Abstract Program Slicing: Abstract interpretation-based approaches to Slicing

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PROGRAM SLICING: BASIC NOTIONS

Program Slicing

A program decomposition technique that extracts from programs statements which affect parameters of interest

Slicing Criterion

Contains different parameters of interest (e.g., $\mathcal{C} = (V, n)$ [Weiser '79])

Program Slice

An executable program obtained that way

PROGRAM SLICING: BASIC NOTIONS

Example

```
1  begin
2    read(x,y);
3    total := 0.0;
4    sum := 0.0;
5    if x <= 1
6    then sum := y;
7    else begin
8              read(z);
9              total := x*y;
10         end;
11   write(total, sum);
12 end.
```

```
begin
  read(x, y);
  if x <= 1
  then
  else
    read(z);
end.
```

(12, z)

```
begin
  read(x, y);
end.
```

(9, x)

```
begin
  read(x, y);
   total := 0.0;
   if x <= 1
  then
  else
   total := x*y;
end.
```

(12, total)

⇒ Slices depend on slicing criterion

THE IDEA

Limitations

- Sometimes standard criteria are too strong

Weakening slicing

- Suppose we want a variable x to have a property ρ at some point n
- The exact value of x can be expressed as $\rho = id = \lambda a.a$
- We are interested in the statements that affect $\rho(x)$ at n
- Abstract slices should be smaller

THE IDEA

Example

$$P \stackrel{\text{def}}{=} \begin{cases} a & := & 1; \\ b & := & b+1; \\ c & := & c+2; \\ d & := & c+b+a-a+c; \end{cases}$$

Abstract criterion: Parity of d

THE IDEA

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THE IDEA

Example

$$P \stackrel{\text{def}}{=} \begin{cases} b & := & b+1; \\ d & := & c+b+a-a+c; \end{cases}$$

Abstract criterion: Parity of *d*

Related works

[Amtoft & Banerjee '07]

Slicing by means of a calculus for independencies

- Syntactic dependencies
- Forward slicing

[Rival '05]

Abstract dependencies

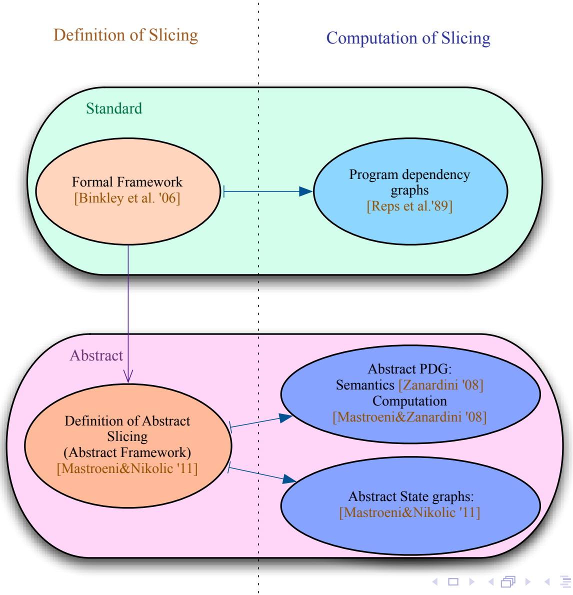
- Mathematical, set theoretic definition of dependencies
- Applied to Alarm diagnosis

[Hong et al. '05]

Abstract Slicing

- Only for predicate abstractions
- Considers a subset of possible executions

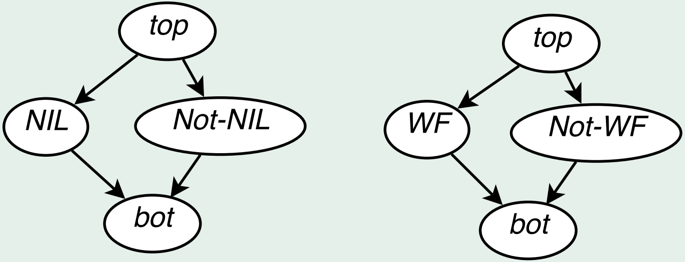
ABSTRACT INTERPRETATION-BASED APPROACHES



REVERSING WELL-FORMED LISTS [Zanardini '08]

Well-formed lists: $\langle 1, 2, 3, 4, [0]\rangle{+}{+}\langle 5, 6, [0]\rangle = \langle 1, 2, 3, 4, 5, 6, [0]\rangle$

The properties of interest are represented by abstract domains for *nullity* and *well-formedness*:



ρ_{nil} ρ_{WF}

$$wellFormed(x) \equiv notNil(x) \wedge lastEl(x).data = 0$$

REVERSING WELL-FORMED LISTS [Zanardini '08]

Reversing the list

```
list rev(list l) {
  list *last;
  list *tmp;
  while (l->next != null){
    tmp = l->next;
    l->next = last;
    last = l;
    l = tmp;
  }
  return last;
}
```

REVERSING WELL-FORMED LISTS [Zanardini '08]

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  }
  return r;
}
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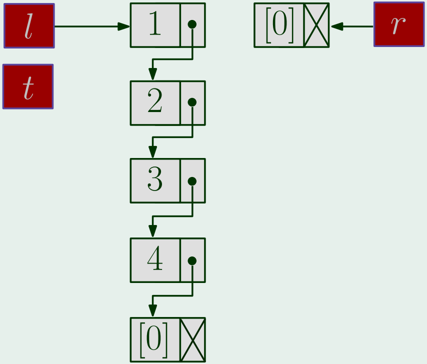

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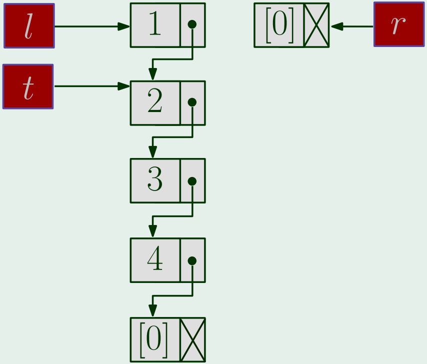

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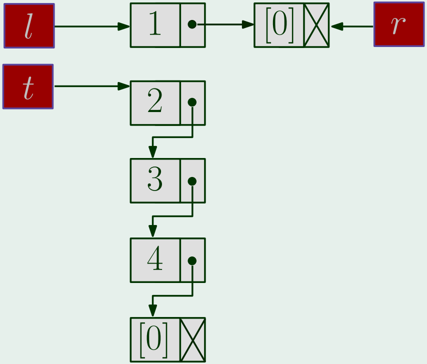

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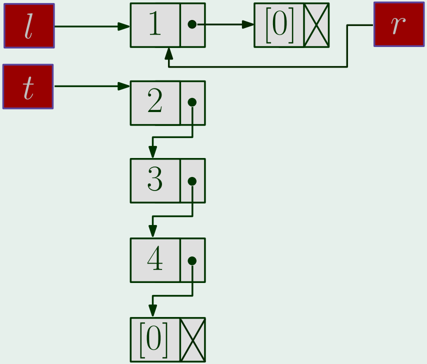

Reversing Well-Formed Lists [Zanardini '08]

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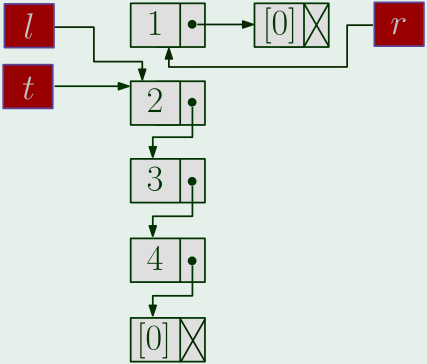

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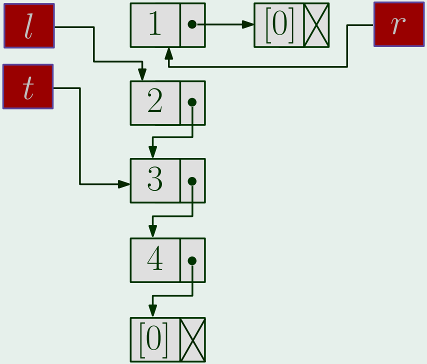

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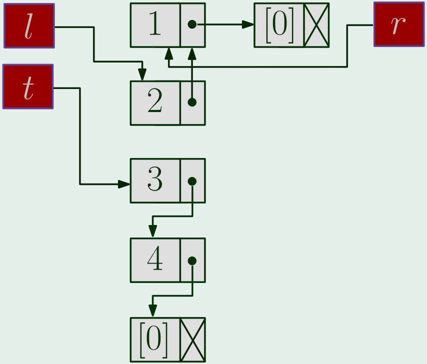

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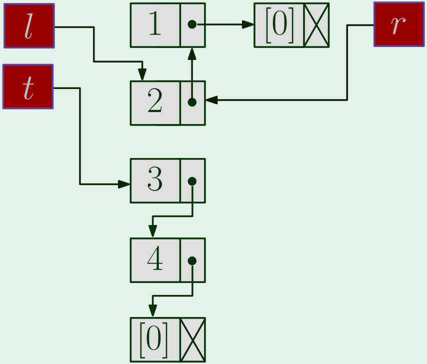

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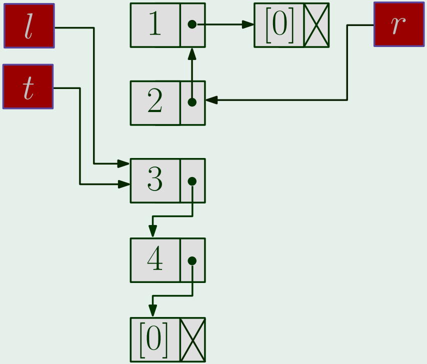

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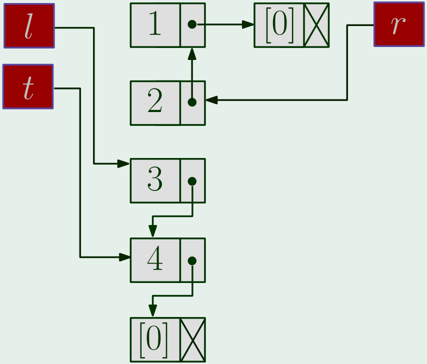

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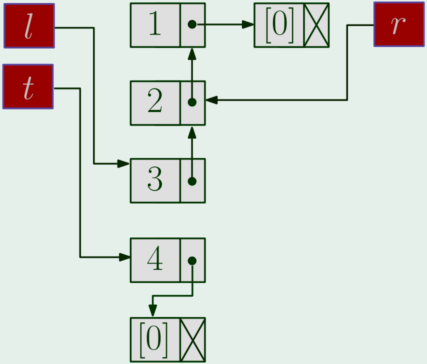

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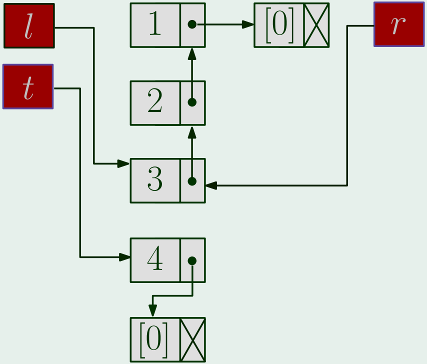

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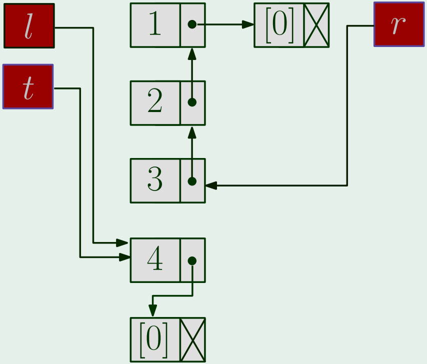

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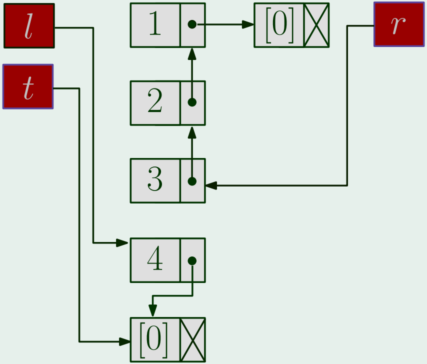

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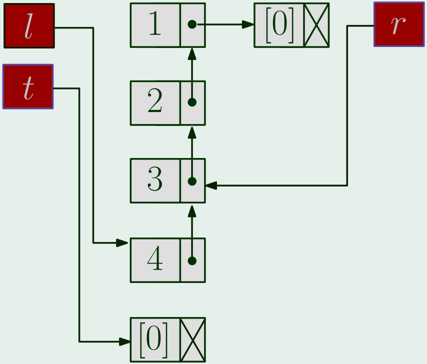

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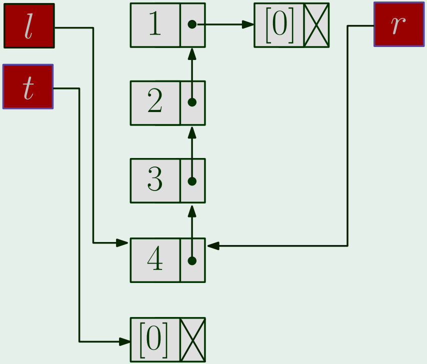

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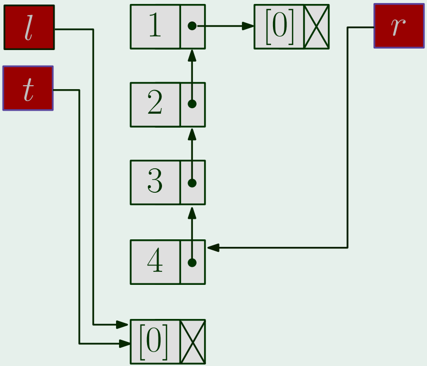

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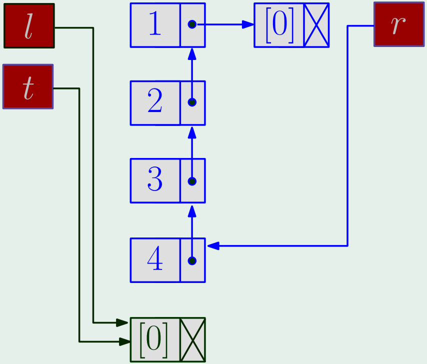

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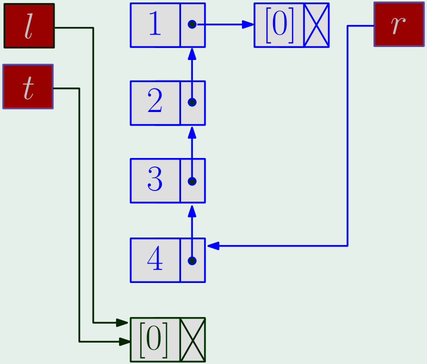
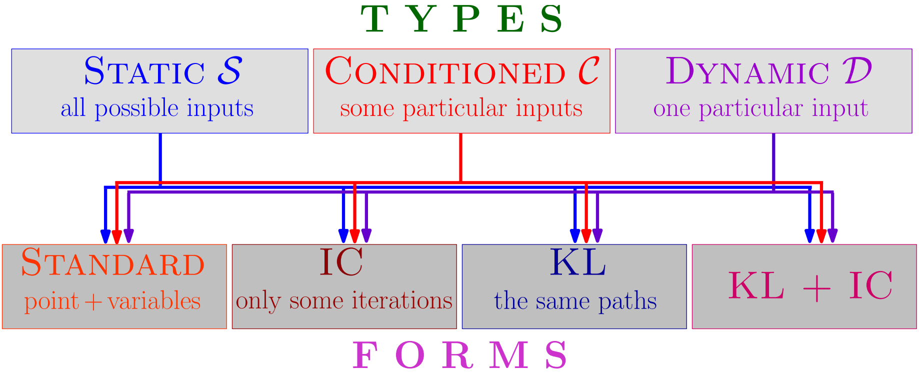


⇒ if r is well-formed before while, it is well-formed after while as well

Formal Framework [Binkley at al. '06]



Formal Framework [Binkley at al. '06]

$$\langle \sqsubseteq, \mathcal{E} \rangle$$

Formal Framework [Binkley at al. '06]

SEMANTIC CONSTRAINTS

maps slicing criteria to semantic equivalence relations

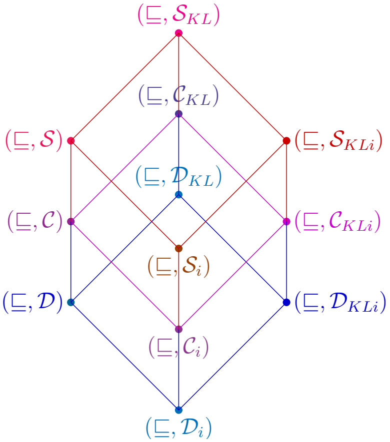
$\langle \sqsubseteq, \mathcal{E} \rangle$

TRADITIONAL SYNTACTIC ORDERING

$$Q \sqsubseteq P \Leftrightarrow \mathcal{F}(Q) \subseteq \mathcal{F}(P)$$

$\mathcal{F}(P)$ maps l to c iff P contains statement c at line l

Formal Framework [Binkley at al. '06]



Hierarchy of Existing Forms of Slicing

C
R
I
T
E
R
I
O
N

VARIABLES OF INTEREST: V

STATIC	CONDITIONED	DYNAMIC
all possible inputs	some particular inputs	one particular input

C
R
I
T
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VARIABLES OF INTEREST: V

STATIC	CONDITIONED	DYNAMIC
all possible inputs	some particular inputs	one particular input
$\mathcal{I} = \mathbb{M}$	$\varnothing \subset \mathcal{I} \subseteq \mathbb{M}$	$\lvert\mathcal{I}\rvert = 1$

CRITERION

- INPUTS OF INTEREST: $\mathcal{I}$
- VARIABLES OF INTEREST: V

C
R
I
T
E
R
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N

Inputs of Interest: $\mathcal{I}$

Variables of Interest: V

Iteration Count only particular iterations

Non-Iteration Count all iterations

CRITERION

- INPUTS OF INTEREST: $\mathcal{I}$
- VARIABLES OF INTEREST: V
- OCCURRENCES OF INTEREST: $\mathcal{O}$

$\mathcal{O} = n \times \wp(\mathbb{N})$

ITERATION COUNT only particular iterations

NON-ITERATION COUNT all iterations

$\mathcal{O} = n \times \mathbb{N}$

C
R
I
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INPUTS OF INTEREST: $\mathcal{I}$

VARIABLES OF INTEREST: V

OCCURRENCES OF INTEREST: $\mathcal{O}$

KL	NON - KL
the same paths	even different paths

CRITERION

- INPUTS OF INTEREST: $\mathcal{I}$
- VARIABLES OF INTEREST: V
- OCCURRENCES OF INTEREST: $\mathcal{O}$
- PATH REQUIREMENT $\mathcal{L}$

$\mathcal{L} = true$

KL
the same paths

$\mathcal{L} = false$

NON - KL
even different paths

STATIC
all possible inputs
$\mathcal{I} = \mathbb{M}$

CONDITIONED
some particular inputs
$\varnothing \subset \mathcal{I} \subseteq \mathbb{M}$

DYNAMIC
one particular input
$|\mathcal{I}| = 1$

CRITERION

- **INPUTS OF INTEREST:** $\mathcal{I}$
- **VARIABLES OF INTEREST:** V
- **OCCURRENCES OF INTEREST:** $\mathcal{O}$
- **PATH REQUIREMENT** $\mathcal{L}$

$\mathcal{O} = n \times \wp(\mathbb{N})$

ITERATION COUNT
only particular iterations

NON-ITERATION COUNT all iterations

$\mathcal{O} = n \times \mathbb{N}$

$\mathcal{L} = true$

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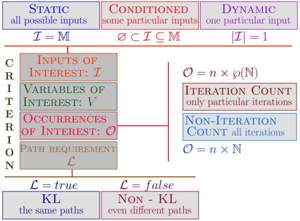


Generalized Slicing Criterion

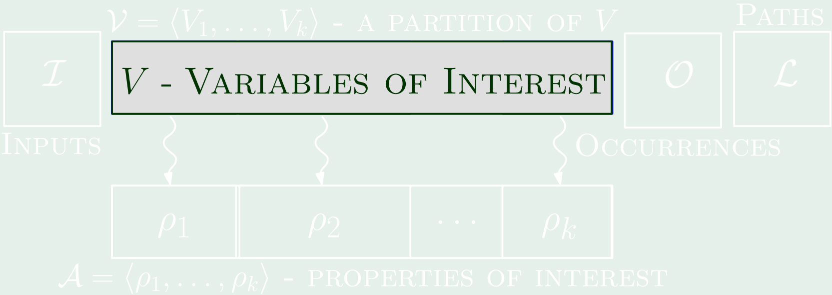
$$\mathcal{C} = \langle \mathcal{I}, \mathrm{V}, \mathcal{O}, \mathcal{L} \rangle,$$

where

- $\mathcal{I} \subseteq \mathbb{M}$ - set of INPUTS of interest,
- V - set of VARIABLES of interest,
- $\mathcal{O} \in n \times \wp(\mathbb{N})$ - set of OCCURRENCES of interest,
- $\mathcal{L} \in \{true, false\}$ - determines a KL form.

ABSTRACTING GENERALIZED SLICING CRITERION

Generalized abstract criterion



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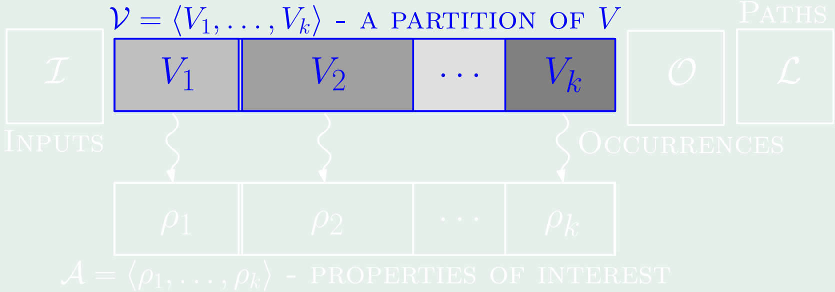
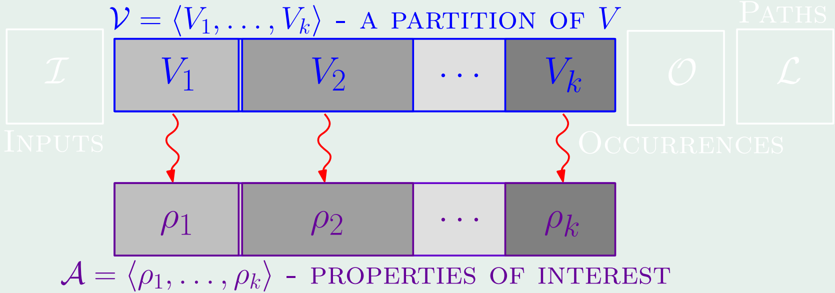


Abstracting generalized slicing criterion

Generalized abstract criterion



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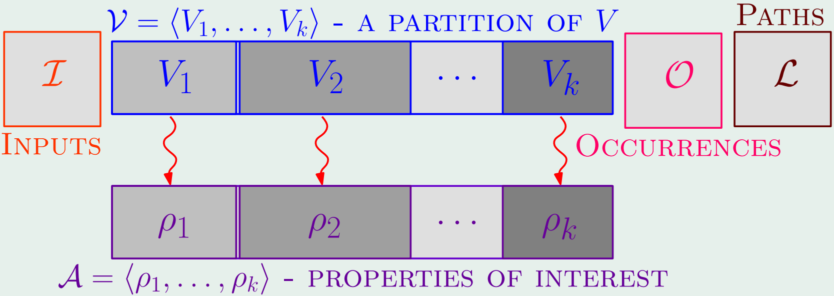

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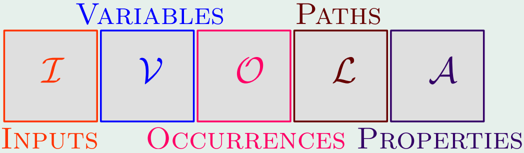
Generalized abstract criterion



Example

$$Var = \{x_1, x_2, x_3, x_4\} \qquad V = \{x_1, x_2, x_3\}$$

Properties of interest: Sign^2 of $x_1 \times x_2$ and Par of x_3

$$\Rightarrow \mathcal{V} = \langle \{x_1, x_2\}, \{x_3\} \rangle \qquad \mathcal{A} = \langle \text{Sign}^2, \text{Par} \rangle$$

$$\text{Sign}^2(x, y) = \begin{cases} \text{POS} & \text{if } x * y > 0 \\ 0 & \text{if } x * y = 0 \\ \text{NEG} & \text{otherwise} \end{cases} \qquad \text{Par}(x) = \begin{cases} \text{EVEN} & \text{if } x \text{ is even} \\ \text{ODD} & \text{otherwise} \end{cases}$$

System States and Traces

```
begin
   read(n);
   i:=1;
   s:=0;
   p:=1;
   while (i<=n) do
   begin
      s:=s+i;
      p:=p*i;
      i:=i+1;
   end;
   write(s);
   write(p);
end;
```

$$\sigma = \{n \leftarrow 2\}$$

SYSTEM STATES AND TRACES

```
1   begin
2     read(n);
3     i:=1;
4     s:=0;
5     p:=1;
6     while (i<=n) do
7     begin
8       s:=s+i;
9       p:=p*i;
10      i:=i+1;
11    end;
12    write(s);
13    write(p);
14  end;
```

$$\sigma = \{n \leftarrow 2\}$$

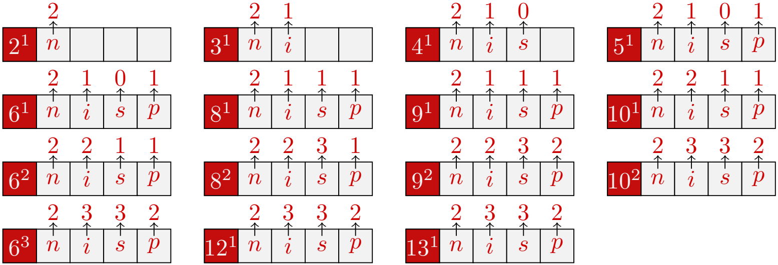
State	n	i	s	p
2^1	2			
3^1	2	1		
4^1	2	1	0	
5^1	2	1	0	1
6^1	2	1	0	1
8^1	2	1	1	1
9^1	2	1	1	1
10^1	2	2	1	1
6^2	2	2	1	1
8^2	2	2	3	1
9^2	2	2	3	2
10^2	2	3	3	2
6^3	2	3	3	2
12^1	2	3	3	2
13^1	2	3	3	2

SYSTEM STATES AND TRACES

L - ADDITIONAL POINTS OF INTERESTED

$$Proj'^{\alpha}_{(\mathcal{V},\mathcal{O},L,\mathcal{A})}(n,k,\sigma) \stackrel{\text{def}}{=} \begin{cases} (n, \sigma \upharpoonright^{\alpha} \mathcal{V}) & \text{if } (n,k) \in \mathcal{O} \\ (n, \sigma \upharpoonright^{\alpha} \varnothing) & \text{if } (n,k) \notin \mathcal{O} \wedge n \in L \\ \lambda & \text{otherwise} \end{cases}$$

$$\mathcal{V} = \langle \{i\}, \{s\} \rangle,\ \mathcal{O} = \{8\} \times \mathbb{N},\ L = \{6\},\ \mathcal{A} = \langle \textsc{Sign}, \textsc{Par} \rangle$$



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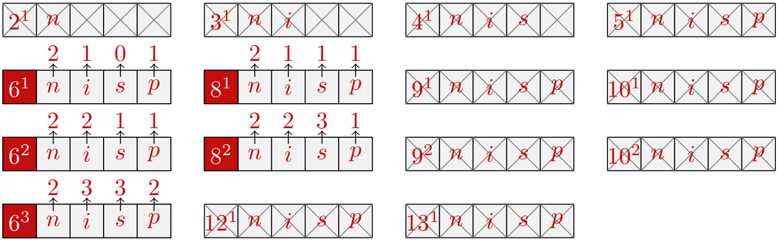

System States and Traces

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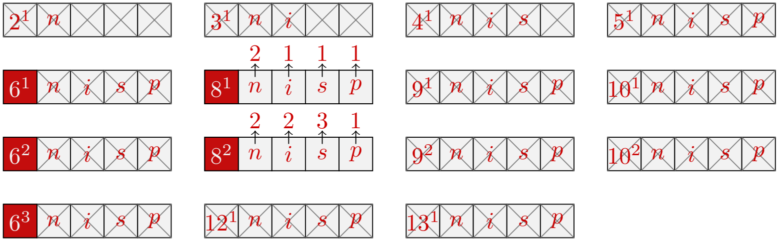

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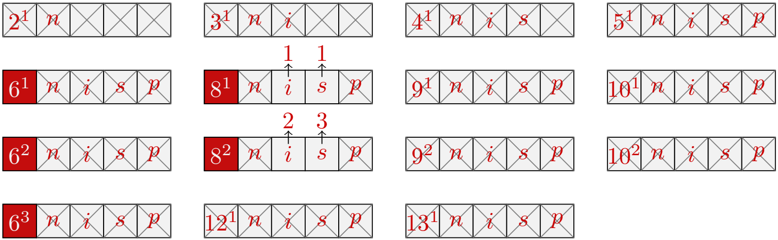

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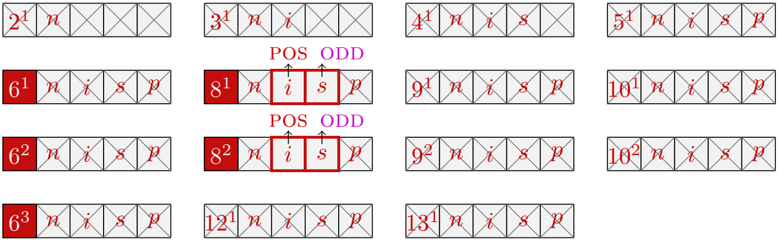

Abstract Unified Equivalence

- P, Q - executable programs,
- l_P, l_Q - sets of line numbers of P and Q
- $\mathcal{C}_{\mathcal{A}} = \langle \mathcal{I}, \mathcal{V}, \mathcal{O}, \mathcal{L}, \mathcal{A} \rangle$ - abstract criterion
- $L_{\mathcal{L}}(P, Q) = \mathcal{L} \ ? \ l_P \cap l_Q \ : \ \varnothing$
- P is Abstract Equivalent to Q ($P \ \ \mathcal{U}^{\mathcal{A}}(\mathcal{I}, \mathcal{V}, \mathcal{O}, L_{\mathcal{L}}, \mathcal{A}) \ \ Q$) iff

$$\forall \sigma \in \mathcal{I}. Proj^{\alpha}_{(\mathcal{V}, \mathcal{O}, L_{\mathcal{L}}, \mathcal{A})}(T^{\sigma}_{P}) = Proj^{\alpha}_{(\mathcal{V}, \mathcal{O}, L_{\mathcal{L}}, \mathcal{A})}(T^{\sigma}_{Q})$$

EXTENDED FRAMEWORK

Semantic Constraint

$$\mathcal{E}_{\mathcal{A}} \stackrel{\text{def}}{=} \lambda(\mathcal{I}, \mathcal{V}, \mathcal{O}, \mathcal{L}, \mathcal{A}).\, \mathcal{U}^{\mathcal{A}}(\mathcal{I}, \mathcal{V}, \mathcal{O}, L_{\mathcal{L}}, \mathcal{A})$$

$\langle \sqsubseteq, \mathcal{E}_{\mathcal{A}} \rangle$ - Representation of Abstract Forms of Slicing

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$\langle \sqsubseteq, \mathcal{E}_{\mathcal{A}} \rangle$ - Representation of Abstract Forms of Slicing

$\Updownarrow$

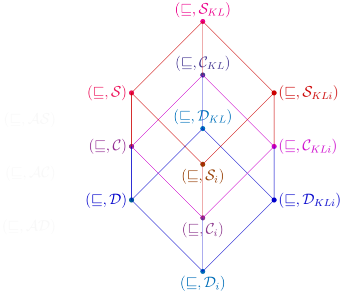
We have inserted Abstract Slicing in Formal Framework

$\Downarrow$ - Extended Theory

Enriched Hierarchy

Extended Framework

$\langle \sqsubseteq, \mathcal{E}_{\mathcal{A}} \rangle$ - Representation of Abstract Forms of Slicing
Enriched Hierarchy



EXTENDED FRAMEWORK

$\langle \sqsubseteq, \mathcal{E}_{\mathcal{A}} \rangle$ - Representation of Abstract Forms of Slicing

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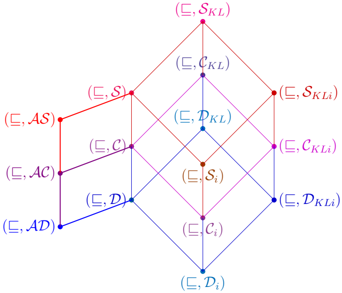

SLICING VS DEPENDENCIES

Slicing

...extracts from programs the statements which are *relevant* for a given behaviour.

Dependency

...defines what *relevant* means.

Slicing vs dependencies

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Example

$$\text{SYNTACTIC DEF-REF}: \begin{cases} x := y + 2z \\ \textbf{x} \text{ depends on } \textbf{y} \text{ and on } \textbf{z} \end{cases}$$

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Example

SYNTACTIC DEF-REF :

$x := y + 2z$
x depends on **y** and on **z**

$x := z + y - y$
x depends on **y** and on **z**

SLICING VS DEPENDENCIES

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...extracts from programs the statements which are *relevant* for a given behaviour.

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SEMANTIC : $\begin{cases} x := z + y - y \\ \textbf{x} \text{ depends on } \textbf{z} \text{ but it does NOT depend on } \textbf{y} \end{cases}$

SLICING VS DEPENDENCIES

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$x := 2y$
x depends on **y**

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ABSTRACT SEMANTIC (PARITY) : $\begin{cases} x := 2y \\ \textbf{x} \text{ does NOT depend on } \textbf{y} \end{cases}$

SLICING VS DEPENDENCIES

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Example

ABSTRACT SEMANTIC (PARITY) :

- $x := 2y$ — **x** does NOT depend on **y**
- $x := 2y + z$ — **x** depends on **z**

SLICING BY PRUNING PDG

Program Dependency Graphs (PDG) are defined by two kind of edges (s_1, s_2):

Control Flow Edge

s_1 represents a control predicate and s_2 represents a component of the program immediately nested within the predicate s_1;

Flow Dependence Edge

s_1 defines a variable x which is used in s_2
i.e., $x \in \mathbf{def}(s_1) \cap \mathbf{ref}(s_2)$,
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$\Downarrow$

Flow dependence edges = DIRECT FLOWS=DEF-REF dependencies
Control flow edges = INDIRECT FLOWS

PRUNING DEPENDENCIES

Kind of dependencies

- Data dependencies (Assignments);
- Control dependencies (Control structures)

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We propose a PRUNING of *data dependencies*!

$\Downarrow$

STILL WE LOSE SOMETHING ABOUT CONTROL DEPENDENCIES!

PRUNING DEPENDENCIES

Kind of dependencies

- Data dependencies (Assignments) $\Rightarrow$ Direct flows;
- Control dependencies (Control structures) $\Rightarrow$ Indirect flows

Example

$$\textbf{if } (y + 2x \; mod \, 2) == 0 \textbf{ then } w := 0 \textbf{ else } w := 0$$

$\Rightarrow$ The guard DOES NOT DEPEND on x: OK

$\Rightarrow$ The variable w DOES NOT DEPEND on y: NO!

DERIVING ABSTRACT DEPENDENCIES

Systematic way to go through the (variables and states)-space

- Incrementally find the set X of variables which are enough to determine the value of e
- X determines e if any change to other variables can be ignored (needs to go into the state space)

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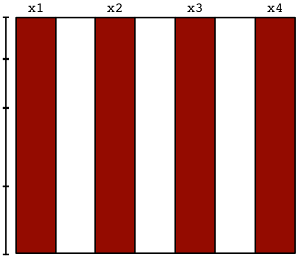

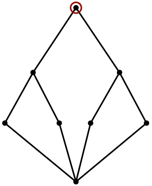

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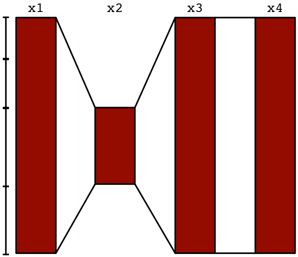

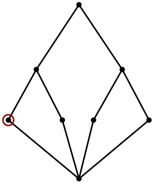

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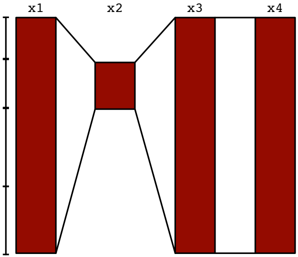

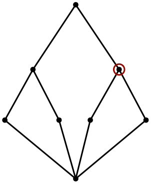

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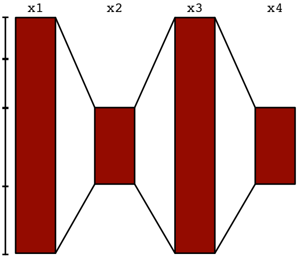

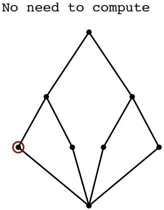

Deriving abstract dependencies

Systematic way to go through the (variables and states)-space

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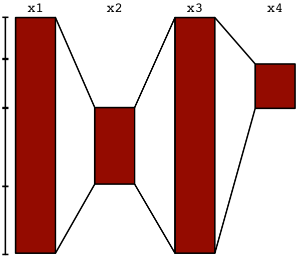
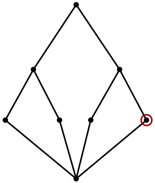
- *X* determines *e* if any change to other variables can be ignored (needs to go into the state space)





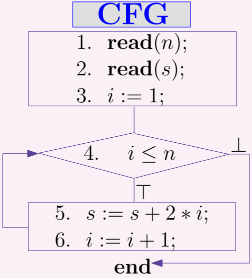
Slicing by abstracting CFG

- Start from a static slice of a program
- Derive an abstraction ρ from $\mathcal{C}_{\mathcal{A}}$ and construct abstract states using ρ
- Determine an abstract state graph ASG
- Abstract Slice corresponds to a pruned ASG

Example

```
read(n);
read(s);
i := 1;
while (i<=n) do
   s := s + 2*i;
   i := i+1;
od
```

Example



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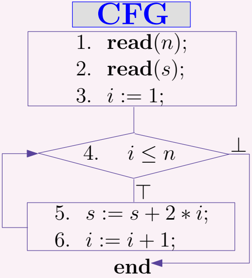
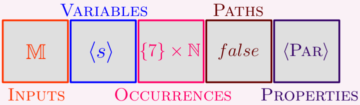
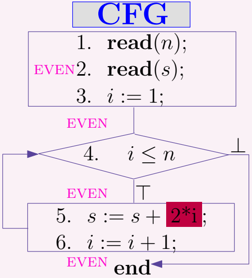


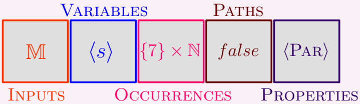


$\mathbb{M}$- all possible inputs

$$\text{PAR}(x) = \begin{cases} \text{EVEN} & \text{if } x \equiv_2 0 \\ \text{ODD} & \text{otherwise} \end{cases}$$

Example





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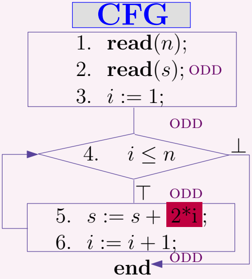

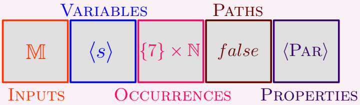

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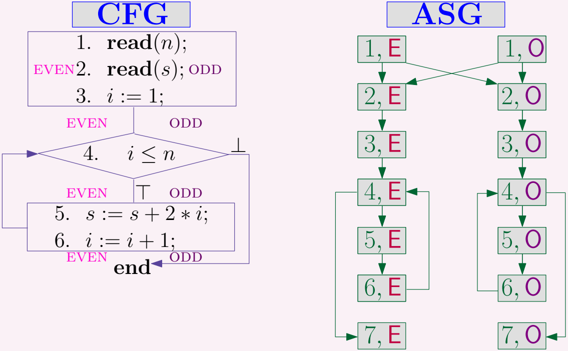
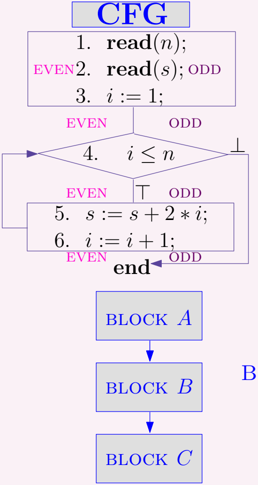
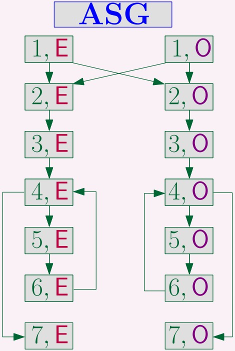

CFG
1. read(n);
EVEN 2. read(s); ODD
3. i := 1;
EVEN
ODD
4. i ≤ n
⊥
EVEN
⊤
ODD
5. s := s + 2 * i;
6. i := i + 1;
EVEN
ODD
end
ASG
1, E
1, O
2, E
2, O
3, E
3, O
4, E
4, O
5, E
5, O
6, E
6, O
7, E
7, O

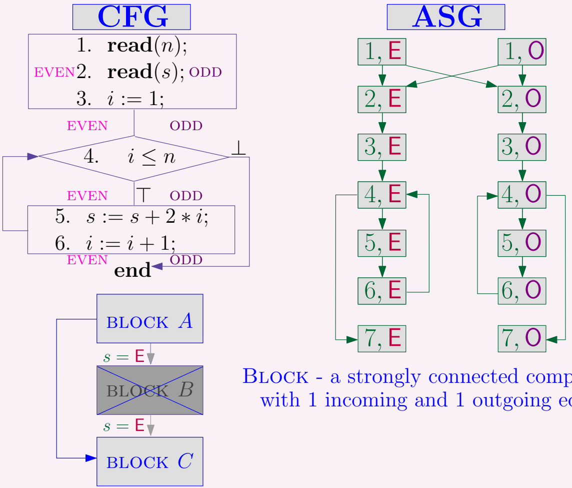
Example





BLOCK - a strongly connected component with 1 incoming and 1 outgoing edge

Example



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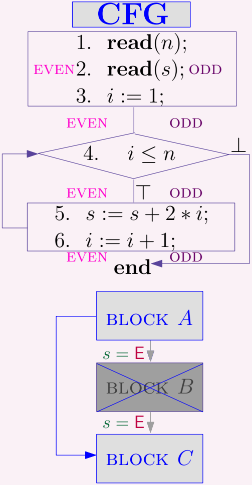
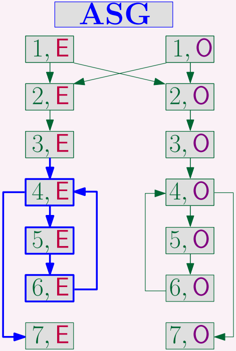


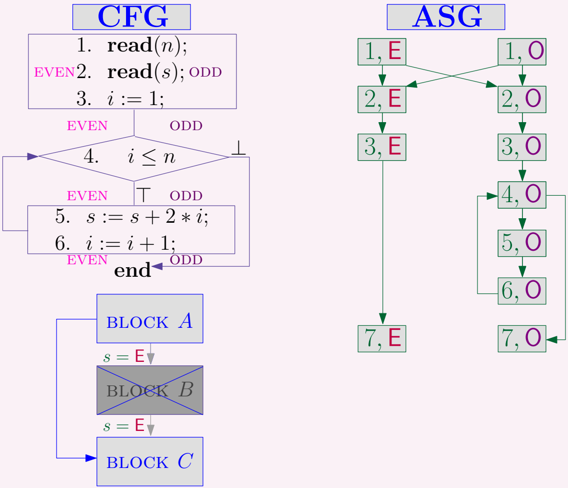


BLOCK ⟨4, E⟩, ⟨5, E⟩, ⟨6, E⟩

Input: $s =$ E Output: $s =$ E

$\Rightarrow$ we can remove it

Example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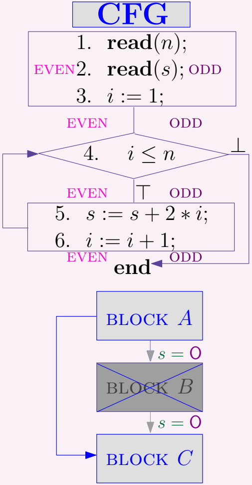
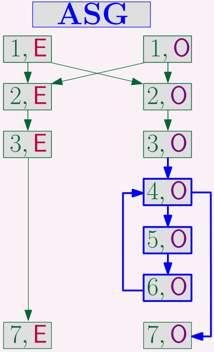
3. i := 1;
EVEN
ODD
4. i ≤ n
⊥
⊤
EVEN
ODD
5. s := s + 2 * i;
6. i := i + 1;
EVEN
ODD
end
BLOCK A
s = E
BLOCK B
s = E
BLOCK C
ASG
1, E
1, O
2, E
2, O
3, E
3, O
4, O
5, O
6, O
7, E
7, O

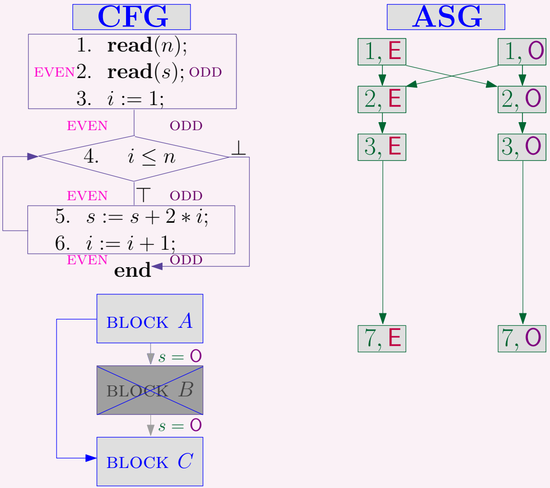
Example





BLOCK $\langle 4, O\rangle, \langle 5, O\rangle, \langle 6, O\rangle$
Input: $s = O$ Output: $s = O$
$\Rightarrow$ we can remove it

Example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EVEN ODD
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⊥
EVEN ⊤ ODD
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EVEN ODD
end
BLOCK A
s = O
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BLOCK C
ASG
1, E
1, O
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2, O
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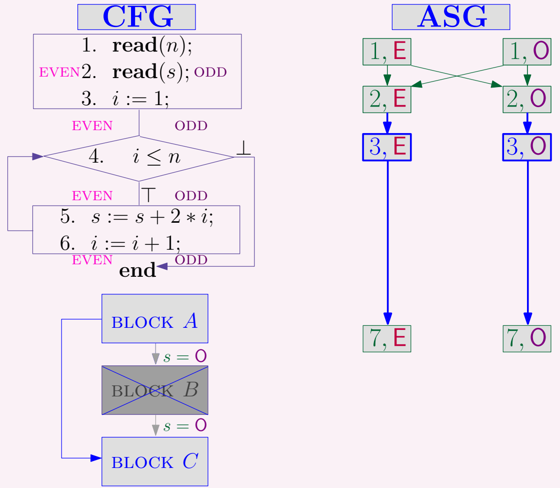
7, E
7, O

Example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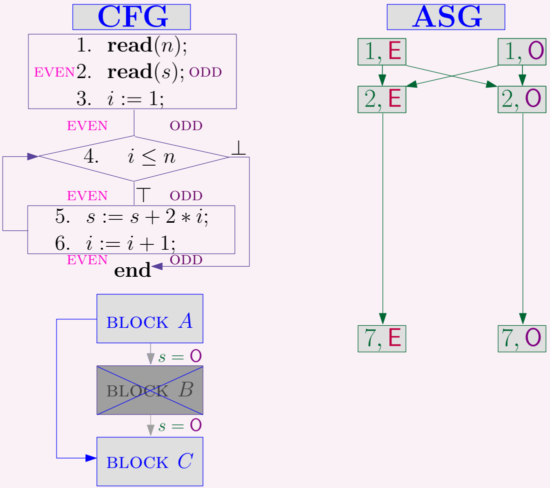
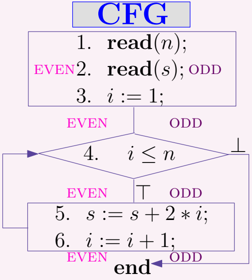
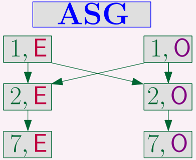
3. i := 1;
EVEN ODD
4. i ≤ n
⊥
EVEN ⊤ ODD
5. s := s + 2 ∗ i;
6. i := i + 1;
EVEN ODD
end
BLOCK A
s = O
BLOCK B
s = O
BLOCK C
ASG
1, E
1, O
2, E
2, O
3, E
3, O
7, E
7, O

Example


CFG
1. read(n);
EVEN 2. read(s); ODD
3. i := 1;
EVEN
ODD
4. i ≤ n
⊥
EVEN
⊤
ODD
5. s := s + 2 * i;
6. i := i + 1;
EVEN
ODD
end
BLOCK A
s = O
BLOCK B
s = O
BLOCK C
ASG
1, E
1, O
2, E
2, O
7, E
7, O

Example





1. **read**(n);
2. **read**(s);
7. **end**

CONCLUSIONS

Putting all together

- Generalized Slicing Criteria (Traditional and Abstract versions)
- Extension of Unified Formal Framework
- Formal definition of Abstract Program Slicing
- Semantic and constructive characterization of abstract dependencies
- First steps towards an implementation of abstract program slicing

Conclusions

Limitations

- If the property used for the construction of ASG is too much abstract, the Simple Approach returns the static slice
- This approach cannot be used for the extraction of dynamic and conditional slices: Extended Approach is one possible refinement of this algorithm
- Still a lot of work to do for obtaining a real implementation
- Also the semantic and constructive characterization of abstract dependencies is still far from its use in a real implementation of abstract slicing

Conclusions

Ideas for the Future

- Improvement and implementation of proposed algorithm(s)
- Obfuscation and Watermarking vs. Abstract Slicing
- Abstract slicing for malware detection