Automatically quantifying information leaks in software
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The problem

An attacker has some a priori knowledge of the secret which is improved by observing the system

measure this improvement: how much did the attacker gain from the observations?
Example: an attacker steals your cash card; he has no idea about your pin (apriori probability to guess it 0.0001)

to randomly try a pin number at a cash machine will generate two possible observations:

1. the pin is accepted (with probability 0.0001),

2. the pin is rejected (with probability 0.9999)
Quantitative analysis of confidentiality according to a measure $F$:
difference of the measure $F$ on the secret $h$
before and after observing the system $P$

$$\Delta_F(P, h) = F(h) - F(h|P)$$

1. $F(h)$ = measure of the secret $h$ before observations

2. $F(h|P)$ measure of the secret $h$ given observations $P$
some possible choices for $F, F(\ldots)$ are:

(A) *Information about the secret:* $F$ and $F(\ldots)$ are Shannon entropy and conditional entropy

$$F(h) = H(h) = \text{entropy of secret } h \text{ before observations} = \text{a priory information about } h$$

$$F(h|P) = H(h|P) = \text{entropy of secret } h \text{ given observations} = \text{information about } h \text{ given observations}$$

$$\Delta_H (\text{Cash machine}, h) = 0.00147 \text{ (bits of information)}$$
(B) : *Probability of guessing in one try*: (introduced by Smith and noted ME)

\[ F(h) = -\log(\max_{x \in h} \mu(h = x)) = \text{a priory probability of guessing } h \]

\[ F(h|P) = -\log(\sum_{y \in P} \mu(y) (\max_{x \in h} \mu(h = x|P = y))) = \text{probability of guessing } h \text{ given observations} \]

\[ \Delta_{ME} (\text{Cash machine}, h) = 1 (= \log(2) : \text{chances have doubled}) \]
(C) Expected number of guesses: (GE)

\[ F(h) = \sum_{x_i \in h, i \geq 1} i \mu(h = x_i) = \text{a priory average number of guesses for } h \]

\[ F(h|P) = \sum_{y \in P} \mu(y)\left(\sum_{x_i \in h, i \geq 1} i\mu(h = x_i|P = y)\right) = \text{av. n. of guesses for } h \text{ given observations} \]

(assume \( i < j \) implies \( \mu(h = x_i) \geq \mu(h = x_j) \))

\[ \Delta_{GE} (\text{Cash machine, } h) = 0.9999 \]
From now on assume:

**System**= deterministic program (e.g. C code),

**Observations**= outputs, return values ... time

Two questions:

1. how these measures $F$ classify threats?

2. what do they have in common?
How do they classify threats?

Define a ”more $F$ secure” ordering between programs $P, P'$ by

”the measure $F$ on $P$ is always less than the measure $F$ on $P'”$:

$$P \leq_F P' \iff \forall \mu(h). \Delta_F(P; h) \leq \Delta_F(P'; h)$$

Does this ”source code secure” ordering depend on the choice of $F$?
remember $F$ can be

1. entropy,

2. probability of guessing,

3. average number of guesses

In general there is no relation between entropy, probability of guessing or average number of guesses (Massey)
but...

All measures give the same ordering:

Teo: $\leq_H = \leq_{ME} = \leq_{GE}$

This answer "what do they have in common?"

They agree on the classification of source code threats
So what is this common order to all measures $F$?

It is the order in the Lattice of Information (LOI)

$\text{LOI} =$ lattice of all partitions (eq. rel.) on a set of atoms. Is a complete lattice with ordering:

$$X \leq_Y Y \Leftrightarrow y \cong_Y y' \Rightarrow y \cong_X y'.$$
assume a distribution on the atoms then we can see LOI as a lattice of random variables....

\[ \mu(X = x) = \sum \{\mu(x_i) | x_i \in x \} \]

strictly speaking is the set theoretical kernel of a r.v. (but as we don’t need the values of the r.v. that will be fine)
associate to a program $P$ the partition $L(P)$ whose blocks are $h$ undistinguishable by the observations:
formally $L(P) = ([|P|])^{-1}$

Teo: $\leq_H \leq_{ME} \leq_{GE} \leq_L$
What do they have in common?...

the channel capacity coincide

i.e. the maximum measure according to entropy and probability of guessing coincide:

$$\max_h \Delta_{ME}(P, h) = \max_h \Delta_H(P, h) = \log_2(|L(P)|)$$

$|L(P)|$ (number of blocks)
Applying these concepts to real code:

"is the channel capacity of this C function $> k$"?

See a C program as a family of equivalence relations (one for each choice of low inputs) and verify whether exists an equivalence relation in this family with $\geq 2^k$ classes (active attacker model e.g. underflow leak CVE-2007-2875)
Linux Kernel analysis verification practicalities:

$h = \text{kernel memory. size} \sim 4 \text{ Gigabits}

\text{low} = \text{C structures. size} \sim \text{arbitrary}

e.g. for a small 5 integer structure and bound $k = 16$ the question is: exists a relation among $2^{160}$ equivalence relations over a space of $2^{64}$ atoms with more than $2^{16}$ equivalence classes?

not easy..

CBMC can help: symbolic+unwinding assertions
(Heusser-Malacaria 2010) use assume-guarantee reasoning and use CBMC for these questions on bounds.

The approach is powerful, e.g. quantifying architecture leaks: CVE-2009-2847 doesn’t leak on a 32 bits architecture but leaks on a 64 bits machine.

It is also the first verification of Linux kernel vulnerability patches.
Current directions

Bit pattern analysis. Meng and Smith 2011

Bit pattern analysis of Linux kernel. Sang and Malacaria 2012

Also work on side channels Kopf et Alt. (Timing + ongoing on Cache leaks)

“Black box” approaches(Chotia work, Side-buster)
Conclusions:

Scientific: different measures of confidentiality are not so different

Engineering: impossible verification tasks are sometimes possible

Testing: David?
<table>
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<tr>
<th>Description</th>
<th>CVE 20-LOC</th>
<th>LOC</th>
<th>$k^*$</th>
<th>Patch</th>
<th>bound</th>
<th>Time</th>
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<td>09-3002</td>
<td>237</td>
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<td>83s</td>
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**Table 1: Experimental Results.** *Number of unwindings†*