When Is a Meta-heuristic Approach Efficient in Search-Based Software Engineering

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Motivation for theoretical analysis of EAs

EAs have many attractive features

- ease of implementation
- applicable in a wide range of domains
- results often competitive with traditional techniques,

but the understanding of how EAs really work is incomplete

- can be highly sensitive to choice of parameter settings
- experimental parameter tuning expensive
- in most cases, run EA and see what happens
- ...

Traditional Investigation of EAs

Run algorithm(s) on "real world" problem instance(s).
Analyse results with some statistical methodology.

How representative are the results?

- Can we make any guarantee about performance?
- What happens on other instances?
- What happens for larger instance sizes?
- What happens for other parameter settings?

How can the results be explained?

- **Why** does/does not the algorithm work?
- Can the algorithm be improved?

$\Longrightarrow$ Why not attempt the well established methodology that exists for analysing classical algorithms?

SEBASE

Outline

Evolutionary Algorithms are Algorithms

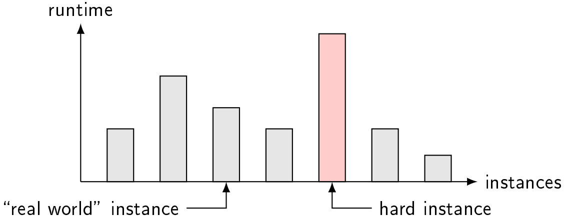
Criteria for evaluating algorithms

1. Correctness.
   - Does the algorithm always give the correct output?
2. Computational Complexity.
   - How much computational resources does the algorithm require to solve the problem?

Same criteria also applicable to search heuristics

1. Correctness.
   - Discover global optimum in finite time?
2. Computational Complexity.
   - Time (number of function evaluations) most relevant computational resource.

Worst Case Computational Complexity



"Real world" runtime: Runtime on "real world" instances

- Are these instances still relevant in 10 years? In 100 years?

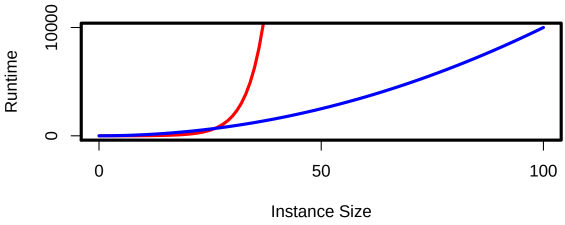
Average case runtime: Runtime averaged over instances

- What is an average input (e.g. average FSM)?

Worst case runtime: Runtime on hardest instance

- Strong guarantee about performance of an algorithm.
- Lower bounds obtained by analysing runtime on specific hard problem instance.

Computational Complexity of Search Heuristics



Prediction of resources needed for a given instance.

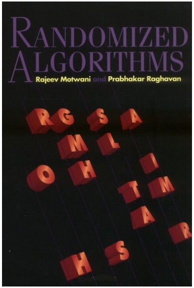
Usually *runtime* as function of *instance size*.

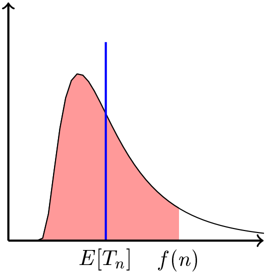
Number of fitness evaluations before finding optimum.

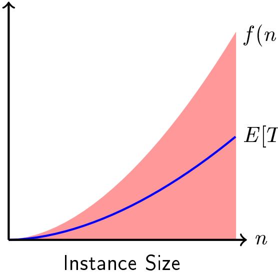
- Exponential runtime $\Longrightarrow$ Inefficient algorithm.
- Polynomial runtime $\Longrightarrow$ "Efficient" algorithm.

Asymptotic notation hides "unimportant" details about runtime.

Search Heuristic are Randomised Algorithms







Search heuristics depend on **random inputs**

- Runtime differs between runs.

Expected runtime

- Runtime averaged over possible random inputs.

Success probability

- Probability of finishing within a specified time $f(n)$.

Research Objectives and Strategy

Runtime analysis of search heuristics on software testing

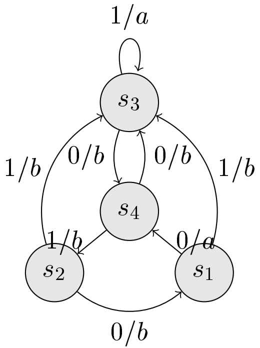
- Understand behaviour of algorithm
- Runtime impact of operators and parameter settings
- Runtime impact of problem instance characteristics

Research strategy

- Start by analysing simple problems and algorithms
- Proceed with more complex scenarios
- Find appropriate mathematical techniques on the way

Conformance testing and UIOs

Conformance testing involves the *state verification problem*, which can be solved using unique input output (UIO) sequences.



Definition

A *unique input output sequence* for a state s is a sequence x st.

- $\forall t \neq s, \;\; \lambda(s,x) \neq \lambda(t,x)$,

where

- $\lambda(s,x)$ is output of FSM on input x, starting in state s.

Example

- 1 is a UIO for state s_3.
- 1 is not a UIO for state s_1.

Previous work

UIOs are fundamental in conformance testing of FSMs.

- Used to solve the *state verification problem*.

Theoretical aspects

- NP-hard to check whether a state has a UIO [Lee and Yannakakis, 1994].
- Shortest UIOs can be exponentially long (empirical results suggest they are often short).

Experimental comparison between random search and GA [Guo et al., 2004] and [Derderian et al., 2006]

- Min. length, max. number of different outputs.
- Similar performance on small FSMs.
- GA better than random search on larger FSMs, especially when long UIOs are needed

(1+1) Evolutionary Algorithm

(1+1) EA

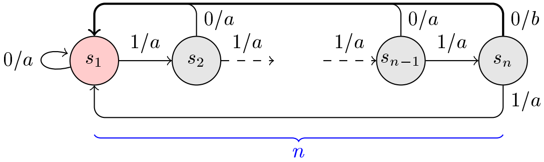
Choose x uniformly from $\{0,1\}^n$.
Repeat
  $x' := x$.
  Flip each bit of x' with probability $1/n$.
  If $f(x') \geq f(x)$,
    then $x := x'$.

Hard instance class - FSM Combination Lock

Theorem

On the instance class below

- *The prob. that (1+1) EA (or RS) finds the UIO for state s_1 within $e^{c \cdot n}$ iterations is exponentially small.*



Proof idea for (1+1) EA:

- All states "collapse" into s_1 on input 0.
- Problem instance is a "needle in the haystack".
- Success probability bounded by drift analysis.

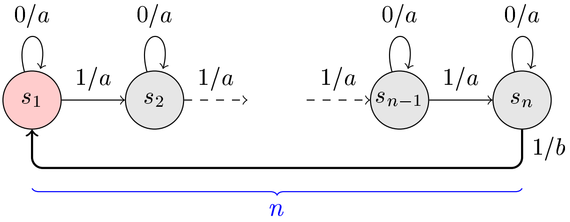
[Lehre and Yao, 2007]

Easy instance class - FSM Counter

Theorem

On the instance class below,

- *(1+1) EA finds the UIO for s_1 in exp. time $O(n \log n)$.*
- *The prob. that random search finds a UIO for s_1 within $e^{c \cdot n}$ iterations is exponentially small $e^{-\Omega(n)}$.*



Proof idea: The problem instance is essentially OneMax.

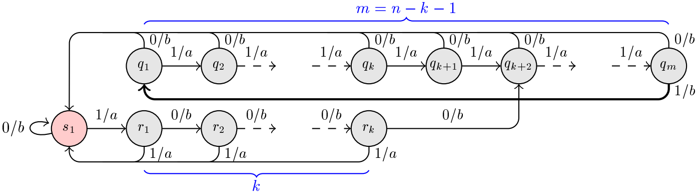
[Lehre and Yao, 2007]

Instances with tunable difficulty

Theorem

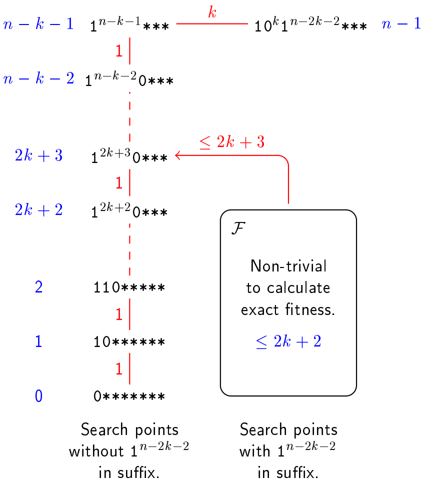
On the instance class below, with $k \geq 2$ any constant,

- *(1+1) EA finds an UIO for s_1 in expected time $\Theta(n^k)$.*



[Lehre and Yao, 2007]

Tunable Difficulty - Proof Idea.



$$\mathbf{Pr}[\mathcal{F}] = e^{-\Omega(n)}$$
$$\mathbf{E}\left[T \mid \overline{\mathcal{F}}\right] = \Theta(n^k)$$
$$\mathbf{E}[T \mid \mathcal{F}] = O(n^{2k+3})$$
$$\mathbf{E}\left[T \mid \overline{\mathcal{F}}\right] = \Omega(n^k)$$

$$\mathbf{E}[T] = (1 - \mathbf{Pr}[\mathcal{F}]) \cdot \mathbf{E}\left[T \mid \overline{\mathcal{F}}\right] + \mathbf{Pr}[\mathcal{F}] \cdot \mathbf{E}[T \mid \mathcal{F}] = \Theta(n^k).$$

Steady State GA with Crossover

(μ+1) SSGA

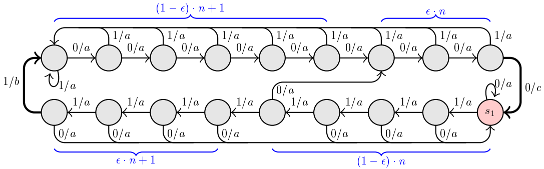
Sample a population P of μ points u.a.r. from $\{0,1\}^n$.
repeat
  with probability $p_c(n)$,
    Sample x and y u.a.r. from P.
    $(x', y') :=$ one point crossover(x, y).
    if $\max\{f(x'), f(y')\} \geq \max\{f(x), f(y)\}$
      then $x := x'$ and $y := y'$.
  otherwise
    Sample x u.a.r. from P.
    $x' :=$ Mutate(x).
    if $f(x') \geq f(x)$
      then $x := x'$.

Effect of Crossover

Theorem

On the instance class below,

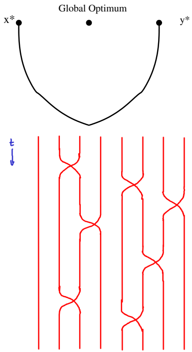
- *(μ+1) SSGA with constant crossover prob.* $p_c > 0$ *finds the UIO for state* s_1 *in* $c\mu^2 n^2$ *generations with probability* $1 - e^{-\Omega(n)} - e^{-\Omega(\mu)}$*.*
- *(μ+1) SSGA without crossover, i.e.* $p_c = 0$*, does not find the UIO for state* s_1 *in time* 2^{cn} *with probability* $1 - e^{-\Omega(n)}$*.*



[Lehre and Yao, 2008]

Proof Idea

$$\textsc{TwoPaths}\,(x) := \begin{cases} 2n & \text{if } x = 1^{(1-\epsilon)\cdot n}0^{\epsilon\cdot n}, \\ \textsc{Lo}(x) + \textsc{Lz}(x) & \text{otherwise.} \end{cases}$$



- Global optimum between two paths.
- Monotonic fitness along lineages.
- Lineages reach a local optimum in

$$O(n^2 \mu \log \mu / (1 - p_c)).$$

- Population divided evenly between paths
- Once on local optima, successful crossover

$$O(n/p_c).$$

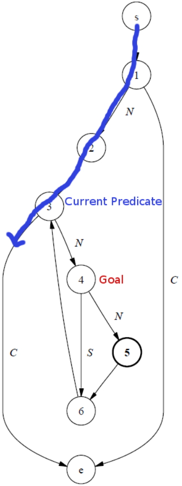
Branch Coverage of Triangle Classification

```
int tri_type(int x, int y, int z) {
  int type;
  int a=x, b=y, c=z;
  if (x > y) {
     int t = a; a = b; b = t;
  }
  if (a > z) { int t = a; a = c; c = t; }
  if (b > c) { int t = b; b = c; c = t; }
  if (a + b <= c) {
      type = NOT_A_TRIANGLE;
  } else {
      type = SCALENE;
      if (a == b && b == c) {
          type = EQUILATERAL;
      } else if (a == b || b == c) {
          type = ISOSCELES;
      }
    }
  return type;
}
```

[McMinn, 2004]

- Testing problem
  - Find x, y, z such that equilateral branch is covered.
- Fitness functions
  - approach level
  - branch distance
- Problem size
  - range of integer variables $x, y, z \in \{-N/2 + 1, ..., N/2\}$.

Fitness Functions (minimisation)



Approach level

- Minimal distance to branch in control flow graph.

Branch Distance

(Approach level, f(curr. predicate))

Predicate	f
`if (a>b)`	$b - a$
`if (a>=b)`	$b - a$
`if (a<b)`	$a - b$
`if (a<=b)`	$a - b$
`if (a==b)`	$\|b - a\|$
`if (a!=b)`	$-\|b - a\|$

McMinn (2004)

Expected runtimes on Equilateral Branch

Algorithms

- RS - Random Search
- HC - Hill Climber (local search)
- AVM - Alternating Variable Method
- (1+1) EA (with unsigned binary integer repr.)

Expected Runtimes

Algorithm	Approach level	Branch distance
RS	$\Theta(N^2)$	$\Theta(N^2)$
HC	$\Theta(N^2)$	$\Theta(N)$
AVM	$\Theta(N^2)$	$\Omega(\log N)$ and $O((\log N)^2)$
(1+1) EA[1]		$\Theta((\log N)^5)$

[Arcuri et al., 2008]

[1] Ongoing work.

Conclusion

Runtime of EAs on **UIO problem**

- (1+1) EA has exponential worst case runtime
- (1+1) EA still efficient on many instances, and outperforms a random search strategy.
- spectrum of increasingly hard instances for (1+1) EA.
- crossover and large population essential on certain instances.

Runtime on **branch coverage** of triangle classification

- AVM $\succ$ (1+1) EA $\succ$ HC $\succ$ RS.
- Theoretically confirmed well known results.

Future Work

Research Questions

- Relationships between problems and heuristics.
- Analysis of other meta-heuristics.
- Analysis of broader problem classes.
- Approximation quality of search heuristics.

Methodology

- Improve mathematical techniques.

Analysis of Other Meta-Heuristics

- Analysis of (1+1) EA necessary to develop techniques
- Lower bounds for population-based EAs
- Estimation of Distribution Algorithms (EDAs)
- Multi-objective EAs

Analysis of Broader Problem Classes

- Know specific instance classes that are easy and hard.
- Which conditions on the instance are sufficient to guarantee polynomial runtime?

Thank you for your attention!

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