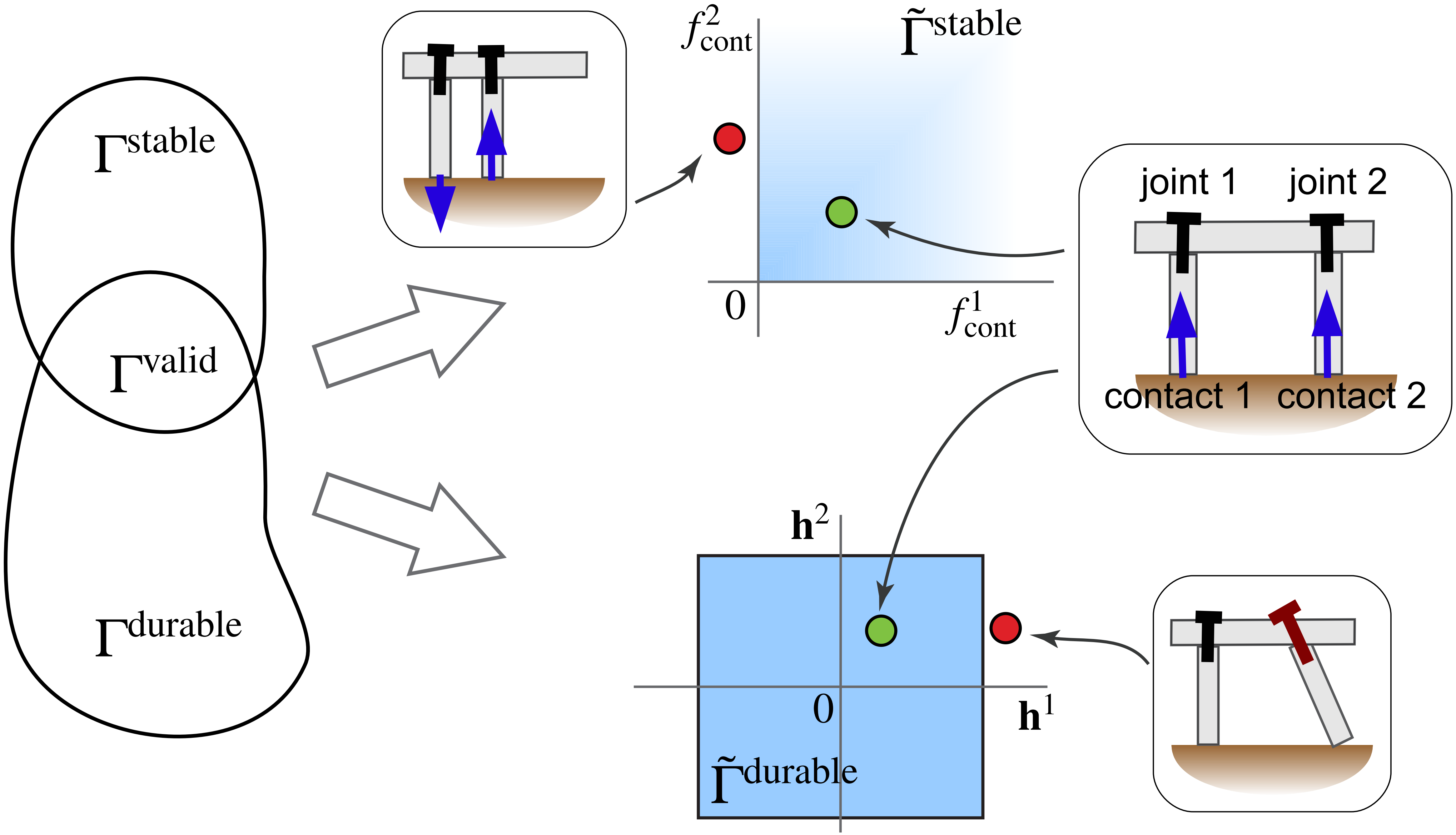


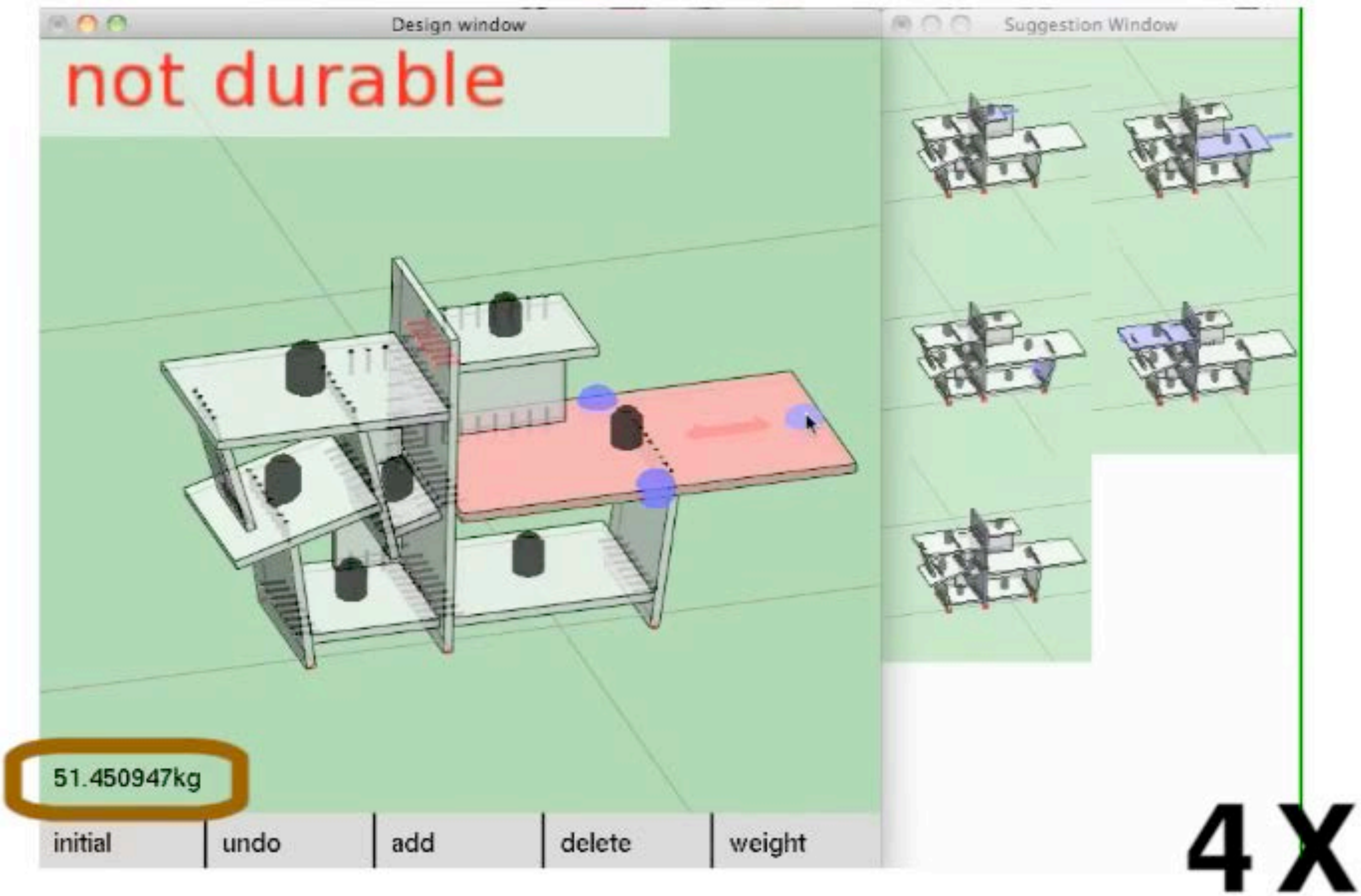

$$\Gamma := \{X_1, X_2, \ldots\}$$

[Siggraph 2012]

# Guided Exploration



# “Forms as Force Diagrams”



# Optimizing Layouts

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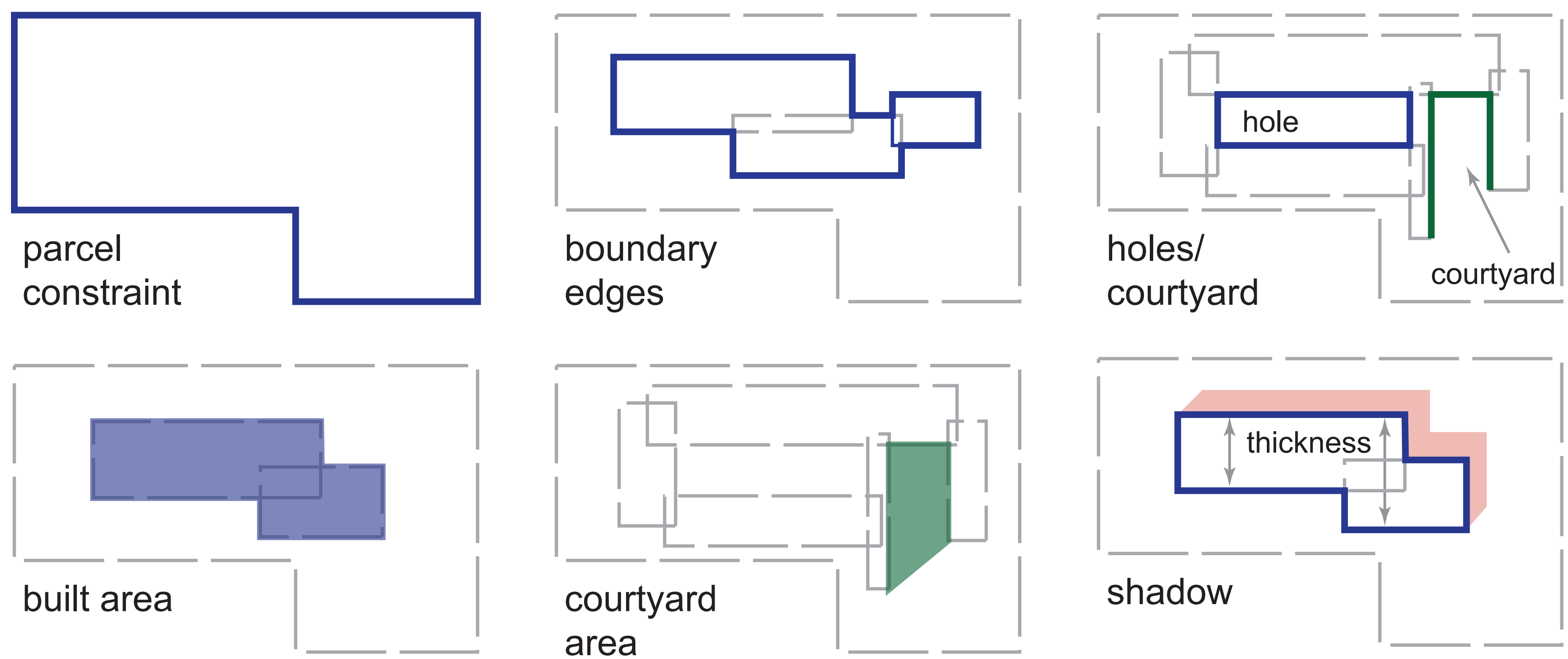

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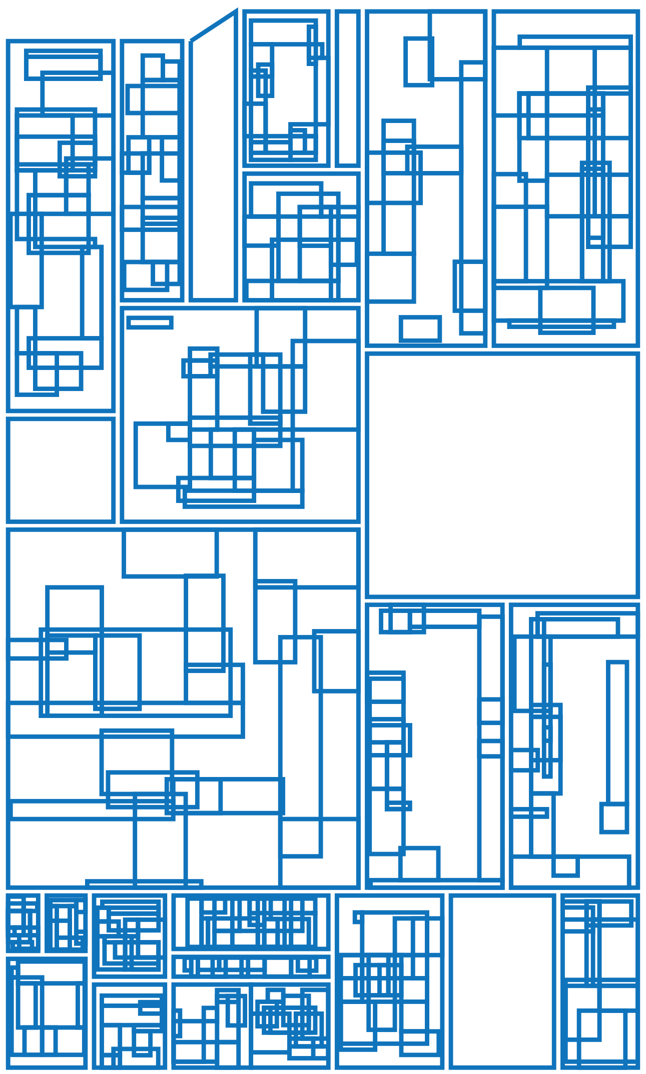

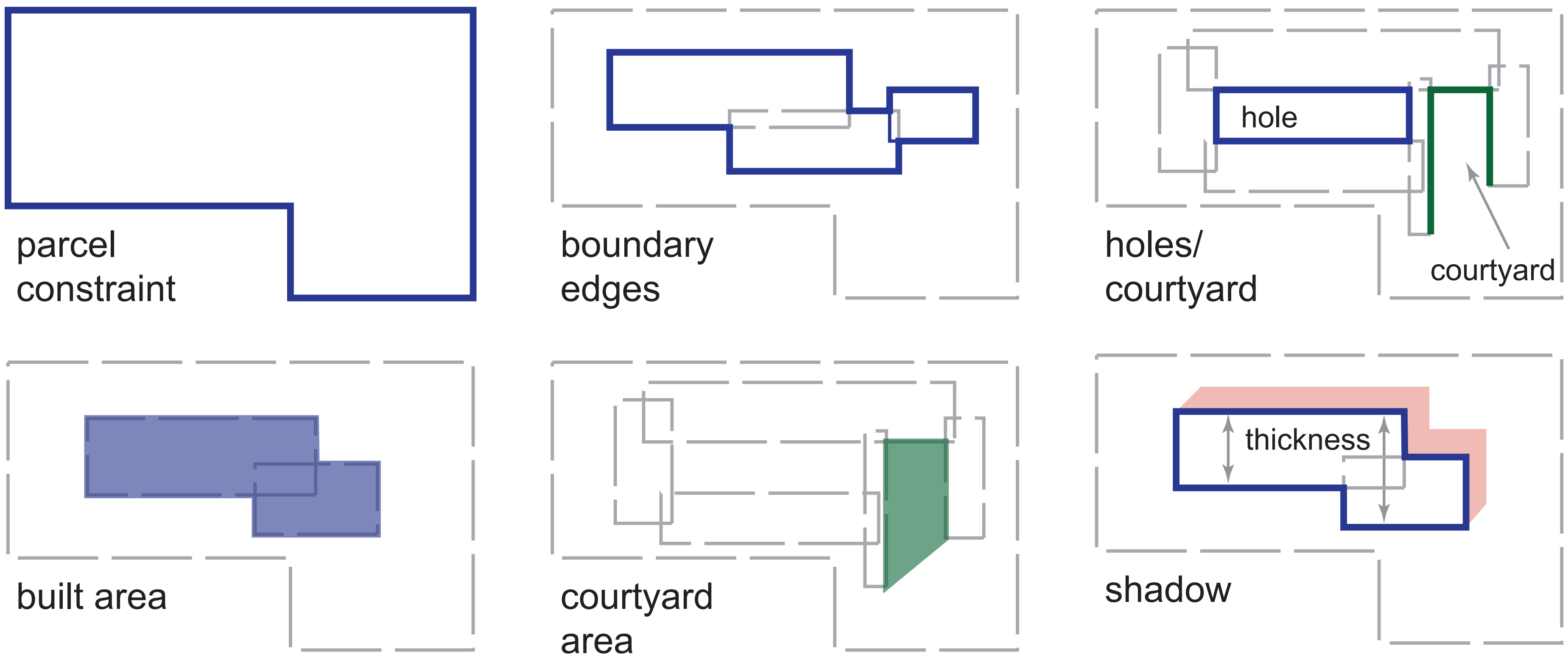

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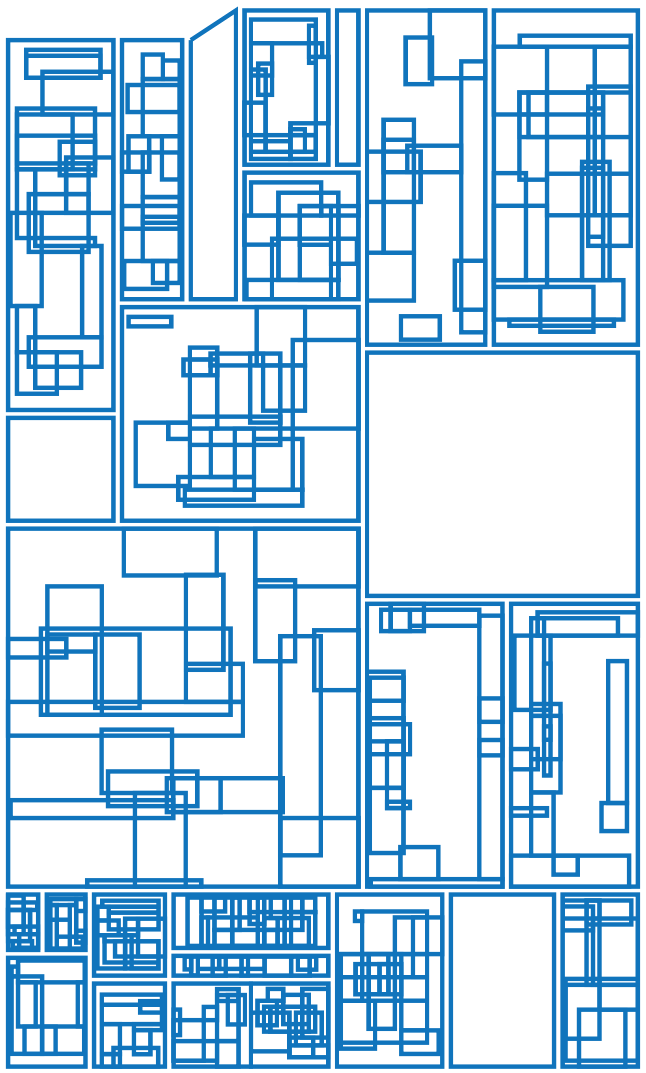
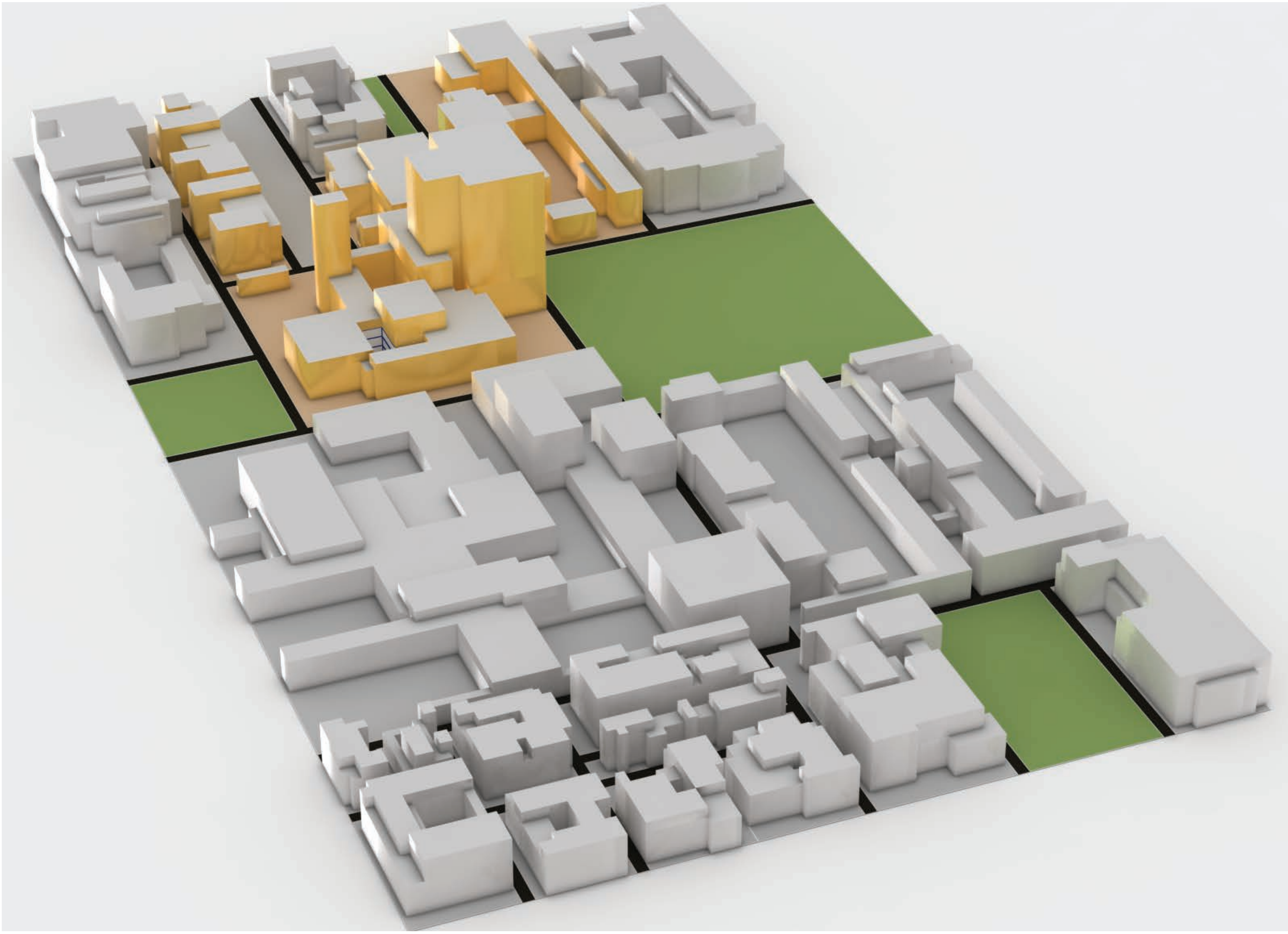
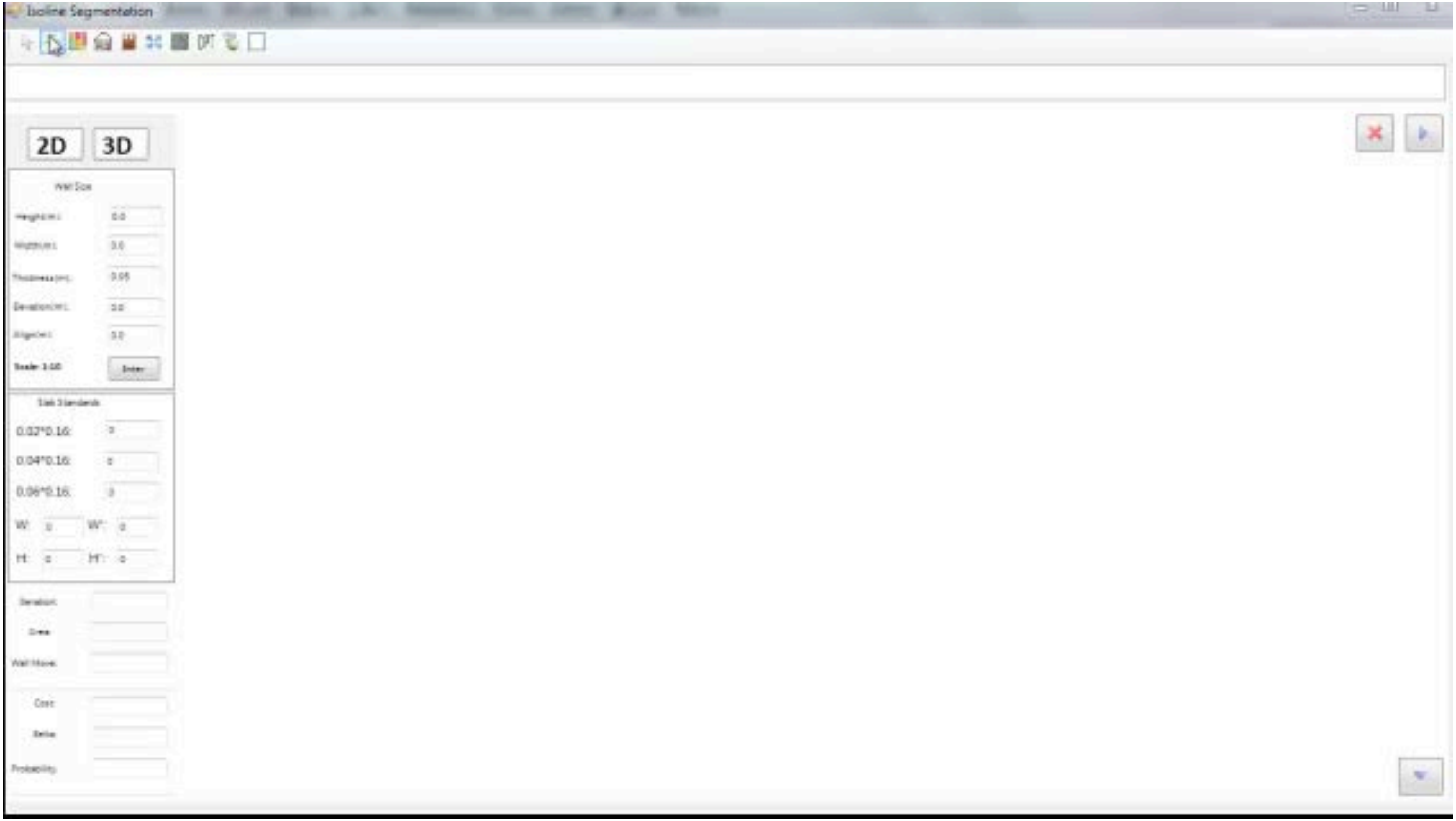




# Same for Indoors



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- symmetry, relations, contacts, etc. are good candidates
- capture the necessary dimensions

- discrete/continuous (global) optimization
- pattern finding in high dimensions

# thank YOU

http://www.cs.ucl.ac.uk/staff/N.Mitra/