Theoretical Foundations of SBSE

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Some Theoretical Foundations of ***SB*****SE**

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Motivation for theoretical analysis of EAs

EAs have many attractive features

- ease of implementation
- applicable in a wide range of domains
- results often competitive with traditional techniques,

but the understanding of how EAs really work is incomplete

- can be highly sensitive to choice of parameter settings
- experimental parameter tuning expensive
- in most cases, run EA and see what happens
- ...

Traditional Investigation of EAs

Run algorithm(s) on "real world" problem instance(s).
Analyse results with some statistical methodology.

How representative are the results?

- Can we make any guarantee about performance?
- What happens on other instances?
- What happens for larger instance sizes?
- What happens for other parameter settings?

How can the results be explained?

- **Why** does/does not the algorithm work?
- Can the algorithm be improved?

$\Longrightarrow$ Why not attempt the well established methodology that exists for analysing classical algorithms?

Evolutionary Algorithms are Algorithms

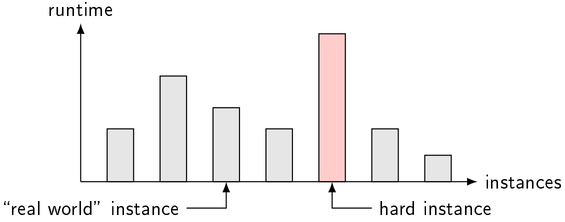
Criteria for evaluating algorithms

1. Correctness.
   - Does the algorithm always give the correct output?
2. Computational Complexity.
   - How much computational resources does the algorithm require to solve the problem?

Same criteria also applicable to search heuristics

1. Correctness.
   - Discover global optimum in finite time?
2. Computational Complexity.
   - Time (number of function evaluations) most relevant computational resource.

Worst Case Computational Complexity



"Real world" runtime: Runtime on "real world" instances

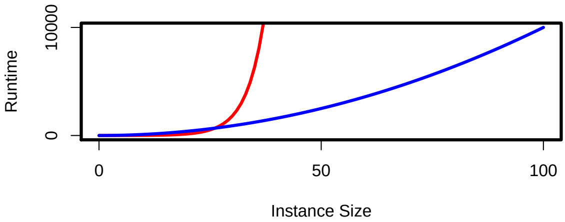
- Are these instances still relevant in 10 years? In 100 years?

Average case runtime: Runtime averaged over instances

- What is an average input (e.g. average FSM)?

Worst case runtime: Runtime on hardest instance

- Strong guarantee about performance of an algorithm.
- Lower bounds obtained by analysing runtime on specific hard problem instance.



Prediction of resources needed for a given instance.

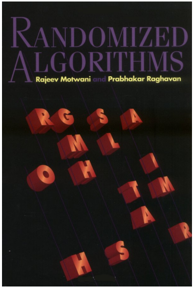
Usually *runtime* as function of *instance size*.

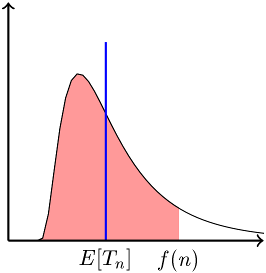
Number of fitness evaluations before finding optimum.

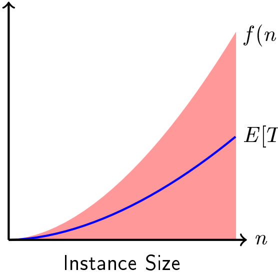
- Exponential runtime $\Longrightarrow$ Inefficient algorithm.
- Polynomial runtime $\Longrightarrow$ "Efficient" algorithm.

Asymptotic notation hides "unimportant" details about runtime.

Search Heuristic are Randomised Algorithms







Search heuristics depend on **random inputs**

- Runtime differs between runs.

Expected runtime

- Runtime averaged over possible random inputs.

Success probability

- Probability of finishing within a specified time $f(n)$.

Research Objectives and Strategy

Runtime analysis of search heuristics on software testing

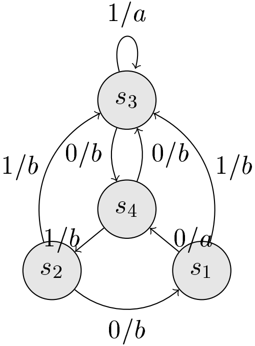
- Understand behaviour of algorithm
- Runtime impact of operators and parameter settings
- Runtime impact of problem instance characteristics

Research strategy

- Start by analysing simple problems and algorithms
- Proceed with more complex scenarios
- Find appropriate mathematical techniques on the way

Conformance testing and UIOs

Conformance testing involves the *state verification problem*, which can be solved using unique input output (UIO) sequences.



Definition

A *unique input output sequence* for a state s is a sequence x st.

- $\forall t \neq s,\ \ \lambda(s,x) \neq \lambda(t,x)$,

where

- $\lambda(s,x)$ is output of FSM on input x, starting in state s.

Example

- 1 is a UIO for state s_3.
- 1 is not a UIO for state s_1.

SEBASE

Previous work

UIOs are fundamental in conformance testing of FSMs.

- Used to solve the *state verification problem*.

Theoretical aspects

- NP-hard to check whether a state has a UIO [Lee and Yannakakis, 1994].
- Shortest UIOs can be exponentially long (empirical results suggest they are often short).

Experimental comparison between random search and GA [Guo et al., 2004] and [Derderian et al., 2006]

- Min. length, max. number of different outputs.
- Similar performance on small FSMs.
- GA better than random search on larger FSMs, especially when long UIOs are needed

(1+1) Evolutionary Algorithm

(1+1) EA

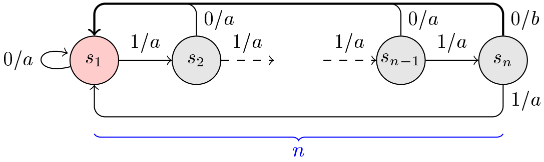
Choose x uniformly from $\{0,1\}^n$.
Repeat
  $x' := x$.
  Flip each bit of x' with probability $1/n$.
  If $f(x') \geq f(x)$,
    then $x := x'$.

Hard instance class - FSM Combination Lock

Theorem

On the instance class below

- *The prob. that (1+1) EA (or RS) finds the UIO for state s_1 within $e^{c \cdot n}$ iterations is exponentially small.*



Proof idea for (1+1) EA:

- All states "collapse" into s_1 on input 0.
- Problem instance is a "needle in the haystack".
- Success probability bounded by drift analysis.

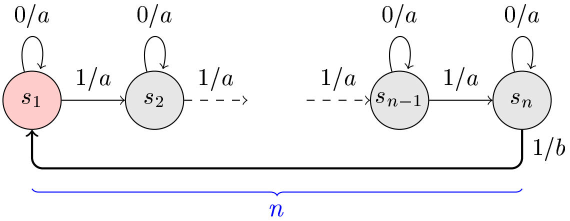
[Lehre and Yao, 2007]

Easy instance class - FSM Counter

Theorem

On the instance class below,

- *(1+1) EA finds the UIO for s_1 in exp. time $O(n \log n)$.*
- *The prob. that random search finds a UIO for s_1 within $e^{c \cdot n}$ iterations is exponentially small $e^{-\Omega(n)}$.*



Proof idea: The problem instance is essentially OneMax.

[Lehre and Yao, 2007]

(1+1) EA? Are you kidding?

- **What about populations?**
- **Well, large populations might not help.**
  - **J. He and X. Yao, 'From an Individual to a Population: An Analysis of the First Hitting Time of Population-Based Evolutionary Algorithms," *IEEE Transactions on Evolutionary Computation*, 6(5):495-511, October 2002.**
  - **T. Chen, K. Tang, G. Chen and X. Yao, "A Large Population Size Can Be Unhelpful in Evolutionary Algorithms," Theoretical Computer Science, accepted on 8/2/2011.**

Operator Interaction

- **We are often concerned about which operators to use. In fact, interactions among operators can be essential. E.g.,**
  - **P. K. Lehre and X. Yao, “On the Impact of Mutation-Selection Balance on the Runtime of Evolutionary Algorithms,” *IEEE Transactions on Evolutionary Computation*, accepted in January 2011.**
- **Even parameter settings.**
  - **T. Chen, J. He, G. Chen and X. Yao, ``Choosing Selection Pressure for Wide-gap Problems," *Theoretical Computer Science*, 411(6):926-934, February 2010.**

Insight into Problems

- **Search algorithms can help us in gaining insight into a problem, e.g., we can use *<u>EDAs (Estimation of Distribution Algorithms)</u>* to find a near optimum while learning a model of the underlying problem --- a wonderful idea!**
- **However,**
  - **. Chen, K. Tang, G. Chen and X. Yao, ``Analysis of Computational Time of Simple Estimation of Distribution Algorithms,'' *IEEE Transactions on Evolutionary Computation*, 14(1):1-22, 2010.**

Future Work

Research Questions

- Relationships between problems and heuristics.
- Analysis of other meta-heuristics.
- Analysis of broader problem classes.
- Approximation quality of search heuristics.

Methodology

- Improve mathematical techniques.

More Practical Considerations

- **Dynamic optimisation: The objective function may change; Fitness evaluation may be noisy; Variable values may be inaccurate**
  - **P. Rohlfshagen and X. Yao, ``Dynamic Combinatorial Optimisation Problems: An Analysis of the Subset Sum Problem,'' *Soft Computing.* Available online.**
- **Robust optimisation: The optimised solution is robust against minor perturbations of the decision variables**
  - **H. Handa, L. Chapman and Xin Yao, ``Robust route optimisation for gritting/salting trucks: A CERCIA experience,'' *IEEE Computational Intelligence Magazine*, 1(1):6-9, February 2006.**
- **ROOT (robust optimisation over time)**
  - **X. Yu, Y. Jin, K. Tang and X. Yao, ``Robust Optimization over Time --- A New Perspective on Dynamic Optimization Problems,'' *Proc. of the 2010 IEEE Congress on Evolutionary Computation (CEC2010)*, Barcelona, Spain, 18-23 July 2010, pp.3998-4003.**

More Practical Considerations

- **Scenario: Given a fixed time budget (say one day), what is the best solution you can generate using whatever algorithms at your disposal?**
- **Should I select one algorithm and allocate all the time to it? Should I divide the time budge among mutiple algorithms? How to allocate the time resources?**
  - **F. Peng, K. Tang, G. Chen and X. Yao, ``Population-based Algorithm Portfolios for Numerical Optimization," *IEEE Transactions on Evolutionary Computation*, 14(5):782-800, October 2010.**

More Practical Considerations

- **Multi-objective formulation can sometimes solve a problem better, even by measuring a single objective only.**
  - **K. Praditwong, M. Harman and X. Yao, ``Software Module Clustering as a Multi-Objective Search Problem,'' *IEEE Transactions on Software Engineering*, 37(2):264-282, March/April 2011.**
  - **Z. Wang, K. Tang and X. Yao, ``Multi-objective Approaches to Optimal Testing Resource Allocation in Modular Software Systems,'' *IEEE Transactions on Reliability*, 59(3):563-575, September 2010.**